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Decentralized interval observer-based control for a class of nonlinear systems

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Abstract: This paper proposes an approach to the design of decentralized interval observer-based output feedback controllers for a class of nonlinear systems. In contrast to observers in the classical sense, the interval observer not only generates signals to stabilize the system based on output measurement, but also provides all times an interval in which the state vector is sure to lie. This paper shows how to introduce decentralized structure to the interval observer-based output feedback for stabilization of large-scale systems with decoupled communication. The notion of integral input-to-state stability is utilized successfully to achieve convergence of the state and the estimated interval to zero in the presence of disturbances.

Keywords: Interval observers; Decentralized control; Output feedback; Integral input-to-state stability; Robust estimation.

1. INTRODUCTION

Estimation for physical and biological processes is one of thematic flagship problems in engineering. In fact, it is difficult to know precisely the value of different quantities evolving in a process based only on available sensors, i.e, output measurements. In the field of control, observers are algorithms to estimate online unknown values based on mathematical models of processes. The standard definition of observers is asymptotic estimation in which an estimated value agrees with its true value as time tends to infinity without any guarantee in transient periods. Although such mechanism is known to be satisfactory for the purpose of feedback control, it does not meet the purpose of monitoring processes. Indeed, the concept of interval observers [7] is more desirable.

Interval observers are relatively new algorithms and developed very quickly in various directions [14], [11], [12], [13], [10], [9], [4] supported by the usefulness of their concept in applications (e.g. biological systems [3], [6]). An interval observer is composed of a dynamic extension with two outputs giving an upper bound and a lower bound for the state of the considered system at each time instant under assumptions of knowing bounds of disturbances and of initial condition. Recently, an interesting mechanism of interval estimation was proposed in the way that the interval observer gives an interval estimate whose end points can be used directly for output feedback control to stabilize the plant [5]. Another advantage of the estimation mechanism is that it can be designed to render the convergence of the interval estimate robust with respect to disturbances and get rid of global Lipschitz assumptions [8].

Recently, control of large-scale systems has regained importance due to the information rich world [16]. There is no doubt that decentralized control is a practically attractive approach to controlling large-scale systems. Although achievable performance and systems admitting decentralization are inherently limited, a variety of approaches are available for stabilization via decentralized controllers, e.g., [17, 18]. The recently developed method [5, 8] of the interval observer for output feedback naturally leads us to the question of whether the method can be modified to give an decentralized interval observer producing a decentralized controller. Although the problem of decentralized observers has been tackled for many decades in the control literature, (see e.g., [1, 2, 15, 17-20] and references therein), to the best of the authors' knowledge, investigation on decentralized interval observers-based control has not been started yet.

Decentralized control is to control plant with many independent sub-controllers which generate control inputs to be applied at their local channels based on their local measurements. For lage-scale systems, it is unreasonable to assume that all information of measurements throughout the entire system is available at each sub-controller. Thus, synthesizing a decentralized controller is nothing but a diagonal information constraint on a controller. The goal of this paper is to meet this constraint in designing interval observer-based output feedback control. To tackle this problem, as introduced in [4, 5, 8], a class of nonlinear system which is affine in the unmeasured part of the state vector is considered. The iSS framework introduced to centralized interval observer design recently by [8] is employed. This paper will show how the method in [8] can be modified successfully to propose a decentralized interval observer-based controller. Design guidelines will be presented for achieving (integral) input-to-state stability ((i)ISS) of the system, but also guaranteeing the estimated intervals of state variables to converge to zero even in the presence of disturbances.

Nomenclature

The set of real numbers is denoted by $\mathbb{R}$. The set of nonnegative real numbers is denoted by $\mathbb{R}_{\geq 0}$, i.e., $\mathbb{R}_{\geq 0} :=$

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$[0, \infty)$. The symbol I denotes the identity matrix in $\mathbb{R}^{n\times n}$ of any dimension n. The symbol $|\cdot|$ denotes Euclidean norm of vectors of any dimension. Inequalities must be understood *component-wise*, i.e., for $x_a = [x_{a,1}, ..., x_{a,n}]^\top \in \mathbb{R}^n$ and $x_b = [x_{b,1}, ..., x_{b,n}]^\top \in \mathbb{R}^n$, $x_a \le x_b$ if and only if, for all $i \in \{1, ..., n\}$, $x_{a,i} \le x_{b,i}$. For a square matrix T_i, block-diag$_{i=1}^N T_i$ is defined as

$$\text{block-diag}_{i=1}^N T_i = \begin{bmatrix} T_1 & 0 & \cdots & 0 \\ 0 & T_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & T_N \end{bmatrix}.$$

For a square matrix $Q \in \mathbb{R}^{n\times n}$, let the matrix $Q^+ \in \mathbb{R}^{n\times n}$ denote $Q^+ = (\max\{q_{i,j}, 0\})_{i,j=1,1}^{n,n}$, where the notation $Q = (q_{i,j})_{i,j=1,1}^{n,n}$ is used. Let $Q^- \in \mathbb{R}^{n\times n}$ be defined by $Q^- = Q^+ - Q$. This notation is limited to square matrices, and the superscripts $+$ and $-$ for other purposes are defined appropriately when they appear. A square matrix $Q \in \mathbb{R}^{n\times n}$ is said to be Metzler if each off-diagonal entry of this matrix is nonnegative. For functions $\alpha, \beta : \mathbb{R}_{\ge 0} \to \mathbb{R}^n$, by $\alpha(s) \equiv \beta(s)$ we mean $\alpha(s) = \beta(s)$ for all $s \in \mathbb{R}_{\ge 0}$. A function $\alpha : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is said to be positive definite and written as $\alpha \in \mathcal{P}$ if α is continuous and satisfies $\alpha(0) = 0$ and $\alpha(s) > 0$ for all $s \in (0, \infty)$. A function $\alpha \in \mathcal{P}$ is said to be of class $\mathcal{K}$ if α is strictly increasing. A class $\mathcal{K}$ function is said to be of class $\mathcal{K}_\infty$ if it is unbounded. A continuous function $\beta : \mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is said to be of class $\mathcal{KL}$ if, for each fixed $t \in \mathbb{R}_{\ge 0}$, $\beta(\cdot, t)$ is of class $\mathcal{K}$ and, for each fixed $s > 0$, $\beta(s, \cdot)$ is strictly decreasing and $\lim_{t\to\infty} \beta(s, t) = 0$. The symbols $\vee$ and $\wedge$ denote logical sum and logical product, respectively.

2. PROBLEM SETUP

We consider the plant described by

$$\dot{x} = A(y(t))x(t) + \beta(y(t), u(t)) + \delta(t), \tag{1a}$$

$$y(t) = Cx(t), \tag{1b}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$ and $y(t) \in \mathbb{R}^{n_y}$. The functions $A : \mathbb{R}^{n_y} \to \mathbb{R}^{n_x\times n_x}$ and $\beta : \mathbb{R}^{n_y} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$ are supposed to be locally Lipschitz. The matrix $C \in \mathbb{R}^{n_y\times n_x}$ is constant. The disturbance $\delta : \mathbb{R}_{\ge 0} \to \mathbb{R}^{n_x}$ is supposed to be any piecewise continuous function. In order to introduce the notion of input-output pairs for decentralized control by output feedback, let u and y be represented by N block components as follows:

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_N(t) \end{bmatrix}, \quad u_i(t) \in \mathbb{R}^{n_{u_i}}, \tag{2a}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_N(t) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix} x, \quad y_i(t) \in \mathbb{R}^{n_{y_i}}. \tag{2b}$$

In order to estimate an interval to which $x(t)$ belongs at each time t, we assume that the vectors $x_0^-, x_0^+ \in \mathbb{R}^{n_x}$ and the piecewise continuous functions $\delta^-, \delta^+ : \mathbb{R}_{\ge 0} \to \mathbb{R}^{n_x}$ satisfying

$$x_0^- \le x_0 \le x_0^+ \tag{3}$$

$$\delta^-(t) \le \delta(t) \le \delta^+(t), \quad \forall t \in \mathbb{R}_{\ge 0} \tag{4}$$

are known. The objective of our control problem is twofold. The first one is to drive $x(t)$ to the origin asymptotically for an arbitrary initial condition satisfying (3) by a decentralized controller consisting of sub-controllers generating u_i from y_i as long as the disturbance $\delta(t)$ converges to zero, i.e., $\delta(t) \to 0$ as $t \to \infty$. The second goal is to estimate an envelope $x^-(t), x^+(t) \in \mathbb{R}_{\ge 0}$ such that

$$x^-(t) \le x(t) \le x^+(t), \quad \forall t \in \mathbb{R}_{\ge 0} \tag{5}$$

holds in the presence of any piecewise continuous disturbance $\delta(t)$ satisfying (4). A system generating such $x^-(t)$ and $x^+(t)$ is called a framer [5]. The system is called an interval observer if the distance between $x^-(t)$ and $x^+(t)$ goes to zero as t tends to ∞ under the assumption of $\delta(t) \equiv 0$. A hidden goal of the design problem tackled in this paper is to use the interval observer directly for constructing a decentralized controller. Thus, we build an interval observer consisting of N components which operate independently. This paper aims at designing a decentralized output feedback controller based on the interval observer.

The traditional observers in control textbooks generate a signal $\hat{x}$ achieving $\lim_{t\to\infty} \hat{x}(t) - x(t) = 0$ instead of (5). The state estimation is achieved only at $t = \infty$. The problem of constructing an interval observer is tougher than that of a traditional observer since (5) should be guaranteed all times from the initial time. On the other hand, it is known (see e.g., [18]) that it is hard to let observers be decentralized in achieving $\lim_{t\to\infty} \hat{x}(t) - x(t) = 0$. In other words, the decentralization is a strong constraint which often disallow $\lim_{t\to\infty} \hat{x}(t) - x(t) = 0$ even if systems are observable. It is worth recalling that the estimation is more demanding than building dynamics for stabilizing feedback. By the N bock-partition in (2), we indicate that the output feedback controller consists of N decoupled components which are called sub-controllers. Each sub-controller is supposed to be furnished with an interval observer to reproduce the state vector. The mathematically easiest formulation would be to reproduce the entire state vector $x(t) \in \mathbb{R}^{n_x}$ at each interval observer. However, it is not only redundant, but also too restrictive in view of observability of the entire x at each output channel y_i. Thus, as one of many possibilities, this paper employs the idea of reproducing block components of the vector x at sub-controllers without any overlapping components. In order to make this idea possible, we make the following assumption on nonlinearities in (1).

Assumption 1: The matrix A and the vector β admit the following partitions:

$$A(y(t)) = \begin{bmatrix} A_{1,1}(y_1(t)) & \cdots\cdots & A_{1,N}(y_1(t)) \\ A_{2,1}(y_2(t)) & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ A_{N,1}(y_N(t)) & \cdots\cdots & A_{N,N}(y_N(t)) \end{bmatrix} = \begin{bmatrix} A_1(y_1(t)) \\ A_2(y_2(t)) \\ \vdots \\ A_N(y_N(t)) \end{bmatrix},$$

$$\beta(y(t), u(t)) = \begin{bmatrix} \beta_1(y_1(t), u_1(t)) \\ \beta_2(y_2(t), u_2(t)) \\ \vdots \\ \beta_N(y_N(t), u_N(t)) \end{bmatrix},$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix},$$

where $x_i(t) \in \mathbb{R}^{n_{x_i}}$, $A_{i,j}(y_i(t)) \in \mathbb{R}^{n_{x_i} \times n_{x_j}}$, $A_i(y_i(t)) \in \mathbb{R}^{n_{x_i} \times n_x}$ and $\beta_i(y_i(t), u_i(t)) \in \mathbb{R}^{n_{x_i}}$ for all $i, j = 1, 2, ..., N$.

The matrix $A_{i,j}$ is the interaction describing how block j affects block i of the state x. The following assumption allows one to construct an interval observer in the required decentralized structure.

Assumption 2: For all $i = 1, 2, ..., N$, there exist $Y_i(w_i) \in \mathbb{R}^{n_{x_i} \times n_{y_i}}$ and $M_{i,i}(w_i) \in \mathbb{R}^{n_{x_i} \times n_{x_i}}$ such that

$$\begin{aligned} &\big[\, 0 \ \cdots \ 0 \ \ \underbrace{A_{i,i}(w_i) + M_{i,i}(w_i)}_{i\text{-th component}} \ \ 0 \ \cdots \ 0 \,\big] \\ &= \big[A_{i,1}(w_i) \quad A_{i,2}(w_i) \quad \cdots \quad A_{i,N}(w_i)\big] \\ &\quad + Y_i(w_i) C_i, \qquad\qquad \forall w_i \in \mathbb{R}^{n_{y_i}} \end{aligned} \tag{6}$$

The condition (6) requires that the information of other block components, i.e, $j \neq i$, is available through the output y_i if those components affect the i-th block component of the state. This can be considered to be a natural assumption for achieving the decentralization. Indeed, unless the plant by itself is decoupled, there is no way for decoupled observers to estimate x_i independently without receiving any information of affecting signals x_j, $j \neq i$.

Remark 1: When the full information x is available at y_i, there always exists $Y_i(w_i) \in \mathbb{R}^{n_{x_i} \times n_{y_i}}$ achieving (6) for any given $M_{i,i}(w_i)$. To see this, suppose that the rank of C_i is n_x, which represents the full information case. Using a left inverse C_i^ℓ, i.e., $C_i^\ell C_i = I$, pick

$$\begin{aligned} Y_i(w_i) = \big(&\big[\cdots 0 \quad A_{i,i}(w_i) + M_{i,i}(w_i) \quad 0 \quad \cdots\big] \\ &- \big[A_{i,1}(w_i) \quad A_{i,2}(w_i) \quad \cdots \quad A_{i,N}(w_i)\big]\big) C_i^\ell. \end{aligned}$$

Then we arrive at (6). In general, the requirement (6) is restrictive. However, we cannot get rid of the natural fact that only a limited class of systems allows the state to be estimated in the forced decentralized structure. There still cases where (6) is fulfilled for non-full rank C_is. A straightforward example fulfilling (6) with a non-full rank C_i is

$$[0, 1-2, 0, 0] - (-1) \cdot [1, 2, 1, 1] = [1, 1, 1, 1],$$

where $A_{2,2} = 1$, $M_{2,2} = -2$, $Y_2 = -1$, $C_2 = [1, 2, 1, 1]$ and $A_2 = [1, 1, 1, 1]$.

Remark 2: There are flexibilities to make use of in fulfilling Assumptions 1 and 2. The pairing of input and output can be often determined by the designer unless there are severe physical constraints. Assumptions 1 and 2 can also provide information about selecting sensors and their locations.

3. DECENTRALIZED INTERVAL OBSERVER AND FEEDBACK GAIN

For interval observer-based output feedback for stabilization of system (1), this paper proposes a candidate of an interval observer consisting of the following N components:

$$\begin{aligned} \dot{\hat{x}}_i^+ &= (A_{i,i}(y_i) + M_{i,i}(y_i))\, \hat{x}_i^+ - Y_i(y_i) y_i \\ &\quad + \beta_i(y_i, u_i) + S_i[R_i^+ \delta_i^+ - R_i^- \delta_i^-] \qquad (7a) \\ \dot{\hat{x}}_i^- &= (A_{i,i}(y_i) + M_{i,i}(y_i))\, \hat{x}_i^- - Y_i(y_i) y_i \\ &\quad + \beta_i(y_i, u_i) + S_i[R_i^+ \delta_i^- - R_i^- \delta_i^+] \qquad (7b) \\ \hat{x}_i^+(0) &= \hat{x}_{0i}^+ := S_i[R_i^+ x_{0i}^+ - R_i^- x_{0i}^-] \qquad (7c) \\ \hat{x}_i^-(0) &= \hat{x}_{0i}^- := S_i[R_i^+ x_{0i}^- - R_i^- x_{0i}^+] \qquad (7d) \\ x_i^+ &= S_i^+ R_i \hat{x}_i^+ - S_i^- R_i \hat{x}_i^- \qquad (7e) \\ x_i^- &= S_i^+ R_i \hat{x}_i^- - S_i^- R_i \hat{x}_i^+, \qquad (7f) \end{aligned}$$

where $S_i = R_i^{-1}$, $\hat{x}_i^+(t), \hat{x}_i^-(t), x_i^+(t), x_i^-(t) \in \mathbb{R}^{n_{x_i}}$, $x_{0i}^-, x_{0i}^+ \in \mathbb{R}^{n_{x_i}}$ and $\delta_i^-(t), \delta_i^+(t) \in \mathbb{R}^{n_{x_i}}$ for $i \in \{1, \ldots, N\}$, and

$$\delta^-(t) = \begin{bmatrix} \delta_1^-(t) \\ \delta_2^-(t) \\ \vdots \\ \delta_N^-(t) \end{bmatrix}, \ \delta^+(t) = \begin{bmatrix} \delta_1^+(t) \\ \delta_2^+(t) \\ \vdots \\ \delta_N^+(t) \end{bmatrix}$$

$$x_0^- = \begin{bmatrix} x_{01}^- \\ x_{02}^- \\ \vdots \\ x_{0N}^- \end{bmatrix}, \ x_0^+(t) = \begin{bmatrix} x_{01}^+ \\ x_{02}^+ \\ \vdots \\ x_{0N}^+ \end{bmatrix}.$$

The N dynamical systems described by (7) operate independently of each other, i.e., decoupled. Each i-th component of the interval observer generates the end points $x_i^+(t)$ and $x_i^-(t)$ of some interval estimate from $y_i(t)$ based on the given envelope of the i-th block component of the disturbance $\delta(t)$. Let the i-th sub-controller consist of the i-th component (7) of the interval observer and the i-th control input u_i to be determined in the form of

$$u_i(t) = u_{si}(y_i(t), \hat{x}_i^+(t)). \tag{8}$$

Note that y_i is available and $\hat{x}_i^+$ is generated at the i-th sub-controller. Thus, the interval observer-based output feedback controller (7)-(8) proposed above is completely decentralized.

The non-singular matrices $R_i \in \mathbb{R}^{n_{x_i} \times n_{x_i}}$ and the matrices $M_{i,i}(y_i) \in \mathbb{R}^{n_{x_i} \times n_{x_i}}$ are supposed to be chosen for $i \in \{1, \ldots, N\}$ so that the following assumptions are fulfilled.

Assumption 3: For each $i = 1, 2, ..., N$, given a locally Lipschitz function $M_{i,i} : \mathbb{R}^{n_{y_i}} \to \mathbb{R}^{n_{x_i} \times n_{x_i}}$, there exists an invertible matrix $R_i \in \mathbb{R}^{n_{x_i} \times n_{x_i}}$ such that, for each fixed $y_i \in \mathbb{R}^{n_{y_i}}$, the matrix

$$\Gamma_i(y_i) = R_i[A_{i,i}(y_i) + M_{i,i}(y_i)]R_i^{-1} \tag{9}$$

is Metzler.

Assumption 4: For each $i = 1, 2, ..., N$, given a locally Lipschitz function $M_{i,i} : \mathbb{R}^{n_{y_i}} \to \mathbb{R}^{n_{x_i} \times n_{x_i}}$, there exist a C^1 function $V_i : \mathbb{R}^{n_{x_i}} \to \mathbb{R}_{\geq 0}$, continuous functions $\underline{\nu}_i, \overline{\nu}_i \in \mathcal{K}_\infty$, $\omega_i \in \mathcal{P}$ and $\eta_i^+, \eta_i^- \in \mathcal{K}$ such that

$$\underline{\nu}_i(|\xi_i|) \leq V_i(\xi_i) \leq \overline{\nu}_i(|\xi_i|) \tag{10}$$

$$\begin{aligned} &\frac{\partial V_i}{\partial \xi_i}(\xi_i)\big\{[A_{i,i}(y_i) + M_{i,i}(y_i)]\xi_i \\ &\quad + S_i[R_i^+\rho_i^+ + R_i^-\rho_i^-]\big\} \\ &\qquad \leq -\omega_i(|\xi_i|) + \eta_i^+(|\rho_i^+|) + \eta_i^-(|\rho_i^-|) \end{aligned} \tag{11}$$

hold for all $\xi_i \in \mathbb{R}^{n_{x_i}}$, $y_i \in \mathbb{R}^{n_{y_i}}$, $\rho_i^+ \in \mathbb{R}^{n_{x_i}}$ and $\rho_i^- \in \mathbb{R}^{n_{x_i}}$.

The above two assumptions are guidelines for the selection of observer gains $M_{i,i}(y_i)$, while the next assumption provides guidelines for the selection of control gains $u_{si}(y_i, \hat{x}^+)$ for $i = 1, 2, ..., N$.

Assumption 5: Given a locally Lipschitz function $u_s : \mathbb{R}^{n_y} \times \mathbb{R}^{n_x} \to \mathbb{R}^{n_u}$, there exist a positive definite radially unbounded C^1 function $U : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, continuous functions $\mu \in \mathcal{P}$ and $\gamma, \zeta \in \mathcal{K}$ such that

$$\begin{aligned} &\frac{\partial U}{\partial x}(x)[A(Cx)x + \beta(Cx, u_s(Cx, x+d)) + \delta] \\ &\qquad \leq -\mu(|x|) + \gamma(|d|) + \zeta(|\delta|) \end{aligned} \tag{12}$$

holds for all $x \in \mathbb{R}^{n_x}$, $d \in \mathbb{R}^{n_x}$ and $\delta \in \mathbb{R}^{n_x}$, where

$$u_s(Cx, x+d) = \begin{bmatrix} u_{s1}(C_1x, x_1 + d_1) \\ u_{s2}(C_2x, x_2 + d_2) \\ \vdots \\ u_{sN}(C_Nx, x_N + d_N) \end{bmatrix}. \tag{13}$$

The above three assumptions are inspired by the similar result developed in [8] for centralized estimation and control. The next section explains this point.

Remark 3: The matrix $M_{i,i}(y_i)$ represents a part of the observer gain for estimating the i-th block component x_i of the plant (1). It cannot be exactly in the form of the Luenberger observer in the decentralized case since the other components x_j $(j \neq i)$ influences x_i. To see this, suppose temporarily that the plant (1) is decoupled, i.e, $A_{i,j} = 0$ for all $i \neq j$, then (6) is fulfilled if and only if $[0, \ldots, 0, M_{i,i}(y_i), 0, \ldots, 0] = Y_i(y_i)C_i$. Thus $Y_i(y_i)$ is exactly the observer gain of Luenberger type. Hence, the pair of $M_{i,i}(y_i)$ and $Y_{i,i}(y_i)$ together with (6) plays the role of the observer gain under the constraint of decentralization.

4. GUARANTEES

For convenience, the remainder of this paper uses the notations

$$z = \begin{bmatrix} \hat{x}^+ \\ \hat{x}^- \end{bmatrix}, \; X = \begin{bmatrix} x \\ z \end{bmatrix}, \; \Delta = \begin{bmatrix} \delta \\ \delta^+ \\ \delta^- \end{bmatrix},$$

$$\hat{x}^+ = \begin{bmatrix} \hat{x}_1^+ \\ \hat{x}_2^+ \\ \vdots \\ \hat{x}_N^+ \end{bmatrix}, \; \hat{x}^- = \begin{bmatrix} \hat{x}_1^- \\ \hat{x}_2^- \\ \vdots \\ \hat{x}_N^- \end{bmatrix}, \; x^+ = \begin{bmatrix} x_1^+ \\ x_2^+ \\ \vdots \\ x_N^+ \end{bmatrix}, \; x^- = \begin{bmatrix} x_1^- \\ x_2^- \\ \vdots \\ x_N^- \end{bmatrix},$$

$$\hat{z} = \begin{bmatrix} \hat{x}^+ - x \\ x^+ - x^- \end{bmatrix}, \; \hat{\rho} = \begin{bmatrix} \delta^+ - \delta \\ \delta^- - \delta \end{bmatrix},$$

$$\eta = \sum_{i=1}^N \eta_i^+ + \eta_i^-, \; \delta^\pm = \delta^+ - \delta^-, \; \overline{\nu} = \sum_{i=1}^N \overline{\nu}_i,$$

$$\omega(s) = \min\left\{\sum_{i=1}^N \omega_i(a_i) : a_i \in \mathbb{R}_{\geq 0}, \; \sum_{i=1}^N a_i^2 = s^2\right\}, \quad \forall s \in \mathbb{R}_{\geq 0},$$

$$\underline{\nu}(s) = \min\left\{\sum_{i=1}^N \underline{\nu}_i(a_i) : a_i \in \mathbb{R}_{\geq 0}, \; \sum_{i=1}^N a_i^2 = s^2\right\}, \quad \forall s \in \mathbb{R}_{\geq 0}.$$

For the proposed decentralized controller candidate consisting of interval observer components (7) and feedback gain components (8), we can prove the following two theorems.

Theorem 1: Assume that $u_{si} : \mathbb{R}^{n_{y_i}} \times \mathbb{R}^{n_{x_i}} \to \mathbb{R}^{n_{u_i}}$ and $M_{i,i} : \mathbb{R}^{n_{y_i}} \to \mathbb{R}^{n_{x_i} \times n_{x_i}}$ are locally Lipschitz for all $i = 1, 2, ..., N$. Suppose that Assumptions 1, 2, 3, 4 and 5 are satisfied with $\mu \in \mathcal{K}$. Then in the case of $\delta(t) \equiv \delta^+(t) \equiv \delta^-(t) \equiv 0$, unique solutions $x(t)$ and $z(t)$ to (1) and (7) exist for all time and satisfy (5) and

$$\lim_{t\to\infty} |x^+(t) - x^-(t)| = 0 \tag{14}$$

for any x_0 satisfying (3), and moreover, $X = 0$ is globally asymptotically stable. If any of the two conditions

$$\omega \in \mathcal{K}_\infty \vee \left[\omega \in \mathcal{K} \wedge \left\{\gamma \notin \mathcal{K}_\infty \vee \lim_{s\to\infty} \omega(s) > \sup_{t \in \mathbb{R}_{\geq 0}} \eta(\sqrt{2}|\delta^\pm(t)|)\right\}\right] \tag{15}$$

$$\mu \in \mathcal{K}_\infty \wedge \omega \in \mathcal{K}_\infty \tag{16}$$

holds, then there exist $\hat{\theta} \in \mathcal{KL}$, $\hat{\psi} \in \mathcal{K}$ and $\hat{\chi} \in \mathcal{K}_\infty$ such that

$$\hat{\chi}\left(|\hat{z}(t)|\right) \leq \hat{\theta}(|\hat{z}(0)|, t) + \int_0^t \hat{\psi}(|\hat{\rho}(\tau)|)d\tau, \quad \forall t \in \mathbb{R}_{\geq 0}. \tag{17}$$

$$\int_0^\infty \hat{\psi}(|\hat{\rho}(\tau)|)d\tau < \infty \ \Rightarrow \ \lim_{t\to\infty} |\hat{z}(t)| = 0 \tag{18}$$

hold for any x_0 and δ satisfying (3) and (4), and moreover, the entire system is iISS with respect to the input Δ and the state X. Furthermore, in the case of (16), there exist $\hat{\theta} \in \mathcal{KL}$ and $\hat{\phi} \in \mathcal{K}$ such that

$$|\hat{z}(t)| \leq \hat{\theta}(|\hat{z}(0)|, t) + \hat{\phi}\left(\sup_{\tau\in[0,t]} |\hat{\rho}(\tau)|\right), \quad \forall t \in \mathbb{R}_{\geq 0} \tag{19}$$

$$\lim_{t\to\infty} |\hat{\rho}(t)| = 0 \ \Rightarrow \ \lim_{t\to\infty} |\hat{z}(t)| = 0 \tag{20}$$

hold for any x_0 and δ satisfying (3) and (4), and moreover, the entire system is ISS with respect to the input Δ and the state X.

Theorem 2: Assume that $u_{si} : \mathbb{R}^{n_{y_i}} \times \mathbb{R}^{n_{x_i}} \to \mathbb{R}^{n_{u_i}}$ and $M_{i,i} : \mathbb{R}^{n_{y_i}} \to \mathbb{R}^{n_{x_i}\times n_{x_i}}$ are locally Lipschitz for all $i = 1, 2, ..., N$. Suppose that Assumptions 1, 2, 3, 4 and 5 are satisfied. If

$$\int_0^1 \frac{\gamma \circ \underline{\nu}^{-1}(s)}{\omega \circ \overline{\nu}^{-1}(s)} ds < \infty \tag{21}$$

holds, then in the case of $\delta(t)\equiv\delta^+(t)\equiv\delta^-(t)\equiv 0$, unique solutions $x(t)$ and $z(t)$ to (1) and (7) exist for all time and satisfy (5) and (14) for any x_0 satisfying (3), and moreover, $X = 0$ is globally asymptotically stable. If $\omega_i \in \mathcal{K}$, $i = 1, 2, ..., N$ and

$$\exists c > 0, k \geq 1 \ \text{s.t.} \quad c\gamma\circ\underline{\nu}^{-1}(s) \leq [\omega\circ\overline{\nu}^{-1}(s)]^k, \forall s \in \mathbb{R}_{\geq 0} \tag{22}$$

are satisfied, then there exist $\hat{\theta} \in \mathcal{KL}$, $\hat{\psi} \in \mathcal{K}$ and $\hat{\chi} \in \mathcal{K}_\infty$ such that (17) and (18) holds for any x_0 and δ satisfying (3) and (4), and moreover, the entire system is iISS with respect to the input Δ and the state X.

The authors do not intend to give proofs here since they are essentially identical to those presented in [8]. Some technical points allowing one to reduce the decentralized case into the general framework of [8] are explained briefly in the rest of this section. First, let $S = R^{-1}$ and

$$\mathcal{A}_{\mathcal{M}}(y) = \text{block-diag}_{i=1}^N A_{i,i}(y_i) + M_{i,i}(y_i),$$
$$R = \text{block-diag}_{i=1}^N R_i, \ Y = \text{block-diag}_{i=1}^N Y_i.$$

Then the N components in (7) are packed into

$$\dot{\hat{x}}^+ = \mathcal{A}_{\mathcal{M}}(y)\hat{x}^+ - Yy + \beta(y, u) \tag{23a}$$
$$+ S[R^+\delta^+ - R^-\delta^-] \tag{23b}$$
$$\dot{\hat{x}}^- = \mathcal{A}_{\mathcal{M}}(y)\hat{x}^- - Yy + \beta(y, u) \tag{23c}$$
$$+ S[R^+\delta^- - R^-\delta^+] \tag{23d}$$
$$\hat{x}^+(0) = \hat{x}_0^+ := S[R^+x_0^+ - R^-x_0^-] \tag{23e}$$
$$\hat{x}^-(0) = \hat{x}_0^- := S[R^+x_0^- - R^-x_0^+] \tag{23f}$$
$$x^+ = S^+R\hat{x}^+ - S^-R\hat{x}^- \tag{23g}$$
$$x^- = S^+R\hat{x}^- - S^-R\hat{x}^+. \tag{23h}$$

Thus, since

$$A_i(y_i)x = [A_{i,i}(y_i)+M_{i,i}(y_i)]x_i - Y_i(y_i)y_i$$

is implied by (6), the matrix $A(y) + \Lambda(y)C$ describing the dynamics of $\hat{x}^+ - x$ and $\hat{x}^- - x$ in [8] is replaced by the block diagonal matrix $\mathcal{A}_{\mathcal{M}}(y)$, which means that the error dynamics are decoupled. For instance, for $p = \hat{x}^+ - x$ we have

$$\begin{aligned}\dot{p} =& \mathcal{A}_{\mathcal{M}}(y)p + S[R^+\delta^+ - R^-\delta^-] - \delta \\ =& \mathcal{A}_{\mathcal{M}}(y)p + S[R^+(\delta^+ - \delta) + R^-(\delta - \delta^-)].\end{aligned}$$

Hence, defining $V = \sum_i^N V_i$, Assumptions 3 and 4 ensure that the corresponding assumptions in [8] are satisfied. Indeed, for each fixed $y \in \mathbb{R}^{n_y}$, we obtain the Metzler matrix

$$\Gamma(y) = R\mathcal{A}_{\mathcal{M}}(y)R^{-1}$$

from (9) satisfied for all $i = 1, 2, ..., N$, where $M = \text{block-diag}_{i=1}^N M_{i,i}$. Due to $|\rho_i^+| \leq |\rho^+|$, $|\rho_i^-| \leq |\rho^-|$ and the definition of ω, the property

$$\frac{\partial V}{\partial \xi}(\xi)\left\{\mathcal{A}_{\mathcal{M}}(y)\xi + S[R^+\rho^+ + R^-\rho^-]\right\} \leq -\omega(|\xi|) + \eta^+(|\rho^+|) + \eta^-(|\rho^-|)$$

follows from (11) satisfied for all $i = 1, 2, ..., N$, where $\eta^+ = \sum_{i=1}^N \eta_i^+$ and $\eta^- = \sum_{i=1}^N \eta_i^-$. On the other hand, Assumption 5 is the same as the one in [8] except that the decentralization is imposed on the control input u_s. Invoking Theorems 1 and 2 in [8] allows one to arrive at Theorems 1 and 2 in this paper.

It is possible to express the conditions on ω in Theorems 1 and 2 in terms of ω_i. For instance, from the viewpoint of the existence of ω_i, using the definition of ω, we can verify that

$$\begin{aligned}&\omega_i \in \mathcal{K}_\infty, \ i = 1, 2, \ldots, N \ \Leftrightarrow \ \omega \in \mathcal{K}_\infty, \\ &\omega_i \in \mathcal{K}, \ i = 1, 2, \ldots, N \quad \Leftrightarrow \ \omega \in \mathcal{K}, \\ &\int_0^1 \frac{\gamma \circ \underline{\nu}^{-1}(s)}{\omega_i \circ \overline{\nu}_i^{-1}(s)} ds < \infty, \ i = 1, 2, \ldots, N \Leftarrow (21), \\ &[\omega\circ\overline{\nu}^{-1}(s)]^k \leq [\omega_i\circ\overline{\nu}_i^{-1}(s)]^k, \imath = 1, 2, \ldots, N.\end{aligned}$$

If one follows the terminology in [8], an observer is said to be an iISS interval observer if (17) and (18) are achieved . An observer is said to be an ISS interval observer if (19) and (20) are achieved.

5. CONCLUDING REMARKS

This paper has proposed a decentralized interval observer-based controller for a class of nonlinear systems. Introducing appropriate constraints to the recent development [8] on centralized interval observer design has led to decentralized structure which is useful in practice for controlling and monitoring at the same time large-scale systems. Guidelines for choosing observer gains and the feedback gains are presented in the iISS framework. They allow the observer to not only estimate intervals of the state variables for any bounded disturbances, but also provide state intervals which are guaranteed to converge to zero in the presence of converging disturbances.

Regardless of linear and nonlinear systems, the class of systems which admits decentralized observation is inherently restrictive [18]. One can never get rid of this fact. In other words, there are a lot of plants which admit centralized observers, but which do not admit any decentralized one. Under such restrictive circumstances, this paper has tackled the tougher problem of estimating intervals of the state throughout the entire time horizon. A very popular approach to design of standard observers is to treat interactions as uncertainties which vanish as time goes to infinity (see e.g., [1, 2, 15, 17-20] and references therein). However, such an approach cannot be simply employed for the interval estimation since the intervals are required to be valid from the initial time to infinity. As in the literature of decentralized control which has been studied extensively for decades, it is natural that this paper has only been able to present a sufficient condition which is not ready to provide an analytical solution explicitly. In practice, the sufficient condition can be checked for a given system. Nevertheless, it would be worth investigating the possibility of reducing conservativeness analytically, although it is undoubtedly very challenging.

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