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On feedback transformation and integral input-to-state stability in designing robust interval observers

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Interval observers offer useful guarantees in monitoring the state of dynamical systems in the presence of large disturbances [1]. This talk addresses the problem of designing such observers for the controlled plant of the form

$$\dot{x}(t) = A(y(t))x(t) + B(y(t))u(y(t), \hat{x}^+(t)) + \delta(t), \quad y(t) = Cx(t) \tag{1}$$

with the state $x(t) \in \mathbb{R}^n$, the output feedback control input $u(y(t), \hat{x}^+(t)) \in \mathbb{R}^q$, the measurement output $y(t) \in \mathbb{R}^p$ and the disturbance $\delta(t) \in \mathbb{R}^n$. The signal $\hat{x}^+(t) \in \mathbb{R}^n$ denotes an estimate of $x(t)$. Assume that $x_0^-, x_0^+ \in \mathbb{R}^n$ and $\delta^-, \delta^+ : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfying $x_0^- \leq x_0 \leq x_0^+$ and $\delta^-(t) \leq \delta(t) \leq \delta^+(t)$ for all $t \in \mathbb{R}_{\geq 0}$ are known, while $x(0) = x_0$ and $\delta(t)$ are unknown. Note that all the inequalities must be understood component-wise. To generate $x^-(t), x^+(t) \in \mathbb{R}^n$ such that

$$x^-(t) \leq x(t) \leq x^+(t), \quad \forall t \in \mathbb{R}_{\geq 0} \tag{2}$$

holds, the following interval observer has been proposed in [2]:

$$\dot{\hat{x}}^+ = A(y)\hat{x}^+ + B(y)u + H(y)[C\hat{x}^+ - y] + S[R^+\delta^+ - R^-\delta^-] \tag{3a}$$

$$\dot{\hat{x}}^- = A(y)\hat{x}^- + B(y)u + H(y)[C\hat{x}^- - y] + S[R^+\delta^- - R^-\delta^+], \tag{3b}$$

where $S = R^{-1}$, $R^+ = (\max\{R_{i,j}, 0\})_{i,j=1,1}^{n,n}$, $R^- = R^+ - R$, $x^+ = S^+R\hat{x}^+ - S^-R\hat{x}^-$ and $x^- = S^+R\hat{x}^- - S^-R\hat{x}^+$. Indeed, if there exists a matrix R such that for each fixed $y \in \mathbb{R}^p$,

$$R[A(y) + H(y)C]R^{-1} \text{ is Metzler} \tag{4}$$

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holds true, then the ordering of solutions to (3a) and (3b) is preserved. In addition to (4), assuming the integral input-to-state stability (iISS) properties of the observer error dynamics and the feedback dynamics separately as

$$\frac{\partial V}{\partial \xi}\left\{[A(y)+H(y)C]\xi+S[R^{+}\rho^{+}+R^{-}\rho^{-}]\right\} \le -\omega(|\xi|)+\eta^{+}(|\rho^{+}|)+\eta^{-}(|\rho^{-}|) \quad (5)$$

$$\frac{\partial U}{\partial x}[A(Cx)x+B(Cx)u(Cx,x+d)+\delta] \le -\mu(|x|)+\gamma(|d|)+\zeta(|\delta|) \quad (6)$$

with appropriate functions V, ω, η^{+}, η^{-}, U, μ, γ and ζ. this present talk first reviews a growth rate condition imposed on ω, μ and γ in [3] to guarantee that the closed-loop system consisting of (1) and (3) is iISS with respect to input $(\delta^{+}, \delta^{-}, \delta)$ and the difference $x^{+} - x^{-}$ converges to zero whenever δ is convergent appropriately. The result in [2] is included as a special case.

Since u can be alway expressed as $u(y, \hat{x}^{+}) = K(y)y + u_a(y, \hat{x}^{+})$ for an arbitrarily given $K(y) \in \mathbb{R}^{q\times p}$, in this talk we replace the observer (3) by

$$\dot{\hat{x}}^{+} = (A(y)+B(y)K(y)C)\hat{x}^{+} + B(y)u_a + H(y)[C\hat{x}^{+} - y] + S[R^{+}\delta^{+} - R^{-}\delta^{-}] \quad (7)$$

and similar equations. This observer mechanism achieves the same properties as (3) by replacing $A(y)+H(y)C$ with $A+H(y)C+B(y)K(y)C$ in (4) and (5).

Property (6) is independent of R, H and K. The state transformation R contributes to only (4), while the gain H contributes to (4) and (5) and has the same effect as BK. The observer (7) depends on the choice of K for a given and fixed u. Thus, the freedom offered by K can change the behavior of the interval estimates x^{+} and x^{-} within the aforementioned guarantees. This change in estimates influences the behavior of x of the plant in the closed loop. The standard Luenberger observer also admits K influencing the closed-loop response. However, the freedom is not much appreciated since the standard observer aims at closed-loop stability and convergence and it is not built for monitoring. In contrast, interval observers provide estimates in the transient phase and the freedom of K matters. Notice that for a given and fixed control law u, the choice of H does not influence u_a in the observer (7), while K does.

This talk also addresses a reduced-order interval observer estimating intervals for only the unmeasured part of x. Since the observer is free from dynamics estimating the measured part, its closed loop can be expected to have swifter response with less effort. Interestingly, attainability of (4) and (5) for the full-order interval observer (7) implies the existence of a reduced-order interval observer unless the state transformation R is fully exploited.

To illustrate ideas and observations, comparative simulations will be shown.

References

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