Supélec, Alcatel-Lucent Chair on Flexible Radio

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Coalitional Network Games in Telecommunications

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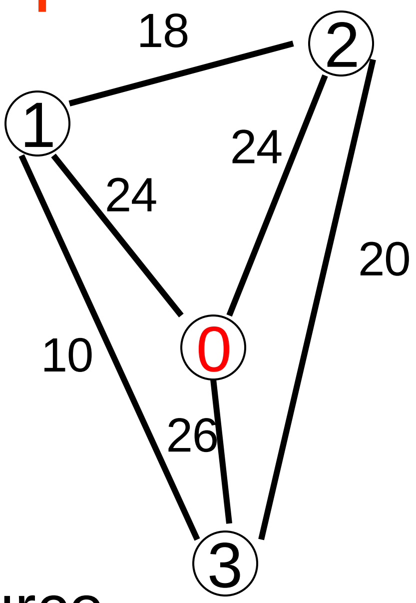
Outline

- Cost allocation problems arising from connection situations
- Criticality index for resource consolidation

Cost allocation problems arising from connection situations

Minimum Cost Spanning Tree Situation

Consider a complete weighted graph



- whose vertices represent agents
- vertex 0 is the source
- edges represent connections between agents or between an agent and the source
- numbers close to edges are connection costs

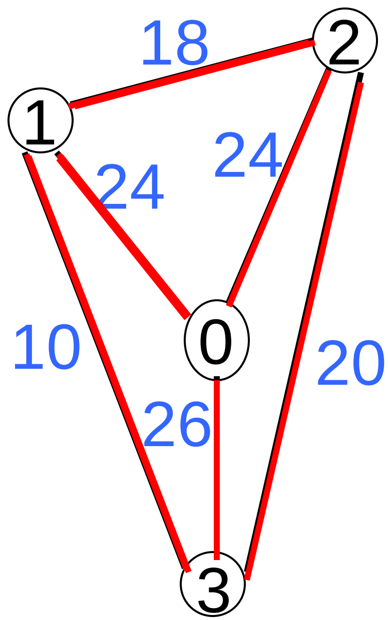
Minimum cost spanning tree (mcst) problem

Optimization problem:

How to connect each node to the source 0 in such a way that the cost of construction of a spanning network (which connects every node directly or indirectly to the source 0) is minimum?

The problem of finding a mcst may be easily solved thanks to different algorithms proposed in literature (Boruvka (1926), Kruskal (1956), Prim (1957), Dijkstra (1959))

Example: The cost game ({1,2,3},c) is defined on the following connection situation:



c(1)=24
c(1)=24

c(2)=24
c(2)=24

c(3)=26
c(3)=26

c(1,3)=34
c(1,3)=34

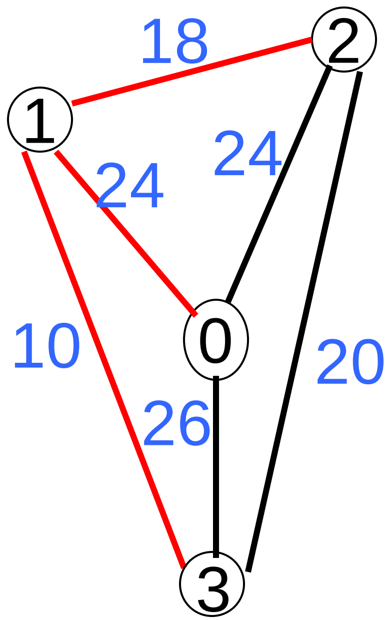
c(1,2)=42
c(1,2)=42

c(2,3)=44
c(2,3)=44

c(1,2,3)=52
c(1,2,3)=52

The game ({1,2,3}, c) is said mcst game (Bird (1976))

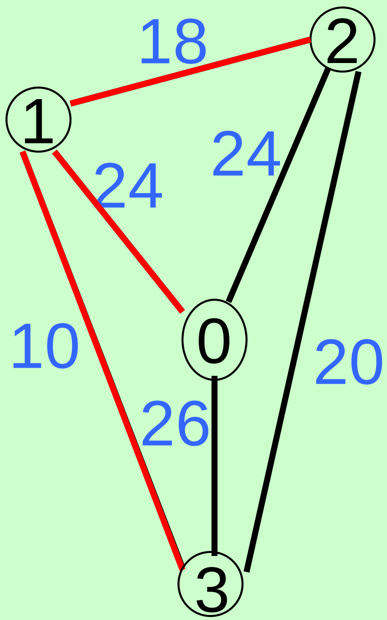
How to divide the total cost? (Bird 1976)

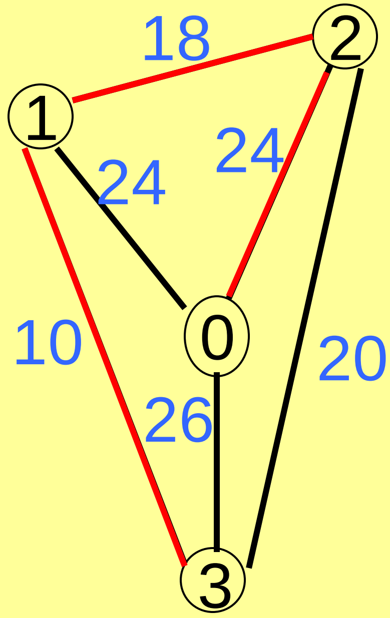


- The predecessor of 1 is 0: the Bird allocation gives to player 1 the cost of {0,1}.
- The predecessor of 2 is 1: the Bird allocation gives to player 2 the cost of {1,2};
- The predecessor of 3 is 1: the Bird allocation gives to player 3 the cost of {1,3}.

$w(\Gamma)=52$

Bird allocation w.r.t. to Γ, $(x_1, x_2, x_3)=(24, 18, 10)$ is in the core of ({1,2,3},c).





The Bird allocation w.r.t .this mcst is

$(x_1, x_2, x_3)=(24, 18, 10)$

The Bird allocation w.r.t. this mcst is

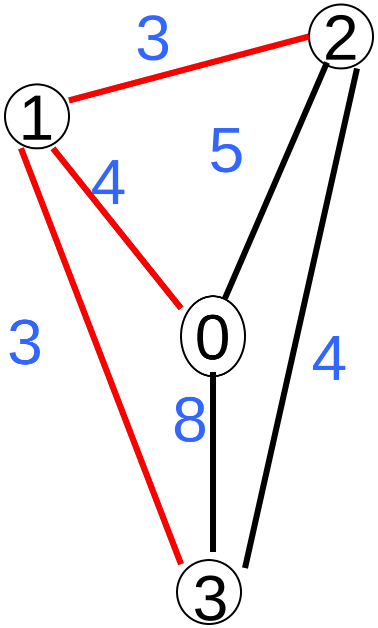
$(x_1, x_2, x_3)=(18, 24, 10)$

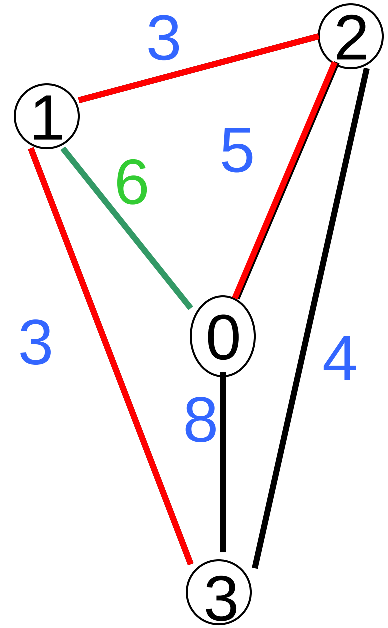
Both allocations belong to the core of the mcst game (and also their convex combination).

What happens when the structure of the network changes?

- Imagine to use a certain rule to allocate costs.
  - The cost of edges may increase: if the cost of an edge increases, nobody should be better off, according to such a rule (*cost monotonicity*);
  - One or more players may leave the connection situation: nobody of the remaining players should be better off (*population monotonicity*).

Cost monotonicity: Bird allocation behaviour



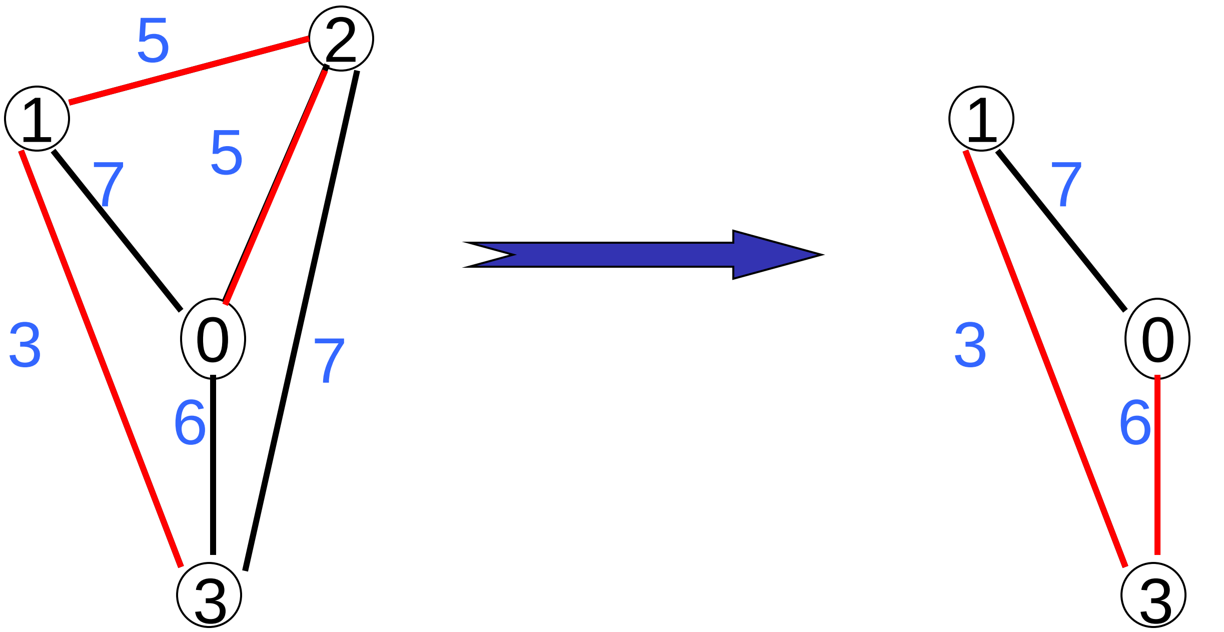


Bird allocation: (4, 3 ,3)

Bird allocation: (3, 5 ,3)

Bird rule does not satisfy cost monotonicity.

Population monotonicity: Bird allocation behaviour



Bird allocation: (5, 5 ,3)

Bird allocation: (3, * ,6)

→ Bird rule does not satisfy population monotonicity

Axiomatic characterization $F : \mathcal{W}^{N'} \to \Re^N$

Property 1. The solution F is *efficient* (EFF) if for each $w \in \mathcal{W}^{N'}$

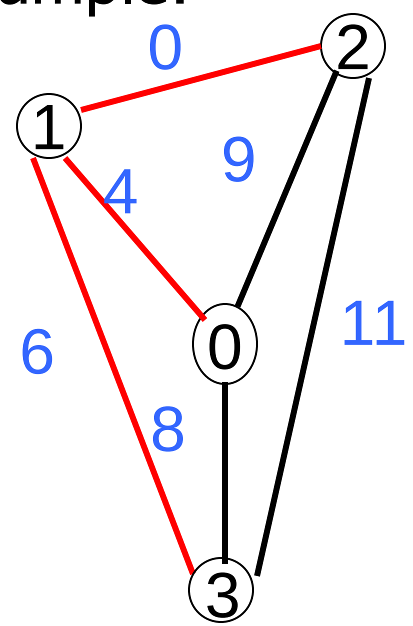
$$\sum_{i \in N} F_i(w) = w(\Gamma),$$

where Γ is a minimum cost spanning network on N'.

Property 2. The solution F has the *Equal Treatment* (ET) property if for each $w \in \mathcal{W}^{N'}$ and for each $i,j \in N$ with $C_i(w) = C_j(w)$

$$F_i(w) = F_j(w).$$

Example:

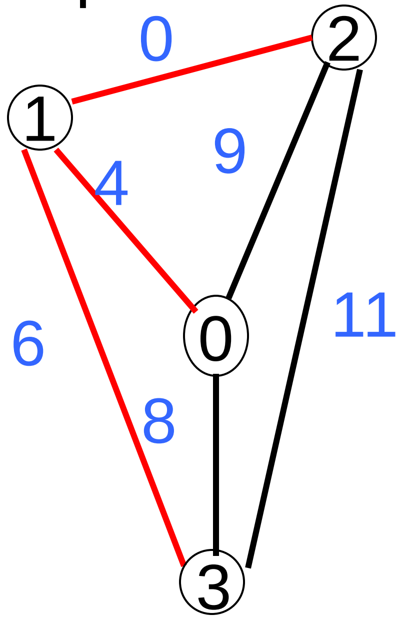


$F(w)=(2,2,6)^t$

Property 3. The solution F has the *Upper Bounded Contribution* (UBC) property if for each $w \in \mathcal{W}^{N'}$ and every (w,N')-component $C \neq \{0\}$

$$\sum_{i \in C \setminus \{0\}} F_i(w) \leq \min_{i \in C \setminus \{0\}} w(\{i,0\}).$$

Example:

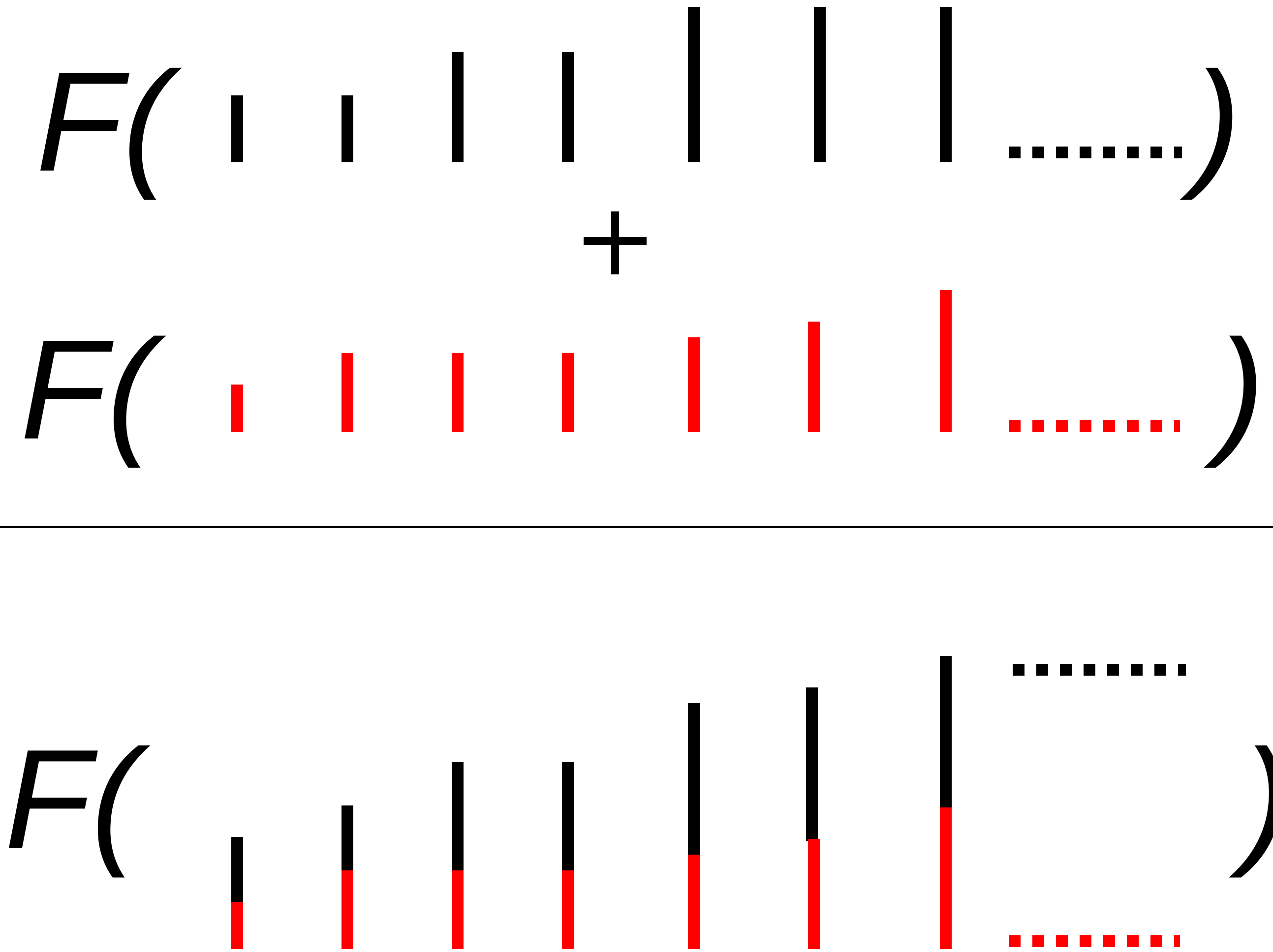


$F(w)=(2,2,6)^t$

Property 4. The solution F has the *Cone-wise Positive Linearity* (CPL) property if for each $\sigma \in \Sigma_{E_{N'}}$, for each pair of mcst situations $w, \hat{w} \in K^{\sigma}$ and for each pair $\alpha, \hat{\alpha} \geq 0$, we have

$$F(\alpha w + \hat{\alpha}\hat{w}) = \alpha F(w) + \hat{\alpha} F(\hat{w}).$$

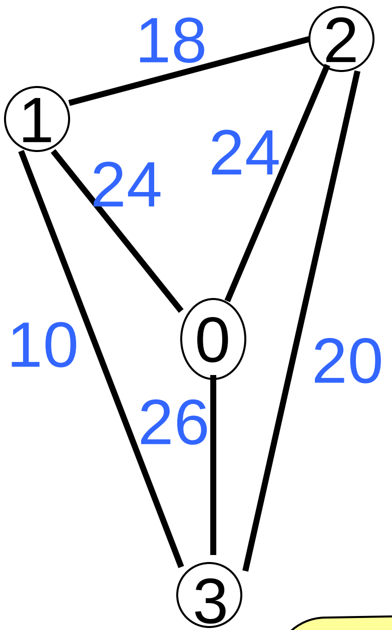
Example:



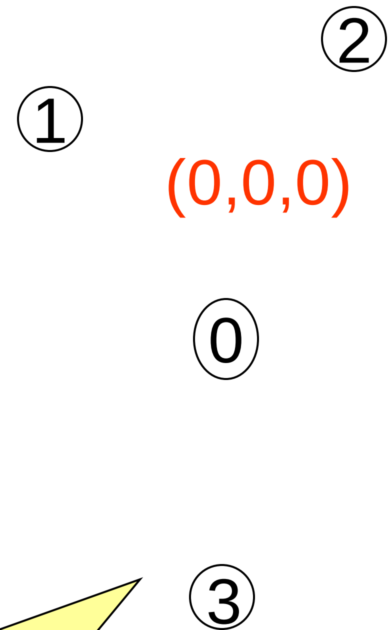
Theorem *The P-value is the unique solution which satisfies the properties EFF, ET, UBC and CPL on the class $\mathcal{W}^{N'}$ of mcst situations.*

- It is possible to prove that the *P*-value satisfies the four properties EFF, ET, UBC and CPL.
- To prove the uniqueness consider a solution for mcst situation *F* which satisfies EFF, ET, UBC and CPL:
  - first look at the **simple mcst situations** (0-1 cost of edges): on such simple situation, EFF, ET and UBC imply *F*=*P*-value;
  - it is possible to decompose each mcst situation as a linear combination of simple mcst problems;
  - by CPL it follows that the *F*=*P*-value on each mcst situation.

P-value: Feltkamp (1994), Branzei et al. (2004), Moretti (2008)

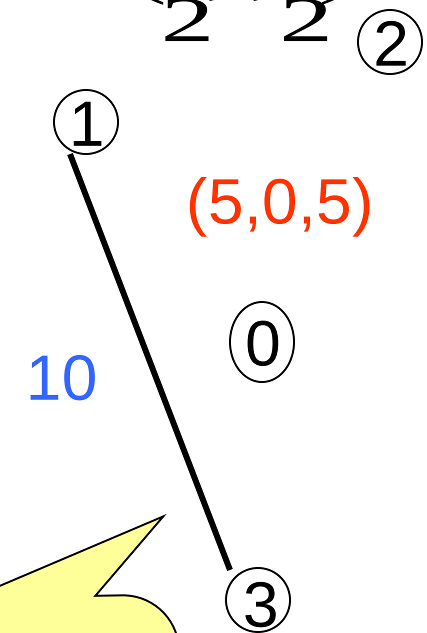


$$b^{\sigma,0} = (1,1,1)^t$$

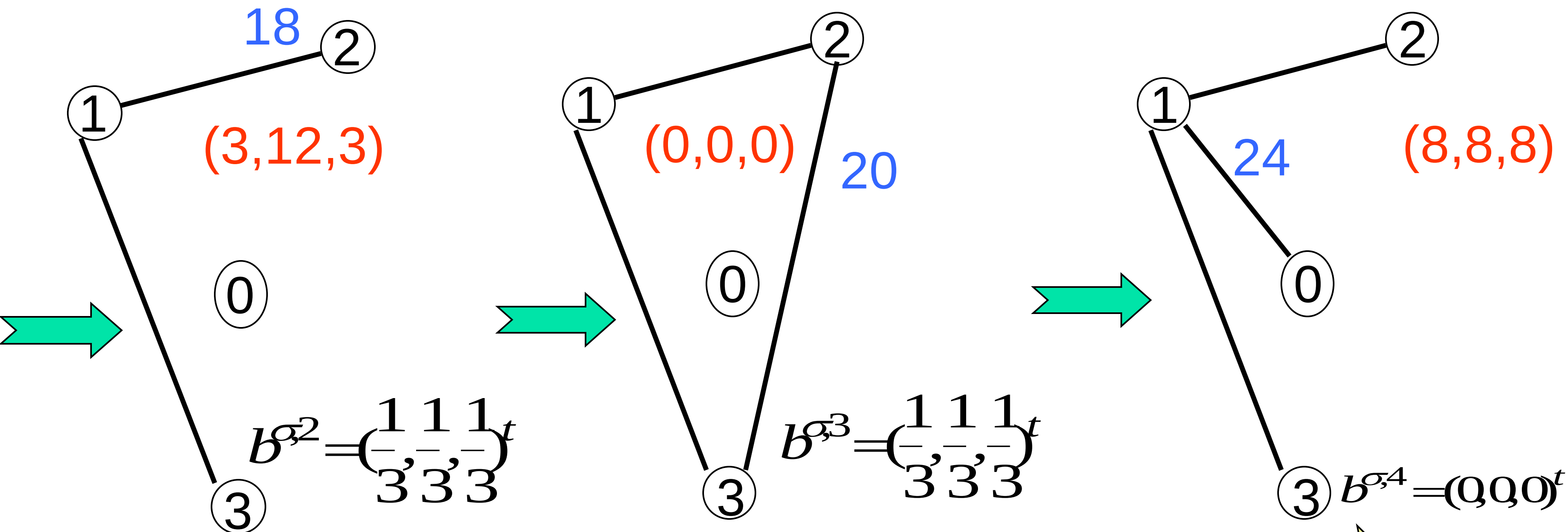


There are no edge costs to share.

$$b^{\sigma,1} = \left(\frac{1}{2},1,\frac{1}{2}\right)^t$$



1 and 3 share cost 10 equally.



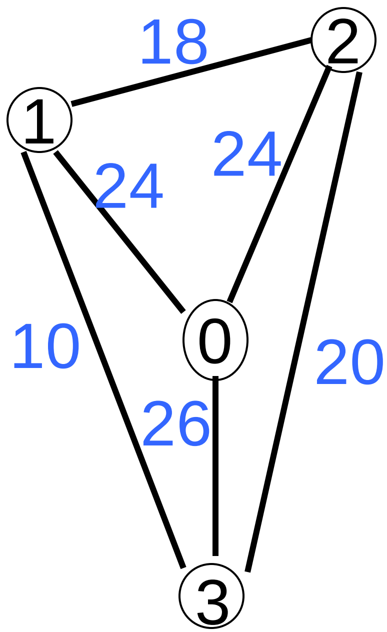
2 is connected to 1 and 3 who were already connected: 2 pays 2/3 of 18 whereas the remaining is shared equally between 1 and 3.

Oops… there is a cycle: nobody want it.

Players are connected to 0: share the total cost of the last edge (=24) equally

P-value

Make the sum of all edge-by-edge allocations:



(0, 0, 0) +
(5, 0, 5) +
(3,12,3) +
(0, 0, 0) +
(8, 8, 8) =

P-value = (16,20,16)

minimum interval cost spanning tree (*micst*) situations

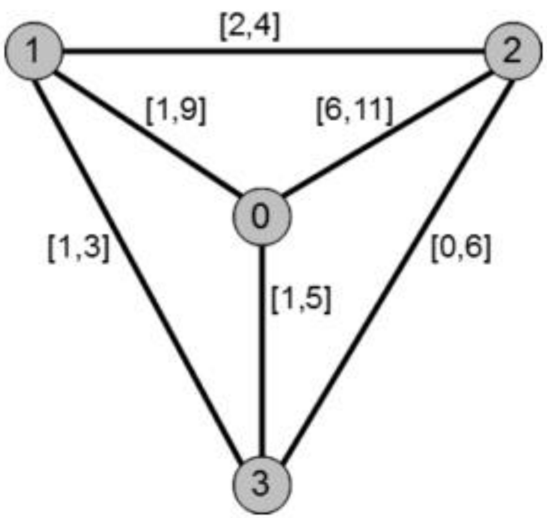


Figure 1: An micst situation $\langle\{0,1,2,3\},W\rangle$.

- We have extended the notion of P-value (and obligation rules) to situations with uncertain costs
- We have analyzed the behavior of such extended rules in correspondence of *pessimistic* and *optimistic* attitude of agents to face uncertainty
- The P-value provides incentives to cooperate even in this context
- *Strategyproof*: to reveal the cost of transmission to the source (if privately known to the owner of the link) is a dominant strategy.

Wireless network

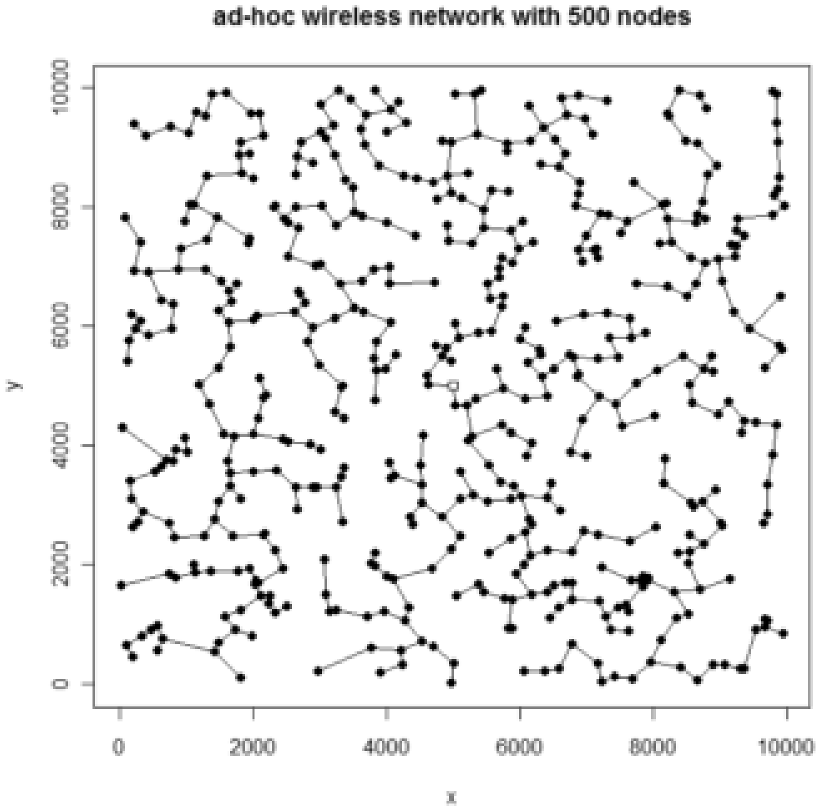
- A *node* in these networks is equipped with an *antenna* which is responsible for sending and receiving signals.
- antennas are unidirectional: problem to assigning to each node an amount of *transmitting power* and a *direction* (toward another node)
- We adopt the model presented in Rappaport (1996): signal *power falls at* $1/d^k$, where d is the distance from the transmitter to the receiver and *k is an environment-dependent coefficient*, typically between 2 and 4.
- the *power requirement* for supporting a link from node i to node j, separated by a distance d_{ij}, is then given by d_{ij}^k
- this problem can be formulated as an micst situation where N is the set of nodes and the *interval cost* for each pair $i,j \in N$ is

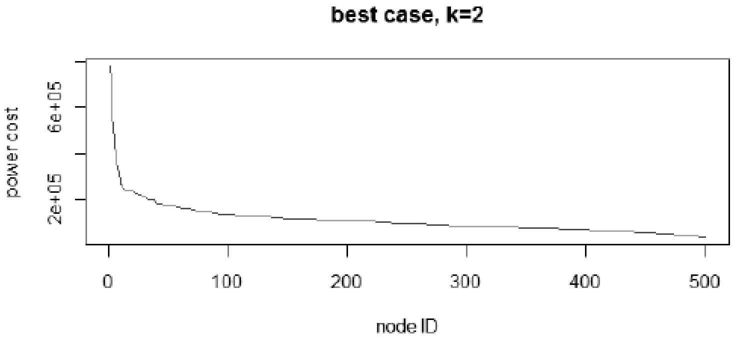
$$W(i,j) = [(d_{ij})^2; (d_{ij})^4]$$

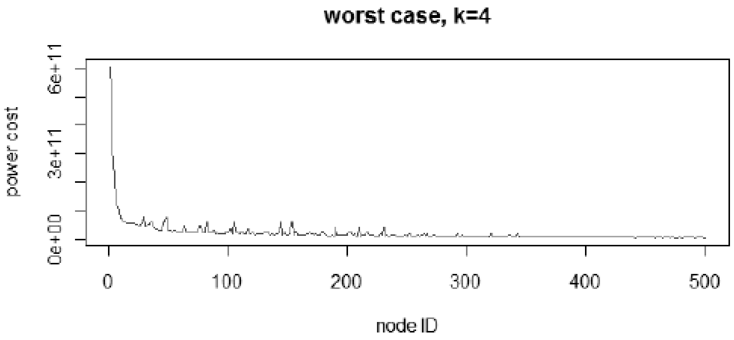
Simulation

- 500 points - they are the nodes of the network - chosen uniformly at random from a grid of size 10000×10000
- a source 0 is placed in the center of the grid
- extended P-value implemented in R language (R Development Core Team (2010))
- The elapsed time required by our software implementation to compute such allocations was less than 17 seconds (on a PC with a 2.93GHz processor, 4GB of memory, 32-bit Operating System and where it was installed the R version 2.11).

Simulation (2)







P-value

- This theoretic problem has been solved:
  - *P–value* provides an allocation that is in the core, tree independent, and also monotonic wrt both costs and population, is an *obligation rule*, on a subclass of connection problems it coincides with the Shapley value of mcst games, nice axioms etc… (http://arno.uvt.nl/show.cgi?fid=80868)
- What about practice?
  - Attempt to find a dynamic justification based on intuitive grounds that directs the people who are engaged with the real topic to the desired solution.

Criticality index for resource consolidation

The Green-Game: Striking a Balance between QoS and Energy Saving

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***Abstract*—The energy consumed by communication networks can be reduced in several ways. A promising technique consists in concentrating the workload of an infrastructure on a reduced set of devices, while switching off the others, and is referred to as resource consolidation. This technique is particularly suited to routing of data traffic when a network is lightly loaded, but the selection of the routers that can be safely switched off requires an accurate evaluation of the criticality of the devices. In this article, we define a measure of the criticality of the different interconnection devices that does not only take into account a network topology but also its traffic matrix. We model the scenario as a coalitional game, and show how the use of the Shapley value associated to the nodes can be used as criticality indexes. This index is compared with other classical indexes on realistic topologies. We find out that our index provides a robust and relevant criticality measure achieving a good tradeoff between energy efficiency and network robustness.**

and energy consumption by expressing network design as an optimization problem whose objective function is a weighted sum of the total energy consumed by the network and of a function of the link utilization level, accounting for robustness.

However these contributions do not propose a definition of the criticality of the network nodes w.r.t. topological aspects and to the traffic that the network effectively conveys. Game theory offers a set of pertinent tools to define such criticality index and subsequently to define heuristics for the resource consolidation problem. In this article, we model the resource consolidation problem as a cooperative Transferable Utility (TU) Game, the *Green-Game* (or G-Game for short). This game takes as its only inputs the network topology, i.e., the set of links and devices, and the traffic matrix (TM), i.e., the

Resource consolidation

- Resource consolidation: to concentrate data traffic over a small subset of the links and devices, allowing the others to enter a power saving mode…

- …preserving the network connectivity and its quality of service through the definition of a minimum path diversity or a maximum link utilization.

Which devices must be switched off?

- A solution purely optimizing the energy consumption does not take into account the *system robustness*
  - There is no control on which network element are switched off

- IDEA: definition of a *criticality index* for the network devices to drive the resource consolidation process taking into account *Energy-saving* and *robustness*

A game theoretical approach

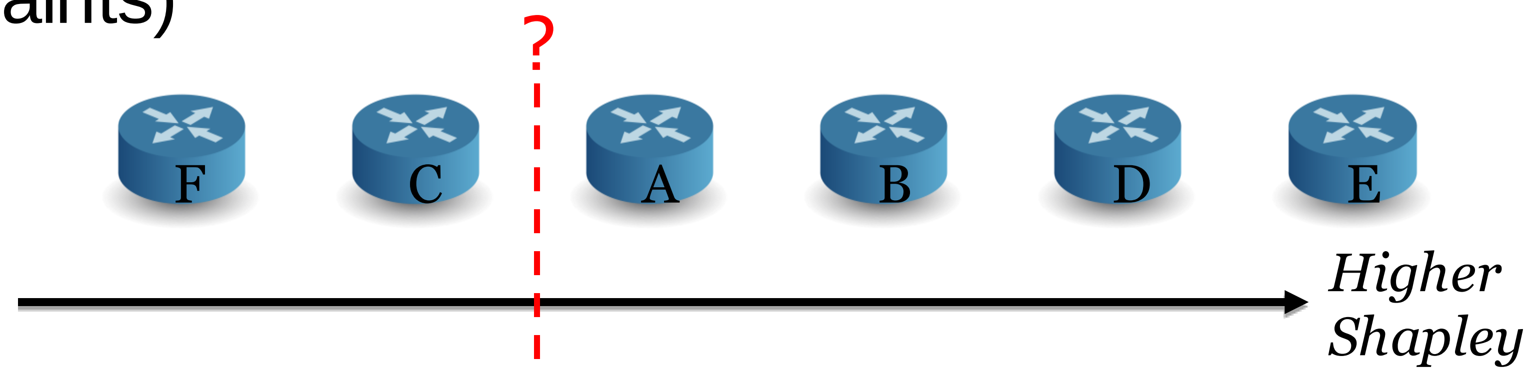
- Modeling the communication network as a cooperative TU-game
- Each node is a player
- Every coalition is a network configuration:
  - Nodes in the coalition -> ON
  - Other nodes -> OFF (or failures)
- The amount of delivered traffic is the revenue of the coalition

A game theoretical approach

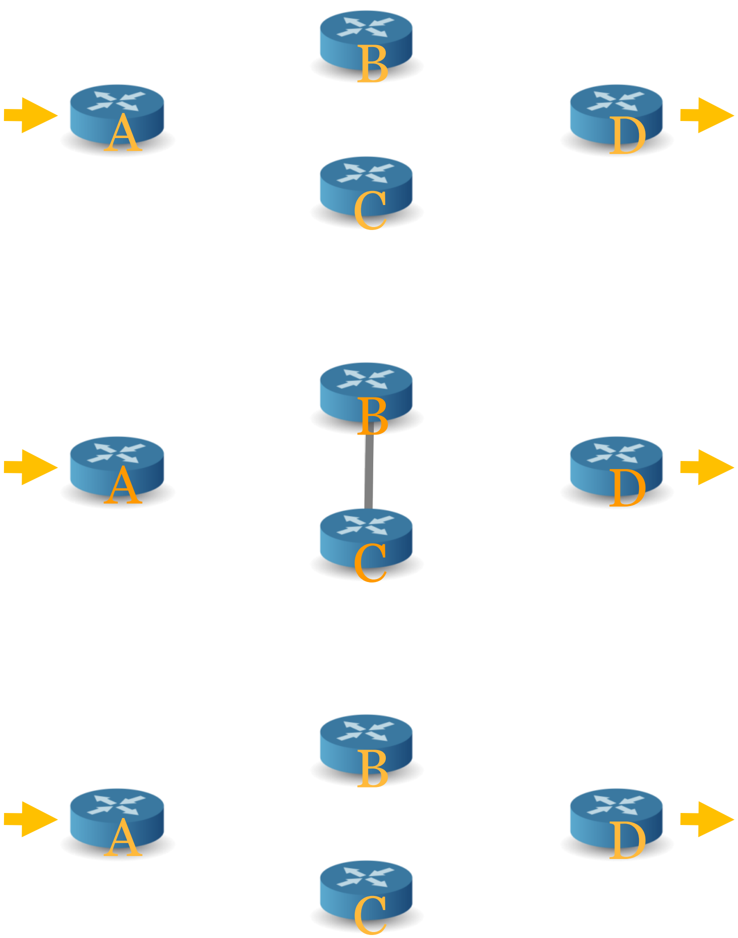
- two games:
- A Traffic Game (*a-priori*) over a full-mash network (accounts only for the Traffic Requests)
- A Topology Game (*a-posteriori*), which is the *restriction* (Myerson (1977)) of the first over the network graph, and accounts for the Topology

The Shapley value ranking

- The Shapley value defines a rank among players (on the basis of the amount of traffic that nodes contribute to carry, and of their criticality while composing the coalition)
- Nodes are progressively switched off (if the all traffic requests are still satisfied, with eventual maximum load constraints)



Example



Traffic Request

C -> D: 1Mbps

Traffic Game:

$v = u(A, D)$

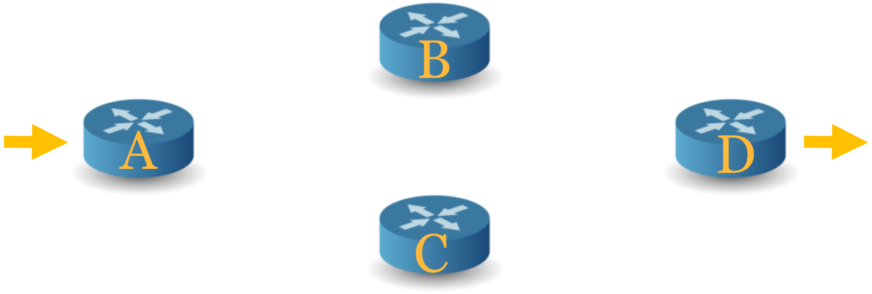
Topology Game:

Minimal connected components containing A, D:

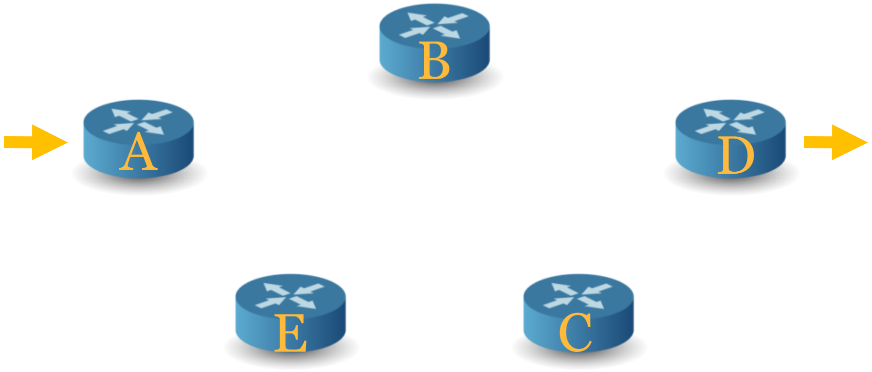
$v^{\Gamma} = u(A,B,D)+u(A,C,D)-u(A,B,C,D)$

Example

Shapley value

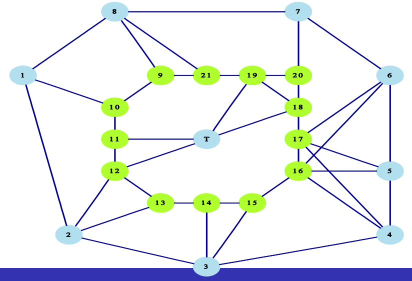


A, D -> 5/12
C, B -> 1/12



A, D -> 23/60
B -> 8/60
C, E -> 3/60

A Real Case Study



➢ The GT method has been tested over a TIGER2 topology, representing a typical access/metropolitan operator network (21 nodes).

➢ The GT method obtains excellent performances, as the distribution of link utilization after consolidation is equivalent to the baseline configuration (where no node is switched off).

➢procedure yielded high energy savings, with little or no impact on the expected QoS levels on the network.