Practical Quantize-and-Forward schemes for the Frequency Division Relay Channel

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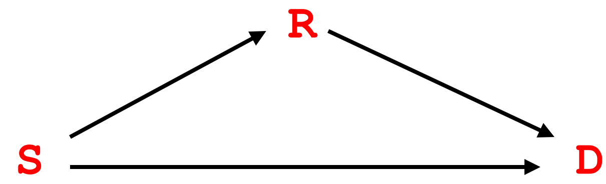
PROBLEM STATEMENT

- Channel description:

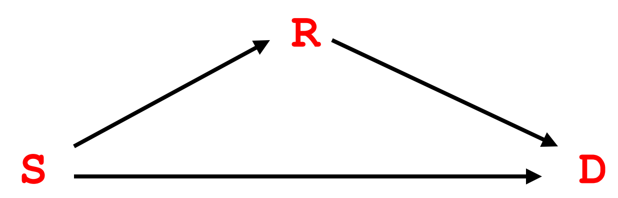


- Information theoretic relay channel (RC)
  - [Cover & El Gamal 1979]: Introduction of Estimate-and-Forward (EF) and Decode-and-Forward (DF) relaying schemes.
  - [El Gamal & Al. 2006]: The optimal relaying scheme depends on channels SNRs.
- Relaying protocols:
  - Decode-and-Forward

    Forward decoding errors when the S-R link is bad.
  - Amplify-and-Forward

    $\alpha[hx + n] \rightarrow$ noise amplification
  - Estimate-and-Forward

PROBLEM STATEMENT



- Main issues
  - Propose two relaying schemes that take SNRs into account:
    - Quantize-and-Forward (QF). Low complexity → scalar
    - Amplify-and-Forward with clipping (C-AF) .
  - Performance analysis for different SNR scenarios.
- Assumptions:
  - Memoryless fading channels.
  - Orthogonality between the downlink and the cooperative link.
  - Coherent reception at R and D.
  - CSI at R and D.
  - SNRs knowledge at R and D.

BACKGROUND

- EF-based protocols for the RC
  - [Liu & Al. 2005] and [Charkrabarti & Al. 2006]: Rates analyses of proposed schemes for the non-orthogonal half-duplex RC.
  - [Hu & Al. 2006]: Practical EF schemes based on Wyner-Ziv coding for the orthogonal half-duplex RC.

- AF-based protocols for the RC
  - Simplicity → Develop advanced coding strategies (ex. Yang & Belfiore): DMT achieving.
  - No loss in terms on information but [analog constraint + noise amplification].
  - Optimal non-linear function at the relay in a multihop relay channel:
    - to minimize the BER [Abou-Faycal & Médard 2004]
      → Complexity for high order modulation
    - To maximize the mutual information [Gomadam & Jafar 2006]

OUTLINE

1. Signal model
2. Quantize-and-Forward
3. Maximum likelihood for the QF protocol
4. Clipped Amplify-and-Forward
5. Simulation results

SIGNAL MODEL

- Source and relay: unit-power constraint

$$E|X|^2 = 1 \qquad E|X_r|^2 = 1$$

- **Block fading channels.**

- Relay: received signal $x_{sr}(n) = h_{sr} \times x(n) + w_{sr}(n)$

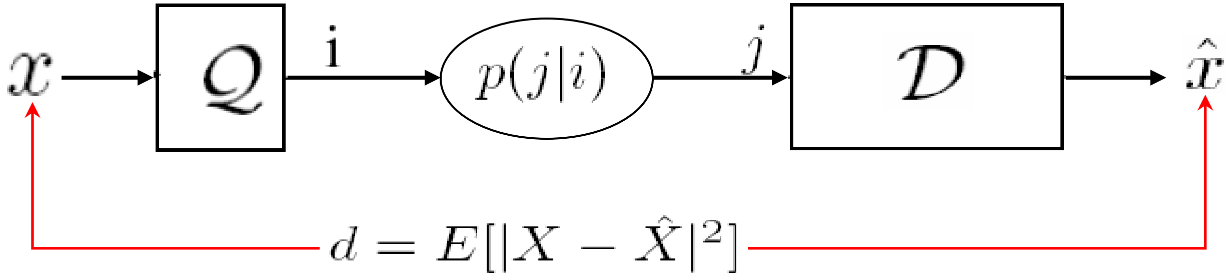
- Destination: orthogonality leads to

$$\begin{cases} y_{sd}(n) = h_{sd} \times x(n) + w_{sd}(n) \\ y_{rd}(n) = h_{rd} \times x_r(n) + w_{rd}(n) \end{cases}$$

Depends on the relaying scheme

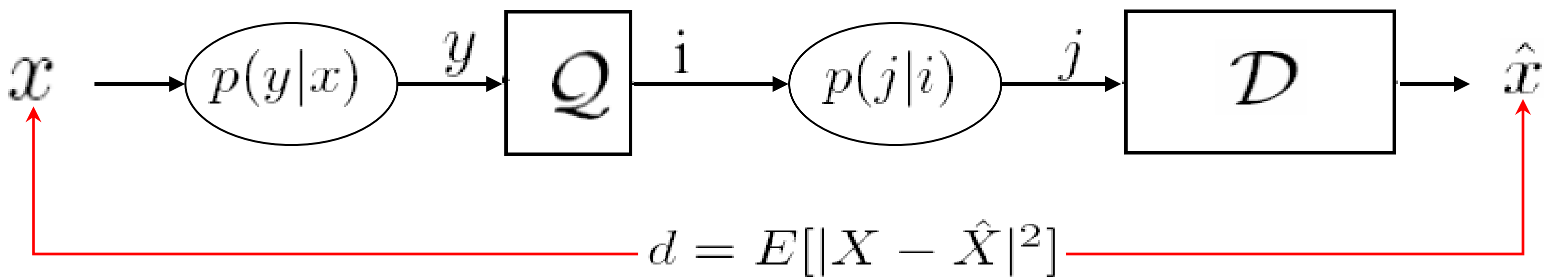
OPTIMIZED QF PROTOCOL (1/5)

- **Kunterbach and Wintz**



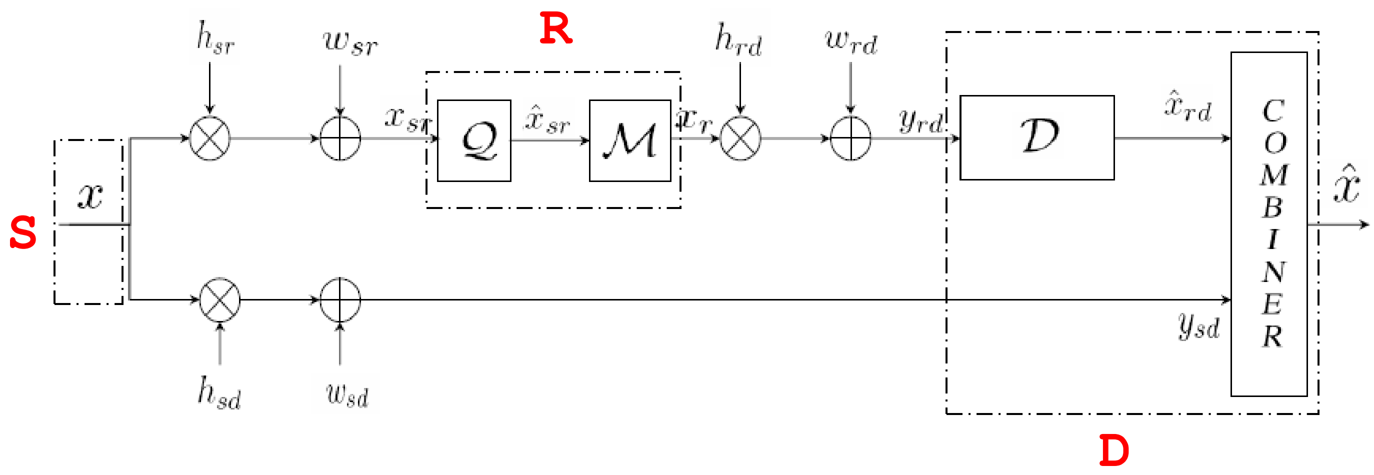
→ **Modify the Lloyd-Max algorithm to take into account channel errors in the quantization optimization**

- **RC: two-hop link S-R-D**



→ **Take into account S-R and R-D channels errors in the quantizer optimization**

OPTIMIZED QF PROTOCOL (2/5)



- **Goal:** Optimize quantization parameters (representatives v_j^R and transition levels u_i^R) to minimize the distortion between the reconstructed signal $\hat{x}_{rd}$ and the original source signal x

$$D_{11} = \underbrace{E\left[(\hat{x}_{rd}^R)^2\right] - 2E\left[\hat{x}_{rd}^R x^R\right] + E\left[(x^R)^2\right]}_{D_{11}^R}$$

Real part of the signal

$$+\underbrace{E\left[(\hat{x}_{rd}^I)^2\right] - 2E\left[\hat{x}_{rd}^I x^I\right] + E\left[(x^I)^2\right]}_{D_{11}^I}$$

- **Uniform quantization:** Numerical optimization (Δ).

OPTIMIZED QF PROTOCOL (3/5)

- **Optimal quantization :**
  - **Explicit expression for optimal representatives no matter the modulation types at S and R.**
  - **Explicit expression for optimal transition levels when using simple modulations at S (ex. of 4-QAM $\left(x^R, x^I\right) \in \{-A,+A\}^2$).**

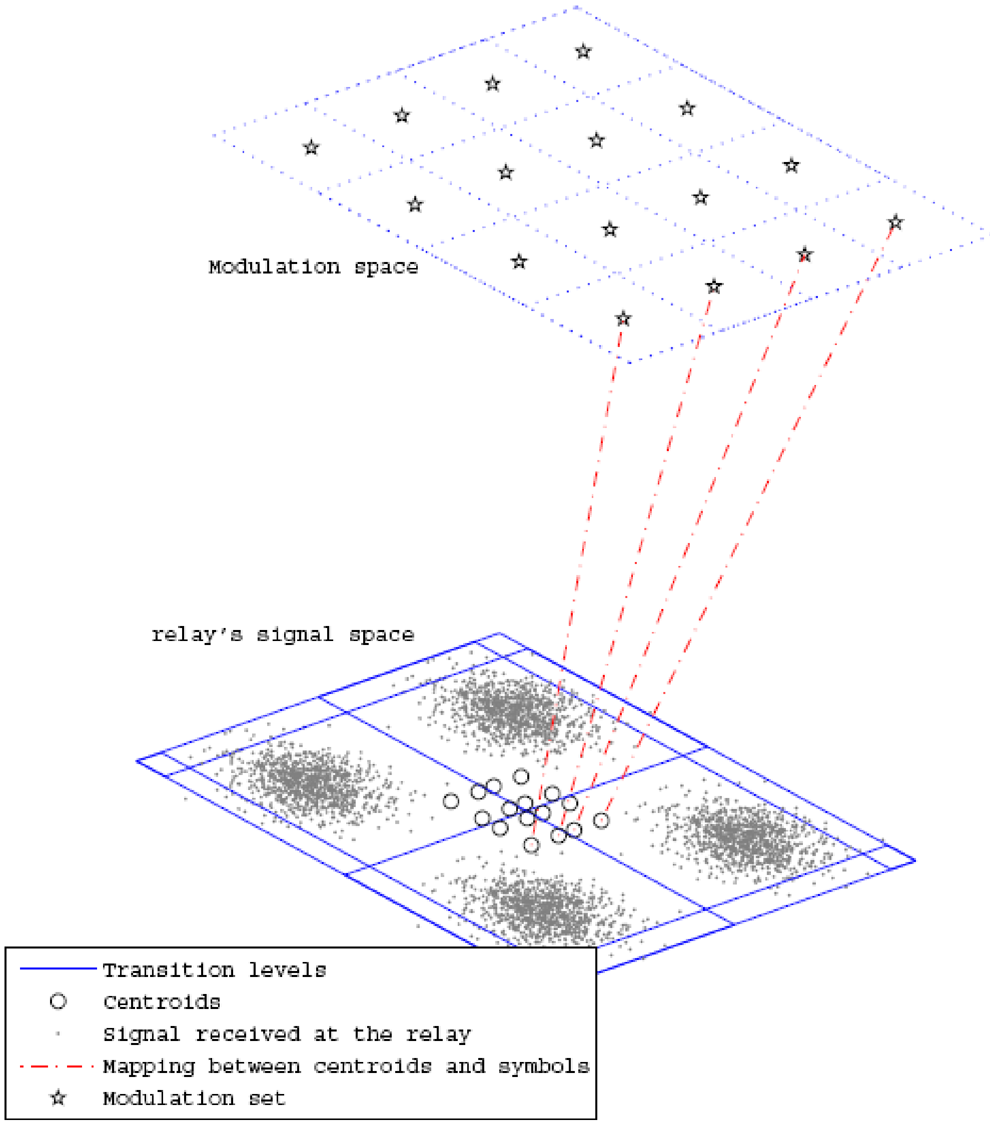
$$v_\ell^{R,*} = A \times \frac{\sum_{k=1}^{L} P_{k,\ell}^R \int_{u_k^R}^{u_{k+1}^R} \phi\left(t-A\right) - \phi\left(t+A\right) dt}{\underbrace{\sum_{k=1}^{L} P_{k,\ell}^R}_{\text{R-D link}} \int_{u_k^R}^{u_{k+1}^R} \underbrace{\phi\left(t-A\right) + \phi\left(t+A\right)}_{\text{S-R link}} dt} \qquad (1)$$

$$u_\ell^{R,*} = \frac{\sigma_{sr}^2}{2A} \ln \left[\frac{\sum_{k=1}^{L} \left(P_{\ell,k}^R - P_{\ell-1,k}^R\right)\left(A + \frac{1}{2}v_k^R\right) v_k^R}{\sum_{k=1}^{L} \left(P_{\ell,k}^R - P_{\ell-1,k}^R\right)\left(A - \frac{1}{2}v_k^R\right) v_k^R} \right] \qquad (2)$$

where $\phi\left(t\right) = \frac{|h_{sr}|}{\sqrt{\pi}\sigma_{sr}} \exp\left(-\frac{|h_{sr}|^2 t^2}{\sigma_{sr}^2}\right)$ **is the PDF of the noise real part on the S-R link.**

OPTIMIZED QF PROTOCOL (4/5)

- **Mapping :**



OPTIMIZED QF PROTOCOL (5/5)

- **Iterative optimization procedure:**
  - **Step 1:** Set $i = 0$. Set $\epsilon = 1$. Initialize $\mathcal{V}^R$ and $\mathcal{U}^R$ with the Kunterbach's algorithm ($\mathcal{V}^R_{(0)}$ and $\mathcal{U}^R_{(0)}$).
  - **Step 2:** Set $i \to i+1$. Fixed partition $\mathcal{U}^R_{(i-1)}$ → eq.(1) $\mathcal{V}^R_{(i)}$. Fixed codebook $\mathcal{V}^R_{(i)}$ → eq.(2) $\mathcal{U}^R_{(i)}$. If the realizability condition $u^R_1 \leq u^R_2 ... \leq u^R_L$ is not met stop the procedure and keep the transition levels provided by the previous iteration.
  - **Step 3:** Update ϵ as follows $\epsilon = \dfrac{\sum_{k=1}^{L} \left| v^R_{k(i)} - v^R_{k(i-1)} \right|}{\sum_{k=1}^{L} \left| v^R_{k(i)} \right|}$. If $\epsilon \geq \epsilon_{max}$ then go to Step 2; stop otherwise.
- **Convergence analysis:**
  - The convergence to at least a local minimum is proved if one assumes a zero-mean channel input and a gaussian noise on the backward channel (S-R) under the following condition

$$\forall \ell, \;\; E[\hat{X}^R_{rd} | \hat{X}^R_{sr} = v^R_{\ell+1}] > E[\hat{X}^R_{rd} | \hat{X}^R_{sr} = v^R_{\ell}]$$

MAXIMUM LIKELIHOOD COMBINER FOR QF(1/2)

- A conventional MRC can lead to a performance loss due to quantization noise when the relay is in bad condition with a low number of quantization bits.

- The likelihood $p_{ML} = p(y_{sd}, \hat{x}_{rd}|x)$ can be factorized as:

$$p_{ML} = p(y_{sd}|x)p(\hat{x}_{rd}|x)$$

- Where $p\left(y_{sd}|x\right) = \frac{1}{\pi\sigma_{sd}^2}\exp\left(-\frac{|y_{sd}-h_{sd}x|^2}{\sigma_{sd}^2}\right)$ and

$$\begin{aligned} p(\hat{x}_{rd} = v_i|x) &= \int_{x_{sr}} p(x_{sr}, \hat{x}_{rd} = v_i|x)\ dx_{sr} \\ &= \int_{x_{sr}} p(x_{sr}|x)\ p(\hat{x}_{rd} = v_i|x_{sr})\ dx_{sr} \\ &= \sum_{\ell=1}^{\sqrt{(M_r)}} \sum_{m=1}^{\sqrt{(M_r)}} P_{j,i} \left[\int_{u_\ell^R}^{u_{\ell+1}^R} \phi\left(t - x^R\right) dt \int_{u_m^I}^{u_{m+1}^I} \phi\left(t' - x^I\right) dt'\right] \end{aligned}$$

MAXIMUM LIKELIHOOD COMBINER FOR QF (2/2)

- **Denote by $\underline{s} = (s_1, ..., s_N)$ the vector of bits associated with the source symbol x.**

- **The log-likelihood ratio associated to the n^th bit:**

$$\lambda(s_n) = \log \left[\frac{\sum_{\underline{s} \in S_1^{(n)}} p(y_{sd} \mid x)\, p(\hat{x}_{rd} \mid x)}{\sum_{\underline{s} \in S_0^{(n)}} p(y_{sd} \mid x)\, p(\hat{x}_{rd} \mid x)} \right]$$

where

$$S_1^{(n)} = \{(s_1, \ldots, s_N) \in \{0,1\}^N \mid s_n = 1\}$$

$$S_0^{(n)} = \{(s_1, \ldots, s_N) \in \{0,1\}^N \mid s_n = 0\}$$

If $\lambda(s_n) > 0$ then $\hat{s}_n = 1$ and $\hat{s}_n = 0$ otherwise

CLIPPED AF PROTOCOL (1/3)

- Advantage: **Simple clipping function depending on only one parameter to be optimized β**

- Clipping function at the relay:

Real part of the signal

$$f_\beta^R(x^R) = \begin{vmatrix} x^R & |x^R| \le \beta \\ \beta \cdot \mathrm{sgn}(x^R) & |x^R| > \beta \end{vmatrix}$$

Transmitted signal at the relay

$$x_r(n) = \alpha \left[f_\beta^R\left(x_{sr}^R(n)\right) + j \cdot f_\beta^I\left(x_{sr}^I(n)\right) \right]$$

Function of β: satisfy the
unit-power constraint

CLIPPED AF PROTOCOL (2/3)

- Optimization criteria: **Minimization of the end-to-end distortion**

$$
\begin{aligned}
J(\beta) &= E\left[\left|\frac{1}{\alpha_\beta}\frac{h_{rd}^*}{|h_{rd}|^2}y_{rd} - x\right|^2\right] \\
&= \frac{2}{\sqrt{M_s}}\sum_{x^R}\left\{E\left[f_\beta^2(x^R + w_{sr}^R/h_{sr}) - 2x^R f_\beta(x^R + w_{sr}^R/h_{sr})\right] + \frac{\sigma_{rd}^2}{\alpha_\beta^2|h_{rd}|^2} + 1\right\}
\end{aligned}
$$

$\Downarrow\Downarrow$

Numerical optimization based on the knowledge of the CSI and SNRs at the relay for its backward and forward channels.

CLIPPED AF PROTOCOL (3/3)

- Let $z \sim \mathcal{N}(\mu, \sigma^2)$ and $f_\beta(\cdot)$ be the clipping function.

  For $\beta = 1$, the first and the second order moments of a clipped gaussian signal are given by:

$$
\begin{aligned}
E[f_1(z)] &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} f_1(z) e^{-\frac{(z-\mu)^2}{2\sigma^2}} dz \\
&= \mu + \frac{1}{\sqrt{2\pi}} \sigma e^{-\frac{(1+\mu)^2}{2\sigma^2}} - e^{-\frac{(1-\mu)^2}{2\sigma^2}} - \mu \left[Q\left(\frac{1+\mu}{\sigma}\right) + Q\left(\frac{1-\mu}{\sigma}\right) \right] - Q\left(\frac{1+\mu}{\sigma}\right) + Q\left(\frac{1-\mu}{\sigma}\right)
\end{aligned}
$$

$$
\begin{aligned}
E[f_1^2(z)] &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} f_1^2(z) e^{-\frac{(z-\mu)^2}{2\sigma^2}} dz \\
&= (\sigma^2 + \mu^2) - \frac{1}{\sqrt{2\pi}} \sigma \left[(1+\mu) e^{-\frac{(1-\mu)^2}{2\sigma^2}} + (1-\mu) e^{-\frac{(1+\mu)^2}{2\sigma^2}} \right] + (1 - \sigma^2 - \mu^2) \left[Q\left(\frac{1+\mu}{\sigma}\right) + Q\left(\frac{1-\mu}{\sigma}\right) \right]
\end{aligned}
$$

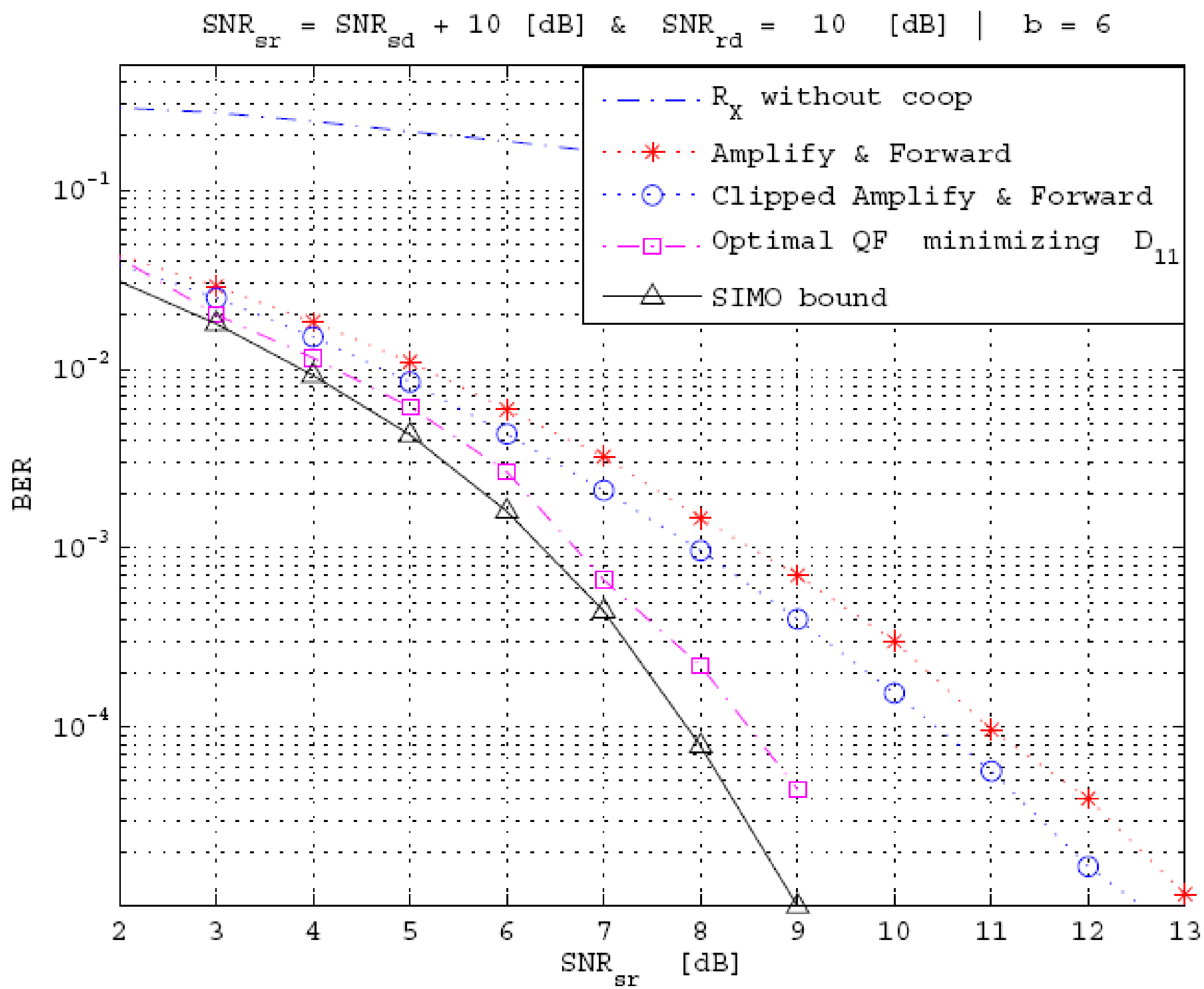
where Q is the classical error function: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2} dt$

SIMULATION RESULTS (1/4)

- **Simulation parameters:**
  - **Constant spectral efficiency.**
  - **We consider a 4-QAM source and static (gaussian or purely rician) or quasi-static (Rayleigh block fading model) channels.**
  - **Number of quantization bits for the QF:** $b \in \{2, 6\}$ → 2^b**-QAM at the relay.**
  - **For reference we consider the case when no relay is available and the full cooperation (**$\sigma_{rd} \rightarrow 0$**).**

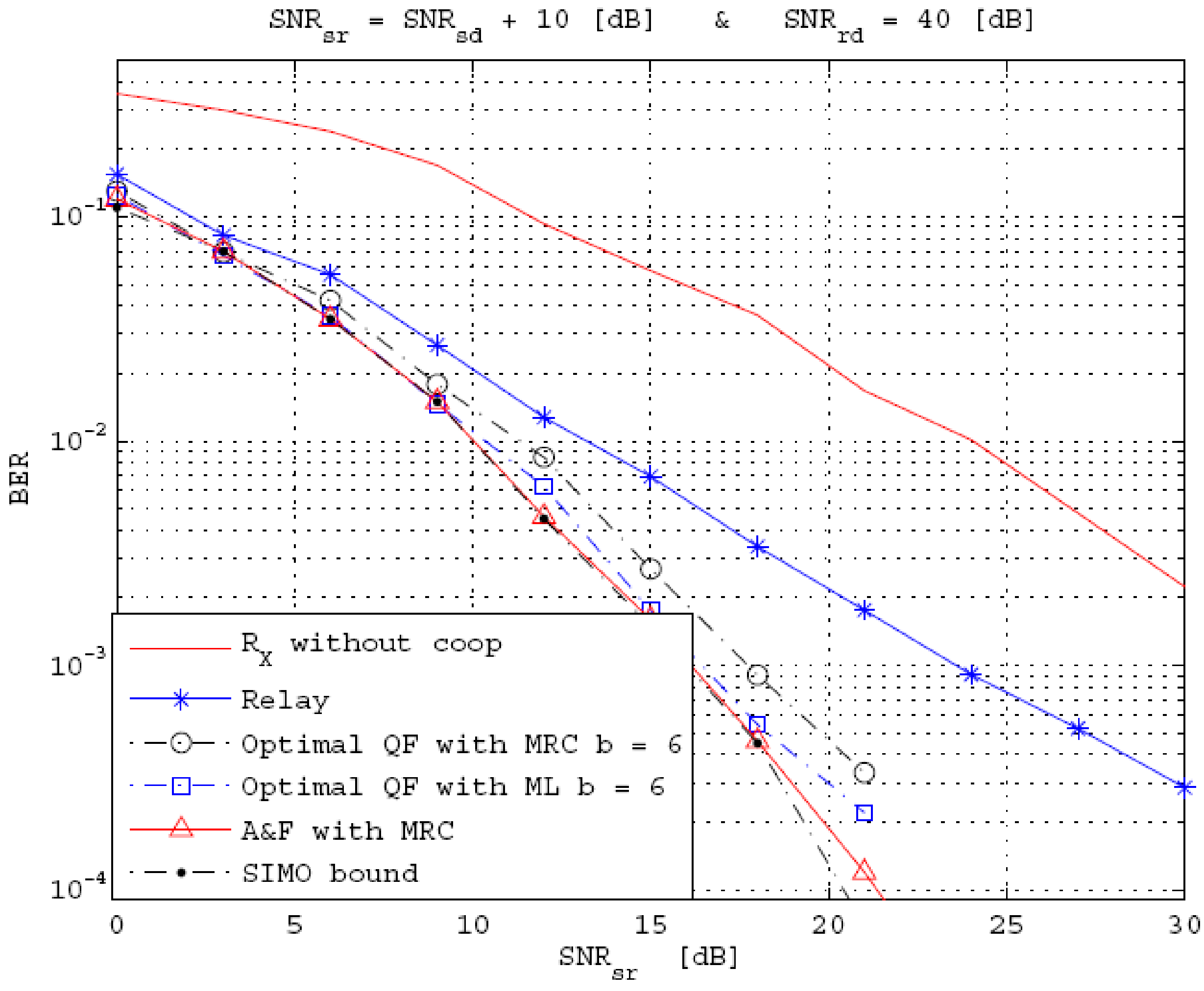
SIMULATION RESULTS (2/4)

- Static channels with MRC combiner for all schemes
  - QF → Significant gain over AF and C-AF
  - Clipping gain



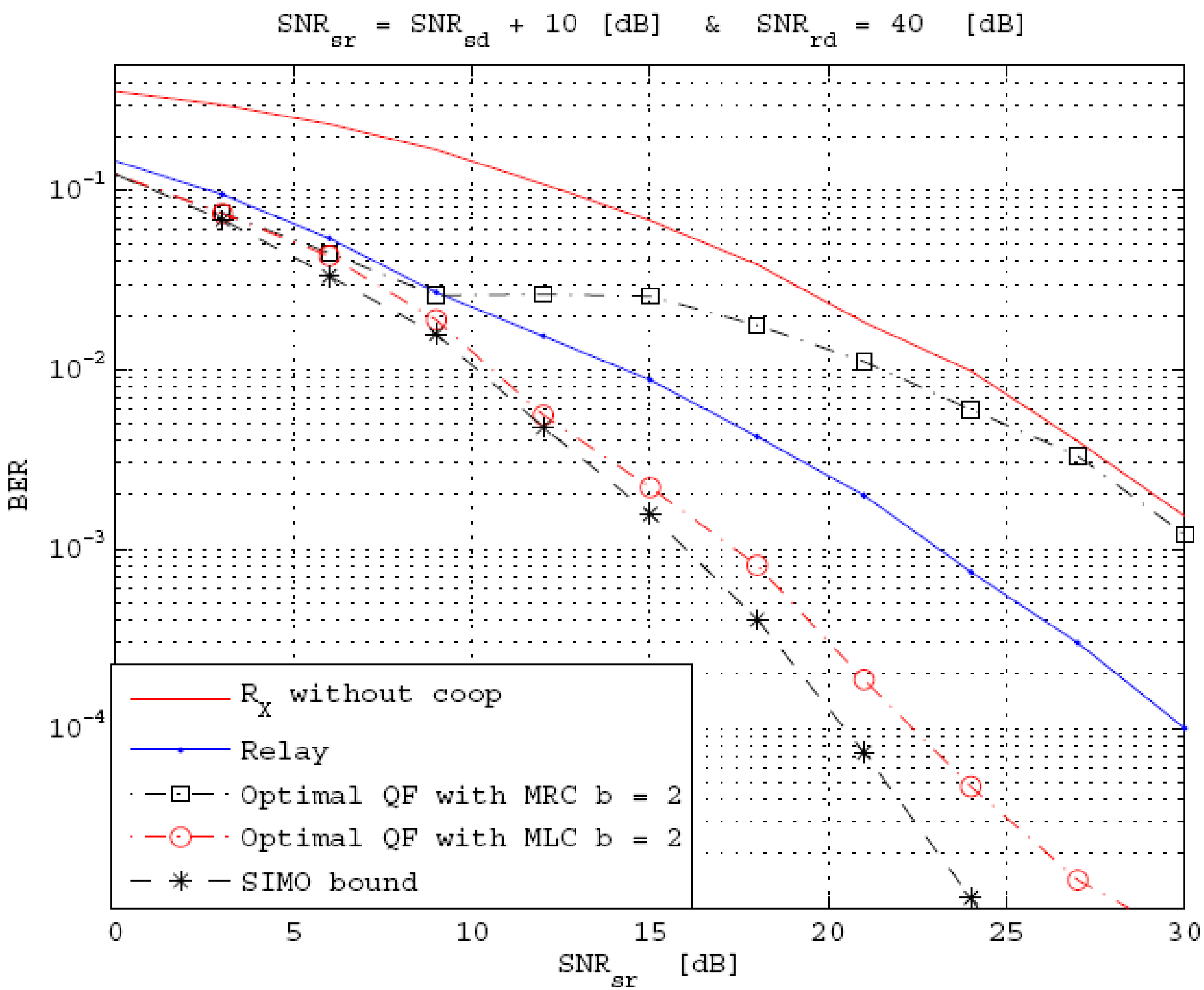
SIMULATION RESULTS (3/4)

- Block fading channels
  - Quasi-similar performance for these relaying schemes



SIMULATION RESULTS (4/4)

- Block fading channels: influence of the combining scheme
  - MLD → significant performance gain
  - Receiver structure adapted to the assumed relaying scheme



CONCLUSION

- **Low-complexity QF:**
  - knowledge of CSIs and SNRs at the relay.
  - Better or similar performance in static channels.

- **Clipped AF**
  - Knowledge of CSIs and SNRs at the relay.
  - Performance gain w.r.t. the conventional AF.

- **ML detector**
  - QF protocol → similar performance in quasi-static fading channel

- **QF protocol** → channel optimized AF-type protocol in a digital relay transceiver

- **Extension:** exploit the structure of channel coding.

COMPLEXITY ANALYSIS

	Optimal Quantizer	**Uniform Quantizer**
Creation*	$\max\{\mathcal{O}(cL^2A^{2/3}), \mathcal{O}(cS\sqrt{M_s}LA^{2/3}), \mathcal{O}(cS\sqrt{M_s}L^2)\}$	$\max\{\mathcal{O}(SL^2\sqrt{M_s}), \mathcal{O}(S\sqrt{M_s}LA^{2/3})\}$
Storage*	$\mathcal{O}(L)$	$\mathcal{O}(L)$
Computation**	$\mathcal{O}(L)$	$\mathcal{O}(L)$

(*): per SNR value; (**): per symbol to quantize

c: nb of iterations, A: accuracy in nb of used digits, S: nb of tested points in the exhaustive search