Energy Efficient Communications in CDMA Networks: A Game Theoretic Analysis Considering Operating Costs

Sharon M. Betz

Supélec

23 September 2008

Work done at Princeton under H. Vincent Poor
Funded by the Intel Corporation

Energy Efficient Communication in Wireless Networks

- Have large network of wireless nodes
- Communication scheme must be
  - Energy-efficient
  - Scalable
  - Simple

Energy Efficient Communications in Multihop and Peer-to-Peer CDMA Networks

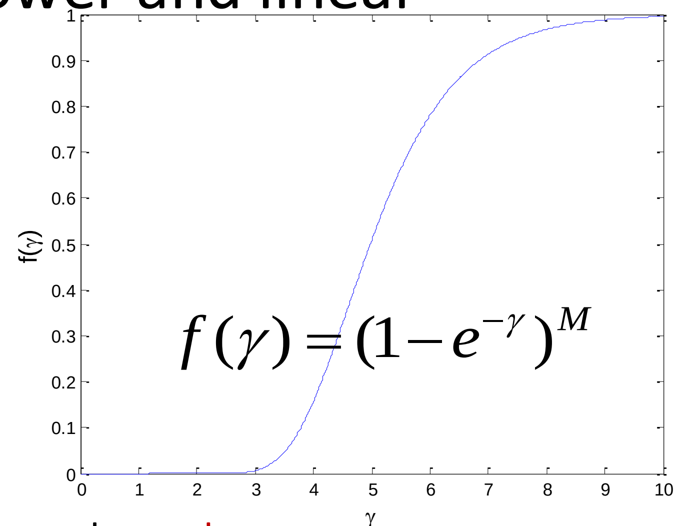
- Problem:
  - Large data network with established multi-hop or peer-to-peer routing
  - Energy-limited nodes
  - Interference
  - **How much power should nodes use to transmit?**
  - **What receiver should nodes use?**
- Game:
  - Distributed non-cooperative game
  - K players: transmitting nodes
  - Strategy set for each node is its transmit power and linear receiver
  - Nodes can choose transmit powers in the set $[0,P_{max}]$
  - Utility in bits per joule
  - Synchronous DS-CDMA with processing gain N
  - Randomly chosen spreading sequences
  - AWGN channels

Utility Function

- Each node k chooses its transmit power and linear receiver to maximize its utility:

$$u_k = R\frac{f(\gamma_k)}{p_k}$$

- u_k: utility for user k
- R: transmission rate
- p_k: transmit power for user k
- γ_k: SINR for user k
- $f(\cdot)$: efficiency function



$$f(\gamma) = (1 - e^{-\gamma})^M$$

- $f(\cdot)$ approximates the packet success rate.
- Any function that is
  - increasing
  - continuously differentiable
  - sigmoidal (S-shaped)
  - $f(0) = 0$
  - $f(+\infty) = 1$

Captures trade-off between throughput & energy consumption (battery life).

Non-Cooperative Games

- Full game: Each node, k, chooses
  - its receiver's linear coefficients, $\boldsymbol{c}_k$, and
  - its transmit power, p_k

  to maximize its own utility, u_k

- Power control games: Each node, k,
  - has a set linear receiver
    - MF: matched filter
    - DE: decorrelating receiver (cancels out all interference)
    - MMSE: minimum mean-squared error receiver
  - chooses its transmit power, p_k

  to maximize its own utility, u_k

Non-Cooperative Power-Control Game

- Nash equilibrium: set of strategies such that no user can unilaterally improve its own utility (stable state)
- For the MF, DE, and MMSE receivers, a unique Nash equilibrium exists
- At equilibrium, all users have equal SINRs, the unique positive number, γ_k^*, that satisfies:

$$f\left(\gamma_k^*\right)=\gamma_k^* f'\left(\gamma_k^*\right)$$

Cooperative Game: Global Optimization Problem

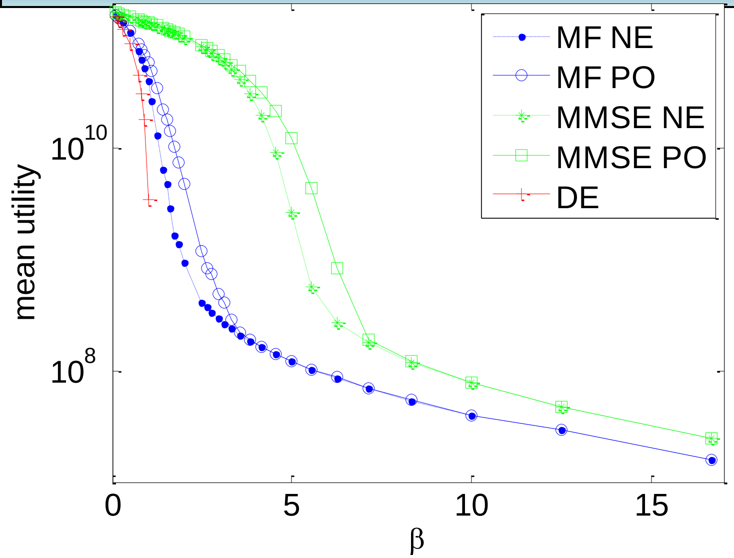
- Maximize the weighted sum of all utilities, where the a_k's are set weighting variables, and the u_k's are the nodes utilities:

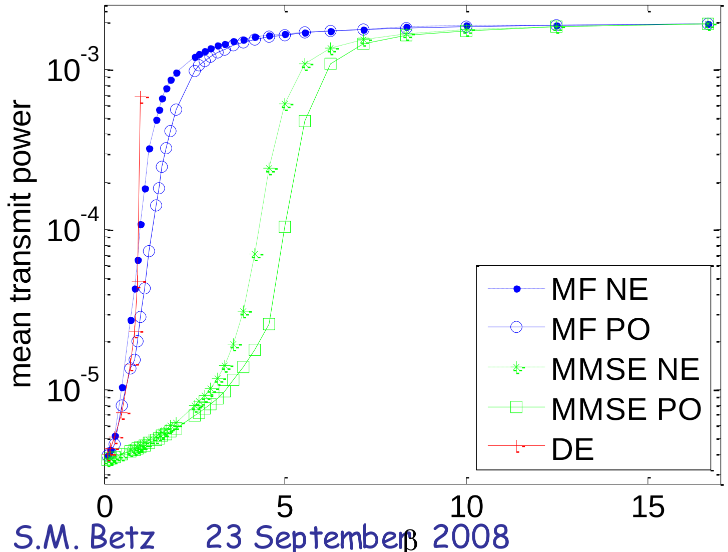
$$\max_{\{p_k\}} \sum_k a_k u_k$$

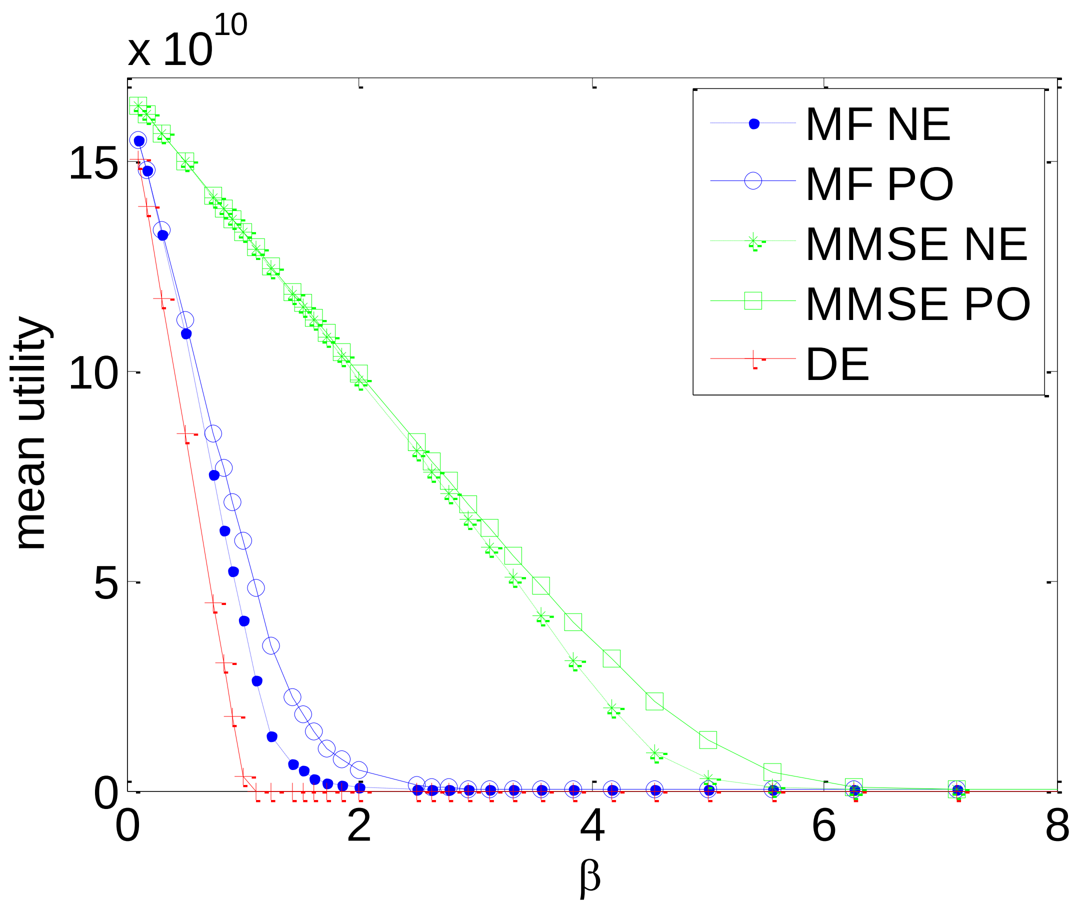
- Pareto optimal: An allocation of resources such that there is no other allocation of resources that is better for some user and at least as good for all users. (The situation cannot be improved for one user without harming another user.) (cooperative / centralized)

- For simplicity and fairness, require that all nodes have equal receiver output SINR. Then the problem is: $\max f(\gamma) \sum_k \frac{a_k}{p_k}$

Simulation Results: Finite Ad Hoc Network







- K = 50 transmitting nodes
- P_{max} = 2 mW
- Load: β=K/N

Utility Function Considering Operating Costs

- Each node *k* chooses its transmit power and linear receiver coefficients to maximize the utility function:

$$u_k = R\frac{f(\gamma_k)}{p_k + q_k}$$

- q_k: non-transmit power for user *k*

Nash Equilibrium Existence and Uniqueness

- **Theorem**: A Nash equilibrium exists. If ***p'*** is a Nash equilibrium point,

$$p_k' = \begin{cases} I_k(\mathbf{p}') & I_k(\mathbf{p}') < P_{\max} \\ P_{\max} & \text{else} \end{cases}$$

  where ***I(p)*** is the unique vector, the interference function [Yates 1995], such that $p_k = \boldsymbol{I_k(p)}$ maximizes $f(\gamma_k)/(p_k+q_k)$ when all other users use the powers in ***p.***

- **Theorem**: If the interference function is monotonic: $\boldsymbol{p} \geq \boldsymbol{p'} \rightarrow \boldsymbol{I(p)} \geq \boldsymbol{I(p')}$ then the Nash equilibrium is unique.
  - **Corollary**: With the efficiency function $f(\gamma) = (1-e^{-\gamma})^M$ for M > 4, the Nash equilibrium is unique.

Optimal SINR

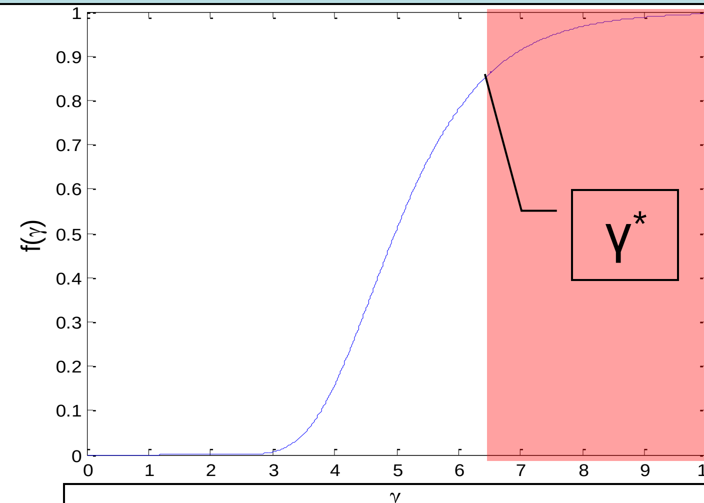
- When operating energy is not taken into account ($\boldsymbol{q}=0$):
  - all users have the same selfishly-optimal SINR
  - independent of the channel
  - unique solution to $f(\gamma)=\gamma \cdot f'(\gamma)$

- When operating energy is taken into account ($q_k > 0$):
  - users have different selfishly-optimal SINRs
  - if the combination of noise and interference seen by user *k* increases, its goal SINR decreases.

Effect of Operating Cost on Performance

- **Lemma**: As a node's operating power (q_k) increases, so does the selfishly-optimal transmit power (p_k) and selfishly-optimal SINR (γ_k).
- **Lemma**: A node's utility increases if it encounters less interference and noise.
- **Theorem**: For any users *j* and *k* (including *j=k*), $du_j/dq_k \leq 0$.
  - The inequality is strict except for the DE with j≠k.

➢ Increasing any node's operating power decreases the utility of all nodes (except with DE, where other nodes are unaffected)



γ* is the solution to $f(\gamma) = \gamma f'(\gamma)$

Pareto Optimality

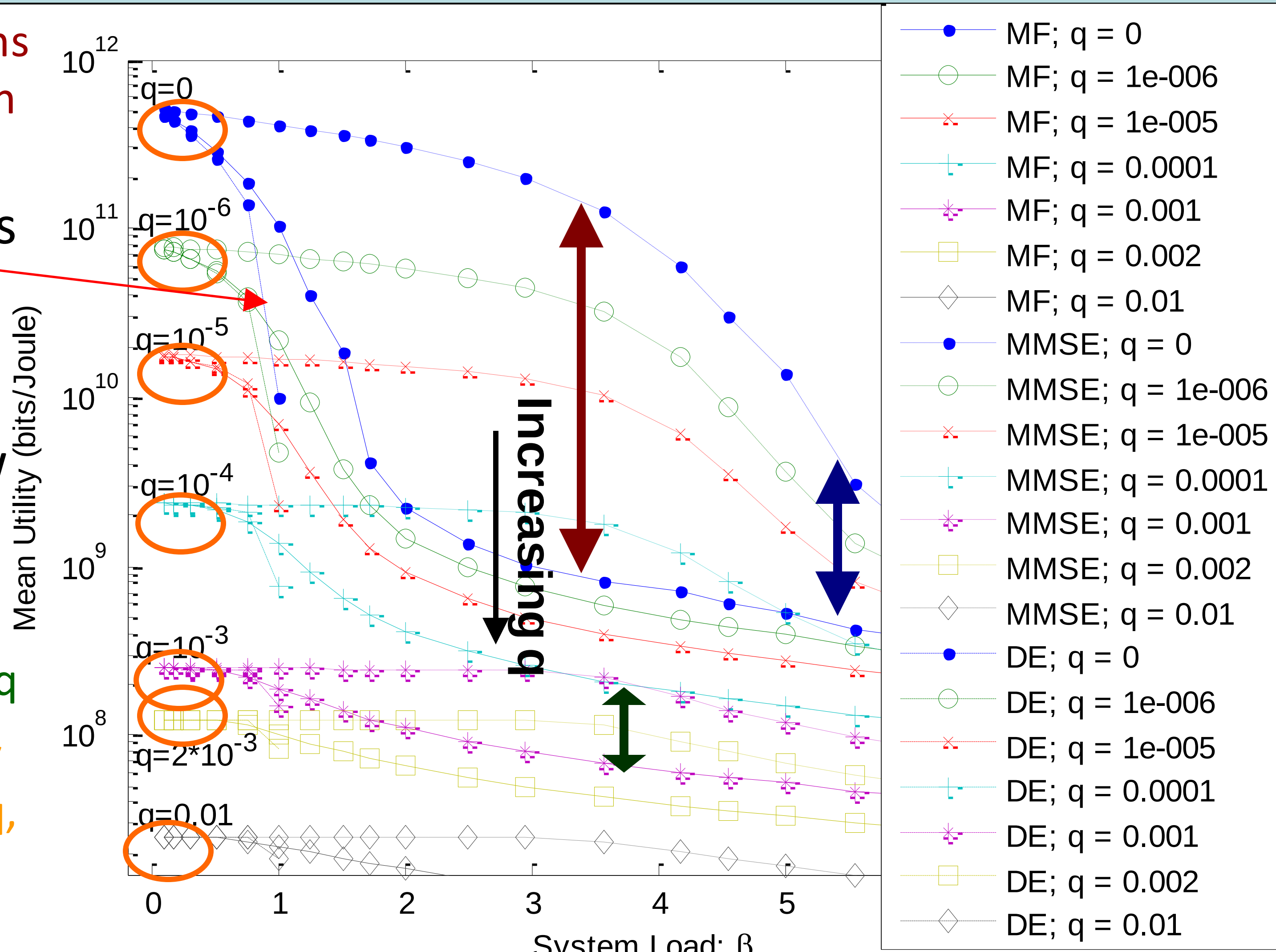
- DE: Nash equilibrium is Pareto optimal
- MMSE & MF: Nash equilibrium is NOT Pareto optimal

- **Theorem**: If every node decreases its transmit power by a small fraction, every node achieves higher utility.

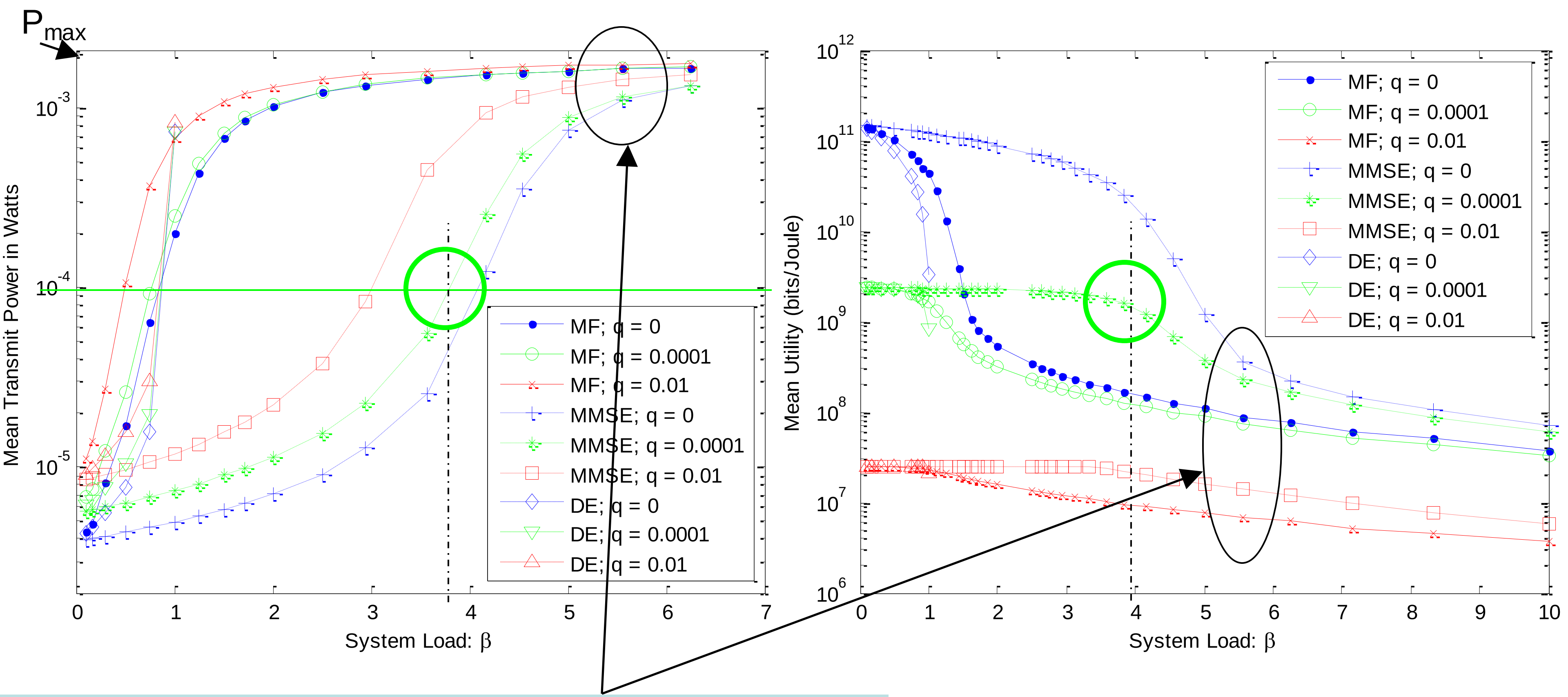
Multihop Simulation Results: Mean Utility With Operating Costs

- K = 50 nodes
- 1 km^2 square
- 100 bit packets
- No overhead
- R = 250 kb/s
- $\sigma^2 = 8 \cdot 10^{-13}$ W
- Line of sight channels

- MMSE outperforms MF, especially with β>1 & small q
- DE most affected by load, esp. for small q
- Performance gap decreases for large β
- Performance gap smaller for larger q
- At low load, utility depends only on q, not receiver type



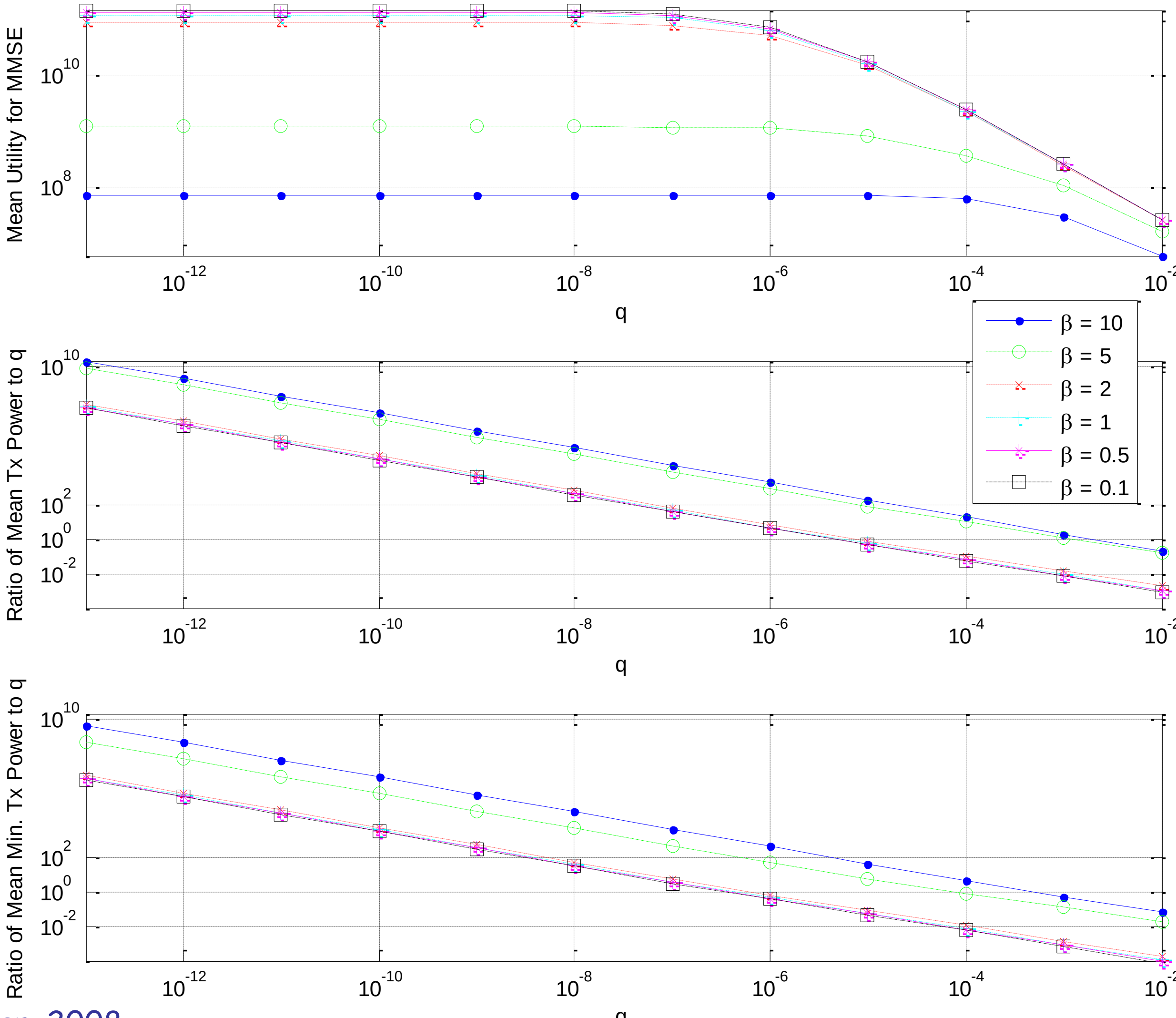
Multihop Simulation Results: Effect of Transmit Power on Utility



As β increases, more nodes transmit at P_{max}: utility gap is mostly due to SINR difference

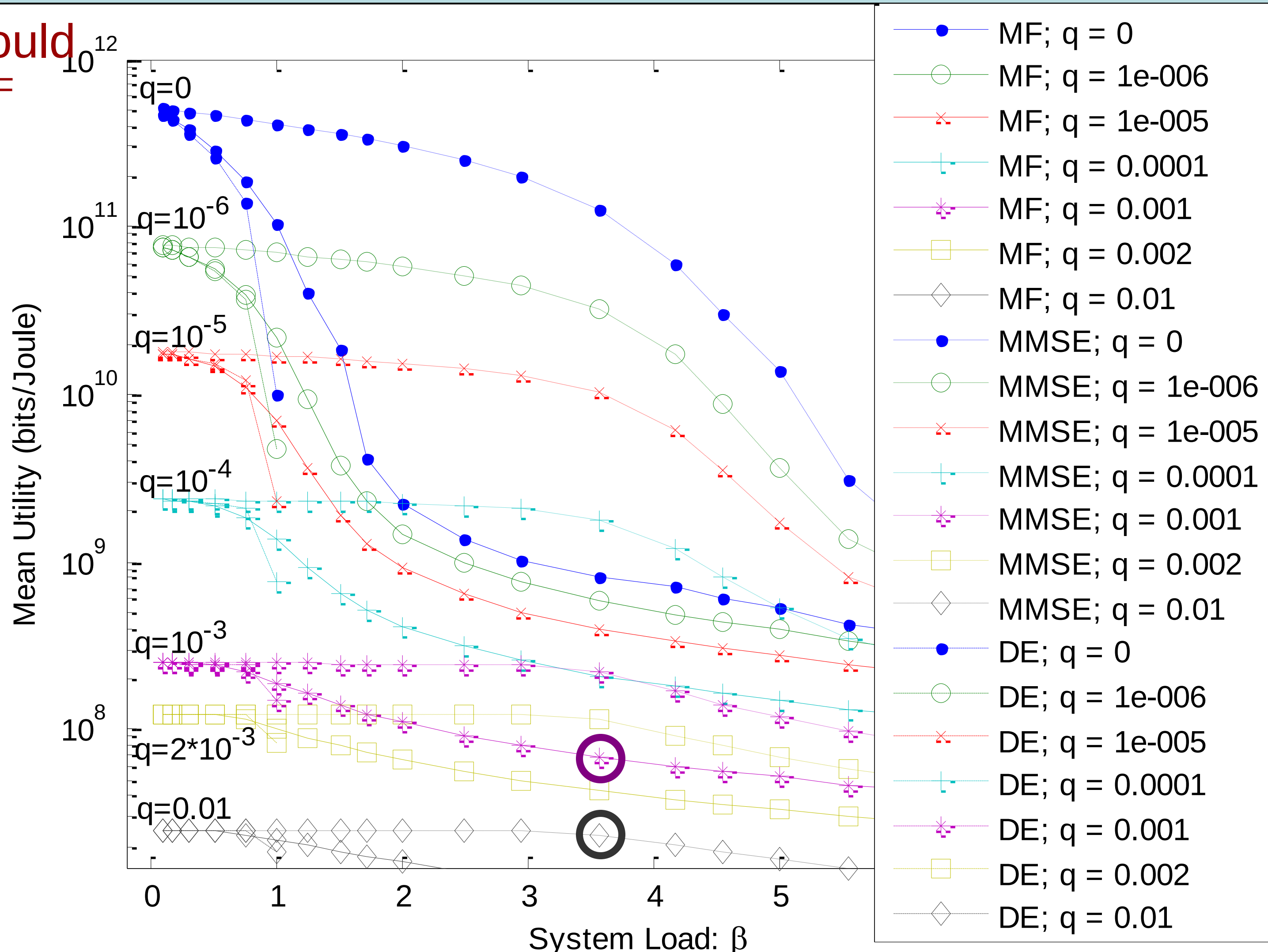
As average transmit power exceeds q, utility decreases

Multihop Simulation Results: Effect of Operating Energy on MMSE performance

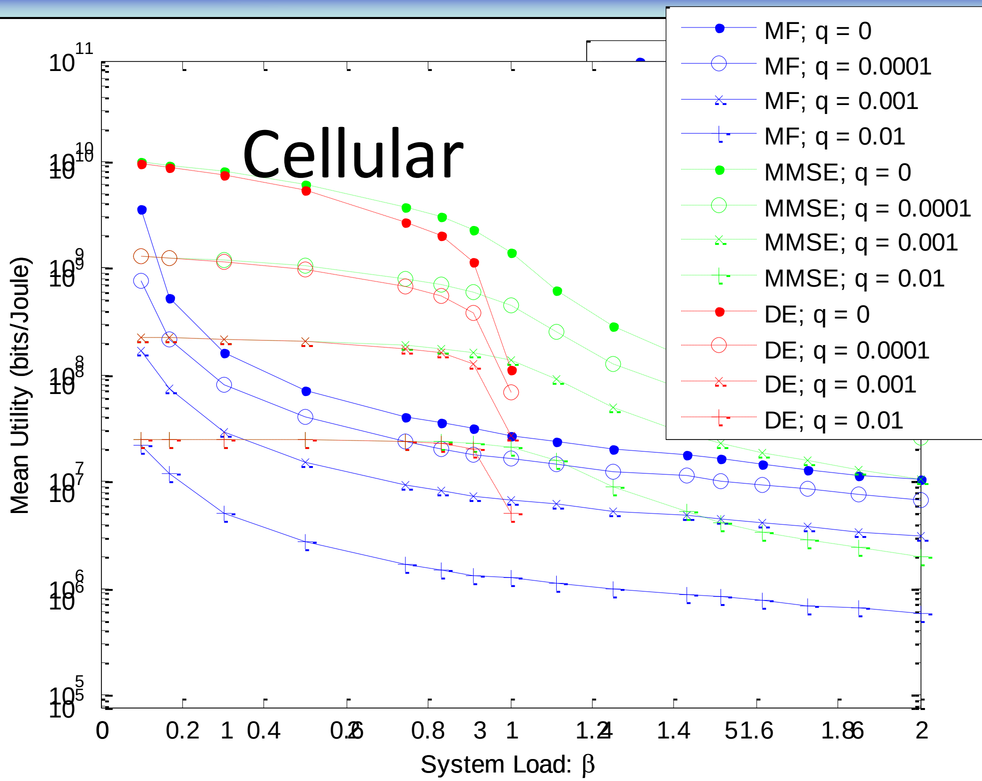


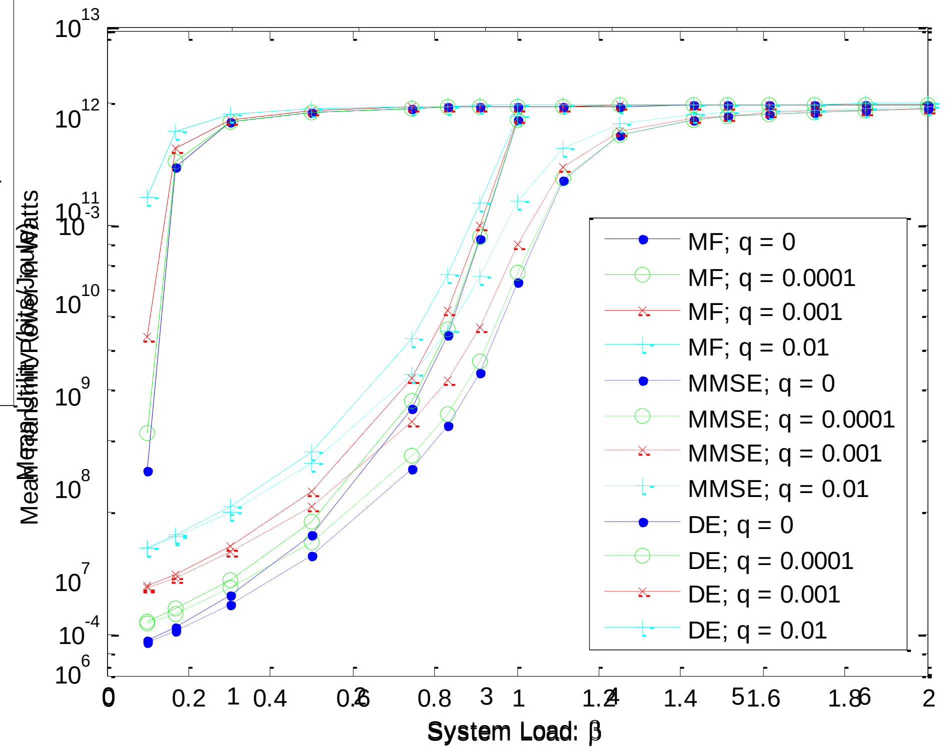
Multihop Simulation Results: Mean Utility With Receiver-Dependent Operating Costs

- If $q^{MMSE} > q^{MF}$, could have $u^{MMSE} < u^{MF}$
- $q^{MMSE} = 0.01$ W
  $q^{MF} = 0.001$ W
  $\beta = 3.5$



Cellular Simulation Results: Mean Utility With Operating Costs





1. Longer distances
2. Increased interference
3. Better relative DE performance

Conclusions

- In the noncooperative game where each node tries to maximize the ratio of its throughput to its total power, a unique Nash equilibrium exists
- For the MMSE and MF receivers, all nodes perform better if all nodes decrease their transmit powers from the Nash equilibrium
- All nodes achieve higher utility when any node's operating energy is decreased
- Minimizing the operating energy may be more important than minimizing the transmit energy
- Multihop networks using MMSE or MF receivers have improved performance for high loads due to decreased interference

Future Work With This Model (1)

- Extend analyses to more complicated systems
  - QoS constraints
  - Fading channels
  - Inter-symbol interference
  - Non-linear multiuser detectors
  - Allow nodes to modify rates, modulation designs, or spreading codes

Future Work With This Model (2)

- Optimize multi-hop networks
  - Optimize routing and power-control
  - Optimize end-to-end utility
- Improve Nash equilibrium using pricing schemes
- Apply Nash bargaining or coalition game theory to improve sum utility
- Analyze Pareto optimal solutions for the various games without requiring SINR-balancing