Talk at Supélec

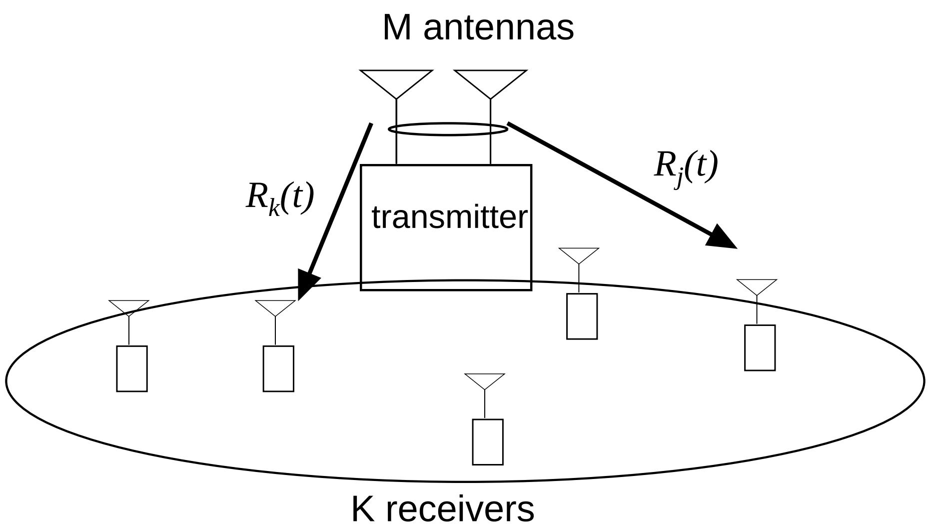
Stability Optimal Resource Allocation in Gaussian MIMO-BC : Theory and Practical Approaches

Mari Kobayashi

Centre Tecnològic de Telecomunicacions de Catalunya, Spain

mari.kobayashi@cttc.es

We consider resource allocation problem in the wireless downlink



Question : What is the optimal scheduling policy ?

Answer : choice depends on the availability of CSIT

Outline

1. Motivation
2. Well-known results
   - MAC-BC duality
   - Stability
3. Perfect CSIT case
   - DPC-based approach
   - Linear beamforming
4. Non-perfect CSIT case
   - Linear beamforming with analog feedback
   - Other practical schemes
5. Conclusions and future works

I.Motivation

1. Motivation (1/3)

- Increasing demand for high-rate data-oriented downlink schemes (e.g. HSDPA, 1xEV-DO)
- Multi-Input Multi-Output (MIMO) has emerged as an attractive solution especially in a multiuser setting $K >> M$
- With channel state information at transmitter, we can exploit
  multiplexing gain : $\lim_{\mathsf{SNR}\to\infty} R_{sum} = M \log \mathsf{SNR}$
  multiuser diversity : $\lim_{K\to\infty} R_{sum} = M \log\log K$
- A resource allocation policy is defined by $\mathcal{S}, \mathcal{A}$
  - signaling scheme $\mathcal{S}$: determine a rate region $\mathcal{R}_{\mathcal{S}}$
  - rate scheduling $\mathcal{A}$: a mapping function $(\mathbf{w}(t), \boldsymbol{\alpha}(t)) \mapsto \mathbf{R}(t)$

    $\mathbf{w}(t)$: QoS parameters

    $\boldsymbol{\alpha}(t)$: channel state information at transmitter (CSIT)

    $\mathbf{R}(t)$: scheduled rate vector $\in \mathcal{R}_{\mathcal{S}}(t)$

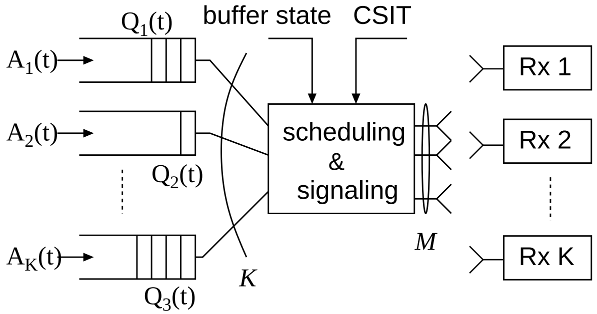
1. Motivation – Two downlink scenarios (2/3)

A queued downlink with random arrivals

- goal : stabilize the K buffer queues
- [Yeh03,Neely03,Boche04] "max-stability policy" maximizing

$$\sum_{k=1}^{K} Q_k(t) R_k(t)$$

achieves the maximum stability region



$Q_k(t)$: queue size of user k

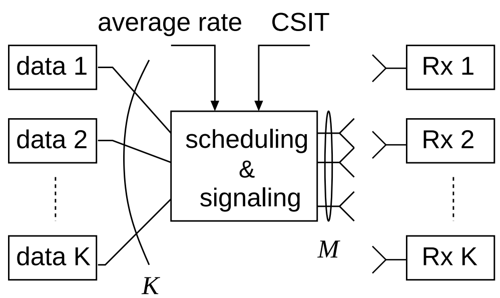
$$Q_k(t+1) = \max\{0, Q_k(t) - TR_k(t)\} + A_k(t)$$

An infinite backlogged downlink

- goal : maximize the throughput constrained to some *fairness*
- [Lau05] Proportional fair scheduling (PFS) maximizes

$$\sum_{k=1}^{K} R_k(t)/\overline{T}_k \approx \sum_{k=1}^{K} R_k(t)/T_k(t)$$

$\overline{T}_k$: the average rate of user k (unknown)



$T_k(t)$: empirical average rate of user k

$$T_k(t+1) = (1 - 1/t_c)\, T_k(t) + 1/t_c\, R_k(t)$$

1. Motivation – Weighted sum rate maximization (3/3)

In both scenarios, the main problem reduces to

Weighted sum rate maximization

$$\max \sum_{k=1}^{K} w_k(t) R_k(t), \quad \text{subject to : } \mathbf{R}(t) \in \mathcal{R}_{\mathcal{S}}(t)$$

for some non-negative time-varying weights $\{w_k(t)\}$

II. Well-known results

2. MIMO-MAC and MIMO-BC (1/7)

MIMO Gaussian Broadcast Channel (BC)

$$y_k(t) = \mathbf{h}_k^H(t)\mathbf{x}(t) + w_k(t), \quad k = 1, \ldots, K$$

where

- $w_k(t) \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ is AWGN, i.i.d over k
- $\mathbf{x}(t) \in \mathbb{C}^M$ is transmit vector
- $\mathbf{h}_k(t) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_M)$ is i.i.d. over k
- Power constraint : total transmit power $\mathbb{E}[\mathbf{x}^H\mathbf{x}] \leq P$

2. MIMO-MAC and MIMO-BC (2/7)

"Dual" MIMO Gaussian Multiple Access Channel (MAC)

$$\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{h}_k(t) x_k(t) + \mathbf{w}(t)$$

where

- K transmitters with a single antenna and M-antenna receiver
- $\mathbf{w}(t) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$ is AWGN
- Power constraint : total power $\sum_k \mathbb{E}[|x_k|^2] \leq P$, or individual power $\mathbb{E}[|x_k|^2] \leq P_k$

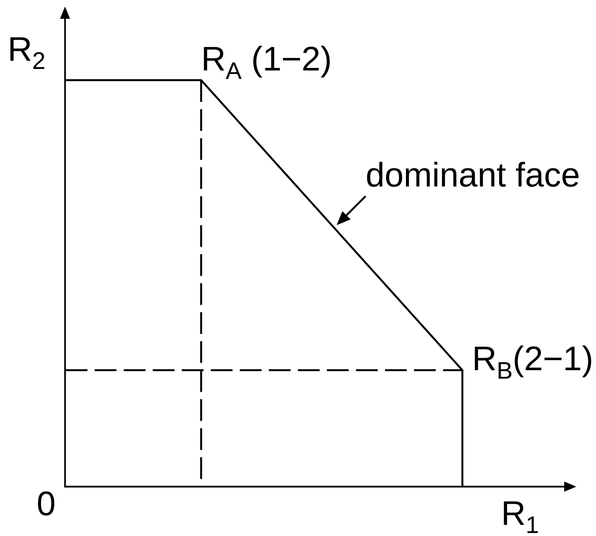
2. MIMO-MAC capacity region $\mathcal{R}_{\mathrm{MAC}}(\mathbf{H};\mathbf{p})$ (3/7)

- $\mathcal{R}_{\mathrm{MAC}}(\mathbf{H};\mathbf{p})$ = set of $\mathbf{R} \in \mathbb{R}^K$ s.t.

$$\sum_{k\in\mathcal{K}} R_k \leq \log \left| \mathbf{I} + \sum_{k\in\mathcal{K}} \mathbf{h}_k \mathbf{h}_k^H p_k \right| \quad \forall \mathcal{K} \subseteq \{1,\ldots,K\}$$

region is convex

- Polymatroid structure ($K!$ vertices on dominant face)



For $K=2$, pentagon

- R_A decoding order 1-2
  $R_{A,1} = \log(1 + p_1 \mathbf{h}_1^H (\mathbf{I} + p_2 \mathbf{h}_2 \mathbf{h}_2^H)^{-1} \mathbf{h}_1)$
  $R_{A,2} = \log(1 + p_2 \mathbf{h}_2^H \mathbf{h}_2)$
- R_B decoding order 2-1
  $R_{B,1} = \log(1 + p_1 \mathbf{h}_1^H \mathbf{h}_1)$
  $R_{B,2} = \log(1 + p_2 \mathbf{h}_2^H (\mathbf{I} + p_1 \mathbf{h}_1 \mathbf{h}_1^H)^{-1} \mathbf{h}_2)$

2. How to achieve vertices of MIMO-MAC capacity region ?(4/7)

- A given vertex satisfies

$$R_k = \log \frac{\det\left(\mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_j \mathbf{h}_j^H p_j\right)}{\det\left(\mathbf{I} + \sum_{j=1}^{k-1} \mathbf{h}_j \mathbf{h}_j^H p_j\right)}$$

  achieved by

  - Gaussian codes at transmitters
  - successive interference cancellation (SIC) decoding
    decoding order : $K, \ldots, 1$
  - MMSE-DFE filtering $\mathbf{F} = (\mathbf{f}_1, \ldots, \mathbf{f}_K) \in \mathbb{C}^{M \times K}$

- User k "sees"

$$\mathbf{f}_k^H \mathbf{y} = \underbrace{\sum_{j=1}^{k-1} \mathbf{f}_k^H \mathbf{h}_j x_j}_{interference} + \underbrace{\mathbf{f}_k^H \mathbf{h}_k x_k}_{useful} + \underbrace{\sum_{j=k+1}^{K} \mathbf{f}_k^H \mathbf{h}_j x_j}_{canceled} + \underbrace{\mathbf{f}_k^H \mathbf{w}}_{noise}$$

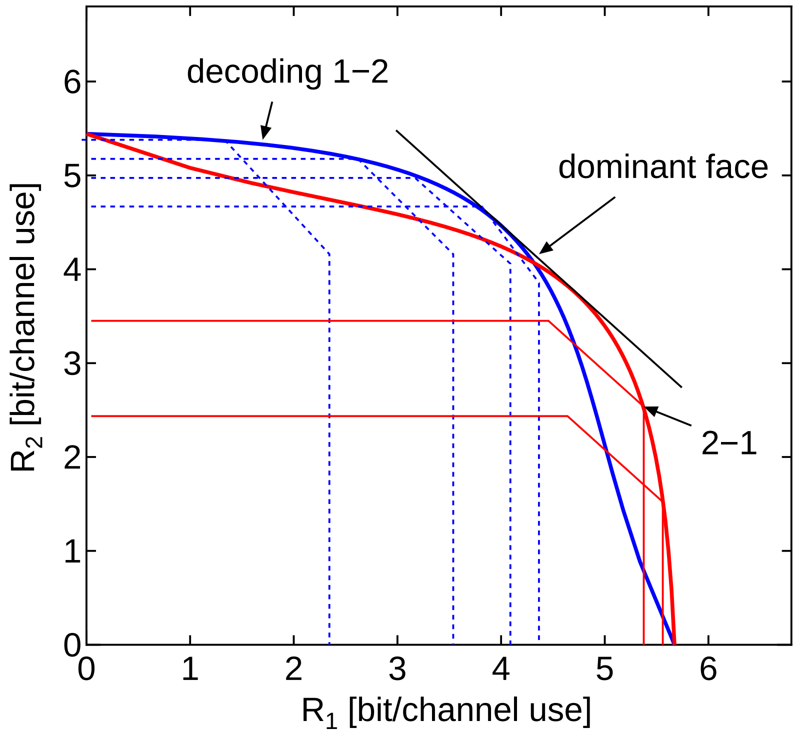
2. MIMO-MAC capacity region under sum-power constraint (5/7)

$$\mathcal{C}_{\text{MAC}}(\mathbf{H}; P) = \bigcup_{\sum_k p_k \le P} \mathcal{R}_{\text{MAC}}(\mathbf{H}; \mathbf{p})$$

Two-user example with $P = 10$ dB and

$$\mathbf{H} = \begin{bmatrix} 2 & -1 \\ -0.5 & 2 \end{bmatrix}$$

Let $p_2 = P - p_1$, and compute vertices R_A, R_B



2. MAC-BC Duality (6/7)

- Capacity region of MIMO-MAC and MIMO-BC coincide [Vishwanath-Tse 03, Jindal-Vishwanath-Goldsmith 03]

$$\mathcal{C}_{\mathrm{MAC}}(\mathbf{H};P) = \mathcal{C}_{\mathrm{BC}}(\mathbf{H}^H;P)$$

- $\mathcal{C}_{\mathrm{BC}}(\mathbf{H}^H;P)$ is achieved by Dirty-paper coding (DPC),i.e.
  - MMSE-DFE beamforming, $\mathbf{x} = \mathbf{F}\mathbf{u}$
  - $\mathbf{u}$ is SIC encoded (Gaussian code) with the reverse order as MAC

- User k sees

$$y_k = \underbrace{\sum_{j=1}^{k-1} \mathbf{h}_k^H \mathbf{f}_j u_j}_{pre-canceled} + \underbrace{\mathbf{h}_k^H \mathbf{f}_k u_k}_{useful} + \underbrace{\sum_{j=k+1}^{K} \mathbf{h}_k^H \mathbf{f}_j u_j}_{interference} + \underbrace{w_k}_{noise}$$

- Duality is useful to cast a downlink problem into a uplink problem, easier to handle

2. Stability and max-stability policy (7/7)

- A system is "stable" if all queues have a finite average size
- Stability region $\Omega =$ set of all arrival rates $\boldsymbol{\lambda}$ for which a causal policy can stabilize the system
- If $\{A_1(t), \ldots, A_K(t)\}$ and $\{\mathbf{H}(t)\}$ are jointly stationary ergodic Markov processes, Ω coincides with ergodic capacity region [Neely 03, Yeh-Cohen 03, Boche-Wiczanowski 03]
- "Max-stability policy" maximizing $\sum_k w_k(t) R_k$ where $w_k(t)$ is queue size of user k can stabilize any arrival rate inside Ω
- The policy needs no priori knowledge on $\boldsymbol{\lambda}$ and guarantees a finite average delay

III. Perfect CSIT case

3. Weighted sum rate maximization for MIMO-MAC (1/10)

Boche-Wiczanowski's result : the original problem can be casted into

$$\max_{\mathbf{R}\in\mathcal{C}_{\mathrm{MAC}}(\mathbf{H};P)} \sum_k w_k R_k = \max_{\mathbf{p}:\sum_k p_k = P} \sum_{k=1}^{K} w_k \log \frac{\det\left(\mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_j \mathbf{h}_j^H p_j\right)}{\det\left(\mathbf{I} + \sum_{j=1}^{k-1} \mathbf{h}_j \mathbf{h}_j^H p_j\right)}$$

where the optimal SIC decoding order $(K \to \cdots \to 1)$ sorts the weights

$$w_1 \geq w_2 \geq \cdots \geq w_K$$

The objective function becomes

$$f(\mathbf{p}) = \sum_{k=1}^{K} \Delta_k \log\det\left(\mathbf{I} + \sum_{j=1}^{k} \mathbf{h}_j \mathbf{h}_j^H p_j\right), \quad \Delta_k = w_k - w_{k+1} \geq 0, \ \forall k$$

convex optimization problem !

3. How to solve this convex problem ? (2/10)

- Define the k-th user's virtual channel gain when users K to $j+1$ are canceled

$$\alpha_{k,j} = \mathbf{h}_k^H \mathbf{\Sigma}_{k,j}^{-1} \mathbf{h}_k, \quad \text{where } \mathbf{\Sigma}_{k,j} = \underbrace{\mathbf{I}}_{noise} + \underbrace{\sum_{i=1, i\neq k}^{j} \mathbf{h}_i \mathbf{h}_i^H p_i}_{interference}$$

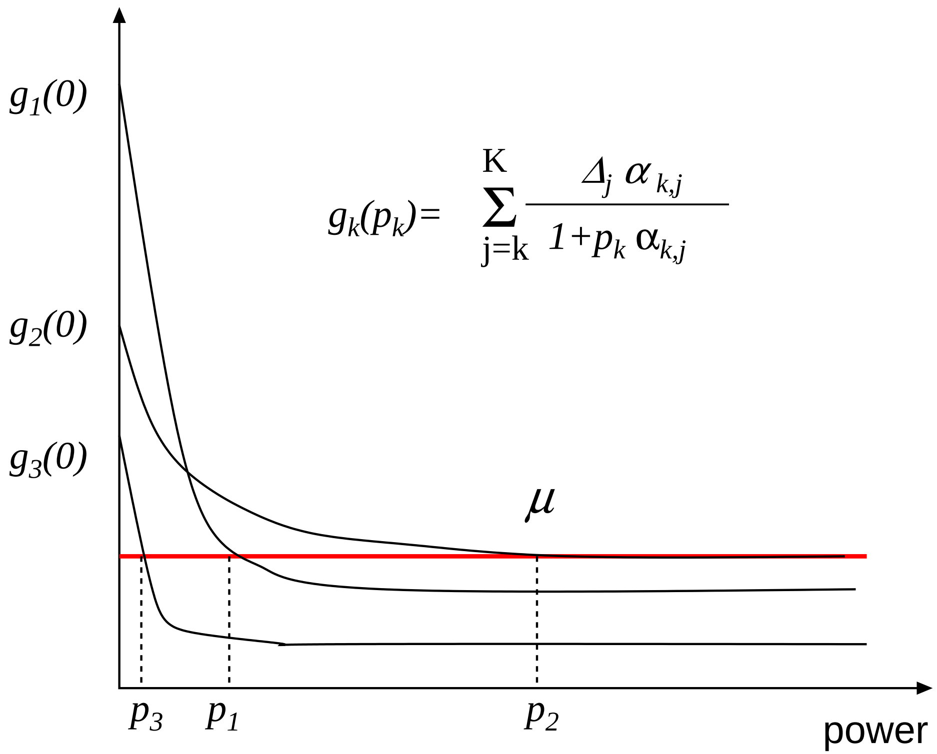
- Treating $\{\alpha_{k,j}\}$ as non-interfered parallel channel gains, consider

$$\max_{\mathbf{p}:\sum_k p_k = P} f_0(\mathbf{p}) \triangleq \sum_{j=1}^{K} \Delta_j \sum_{k=1}^{j} \log(1 + p_k \alpha_{k,j})$$

- KKT equality condition of $f_0(\mathbf{p})$ and $f(\mathbf{p})$ is identical, given by

$$\sum_{j=k}^{K} \frac{\Delta_j \alpha_{k,j}}{1 + p_k \alpha_{k,j}} = \mu > 0 \ \text{ for } k = 1, \dots, K$$

3. Waterfilling approach to solve KKT condition (3/10)



- objective : find a water level $\mu^\star > 0$ satisfying total power constraint $\sum_k p_k = P$
- $g_k(p_k)$ is a monotonic decreasing function of p_k, $\mu^\star$ is unique
- $\mu^\star$ can be found by line-search (bisection)

3. Iterative water-filling algorithm for WSR maximization (4/10)

1. Initialize $\mathbf{p}^{(0)} = \mathbf{0}$
2. At iteration n, compute for $k = 1, \dots, K$ and $j \geq k$ the coefficients

$$\alpha_{k,j}^{(n)} = \mathbf{h}_k^H \left(\mathbf{I} + \sum_{i=1, i \neq k}^{j} p_i^{(n-1)} \mathbf{h}_i \mathbf{h}_i^H \right)^{-1} \mathbf{h}_k$$

3. water-filling step: let $\boldsymbol{\gamma}^{(n)}$ be the solution of

$$\boldsymbol{\gamma}^{(n)} = \text{arg} \max_{\boldsymbol{\gamma} \geq \mathbf{0}, \sum_k \gamma_k \leq P} \sum_{j=1}^{K} \Delta_j \sum_{k=1}^{j} \log(1 + \gamma_k \alpha_{k,j}^{(n)})$$

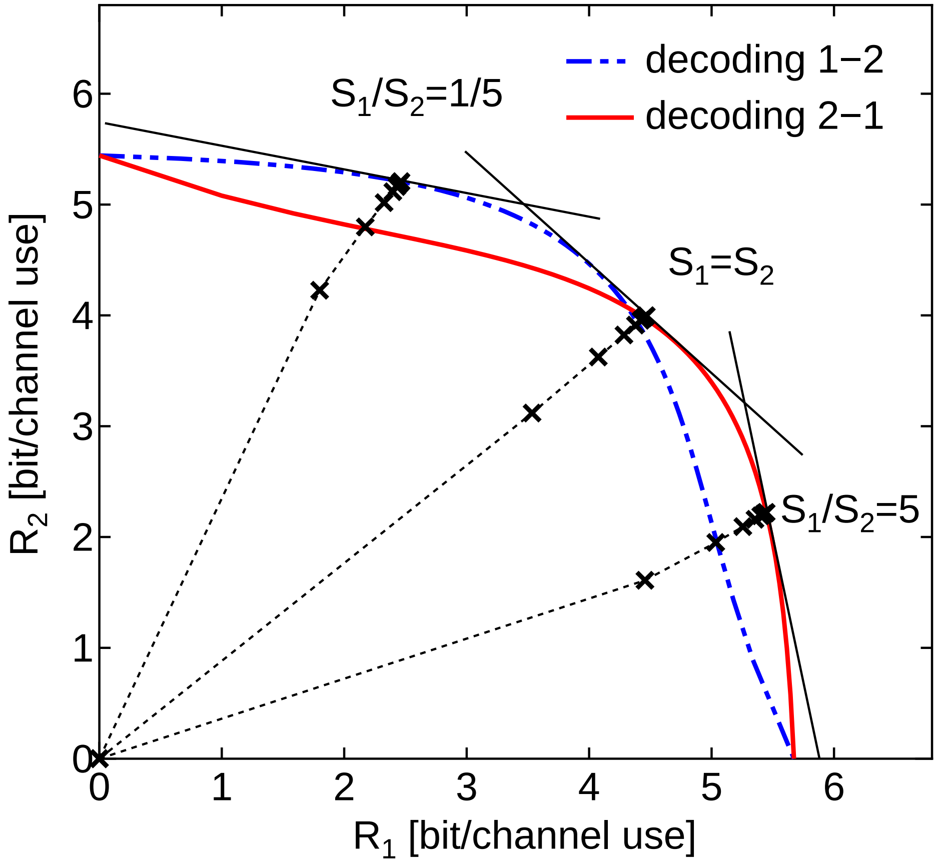
4. Update step: let $\mathbf{p}^{(n)} = \theta^\star \boldsymbol{\gamma}^{(n)} + (1 - \theta^\star) \mathbf{p}^{(n-1)}$ where

$$\theta^\star = \text{arg} \max_{\theta \in [1/K, 1-1/K]} f_0 \left(\theta \boldsymbol{\gamma}^{(n)} + (1 - \theta) \mathbf{p}^{(n-1)} \right)$$

Generalization of iterative MUW for sum-rate maximization [Jindal et al. 03]

3. Convergence of iterative waterfilling algorithm (5/10)

Two-user example with $P = 10$ dB and $\mathbf{H} = [2\ -1; -0.5\ 2]$



3. Proposed linear beamforming (6/10)

- The transmit vector is formulated by a unnormalized $\mathbf{G} \in \mathbb{C}^{M\times K}$

$$\mathbf{x} = \sqrt{\frac{P}{\mathrm{tr}(\mathbf{G}^H\mathbf{G})}}\mathbf{G}\mathbf{u}, \quad \text{with } \mathbb{E}[\mathbf{u}\mathbf{u}^H] = \mathbf{I}$$

- User k "sees"

$$y'_k = \underbrace{\mathbf{h}_k^H\mathbf{g}_k u_k}_{useful} + \underbrace{\sum_{j\neq k}\mathbf{h}_k^H\mathbf{g}_j u_j}_{interference} + \underbrace{z'_k}_{noise}, \quad z'_k \sim \mathcal{N}_{\mathbb{C}}(0, \mathrm{tr}(\mathbf{G}^H\mathbf{G})/P)$$

- Objective function : maximize

$$f_1(\mathbf{G}) = \sum_k w_k \log_2\left(1 + \frac{|\mathbf{h}_k^H\mathbf{g}_k|^2}{\mathrm{tr}(\mathbf{G}^H\mathbf{G})/P + \sum_{j\neq k}|\mathbf{h}_k^H\mathbf{g}_j|^2}\right)$$

non-convex optimization !

3. Proposed linear beamforming (7/10)

- Consider heuristics for the weighted sum rate maximization
- By letting the gradient $\nabla f_1(\mathbf{G}) = \mathbf{0}$, we obtain [Stojnic Vikalo Hassibi 04]

Iterative beamforming algorithm (SHV)

1. Initialize $\mathbf{G}^{(0)} = \mathbf{H}^H$
2. At iteration n, update $\mathbf{G}^{(n)}$

$$\mathbf{G}^{(n)} = \left(\mathbf{H}^H \mathbf{D}^{(n-1)} \mathbf{H} + \frac{1}{P} \mathrm{tr}(\mathbf{D}^{(n-1)}) \mathbf{I}_M \right)^{-1} \mathbf{H}^H \mathbf{\Delta}^{(n-1)}$$

3. Repeat $n = n + 1$

- The algorithm might converge to bad local maxima !!

3. Greedy user selection (8/10)

- It is essential to select a subset of linearly independent users
- Exhaustive search for optimal user selection is too complex

Greedy user selection algorithm

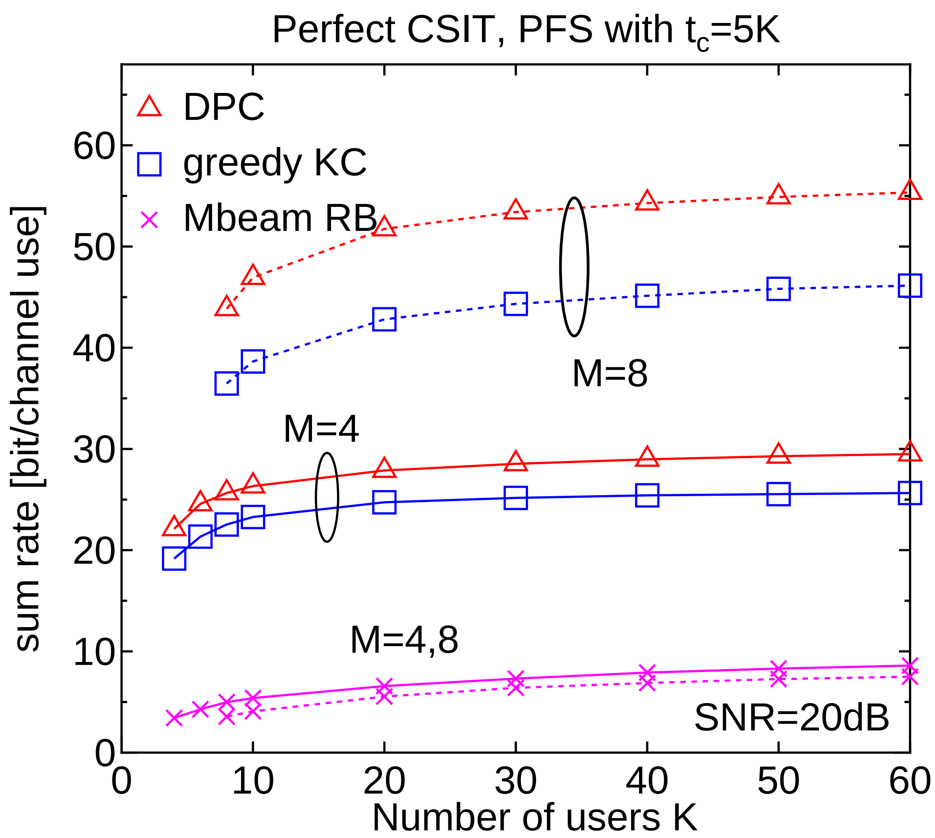
1. Initialize $\mathcal{K}_0 = \{k : w_k > 0\}$ and $\mathcal{K}_1 = \emptyset$
2. For $B = 1, \ldots, M$ repeat

$$k_B = \arg \max_{j \in \mathcal{K}_0} f_B(j, \mathcal{K}_1)$$

$$\mathcal{K}_1 = \mathcal{K}_1 + \{k_B\}$$

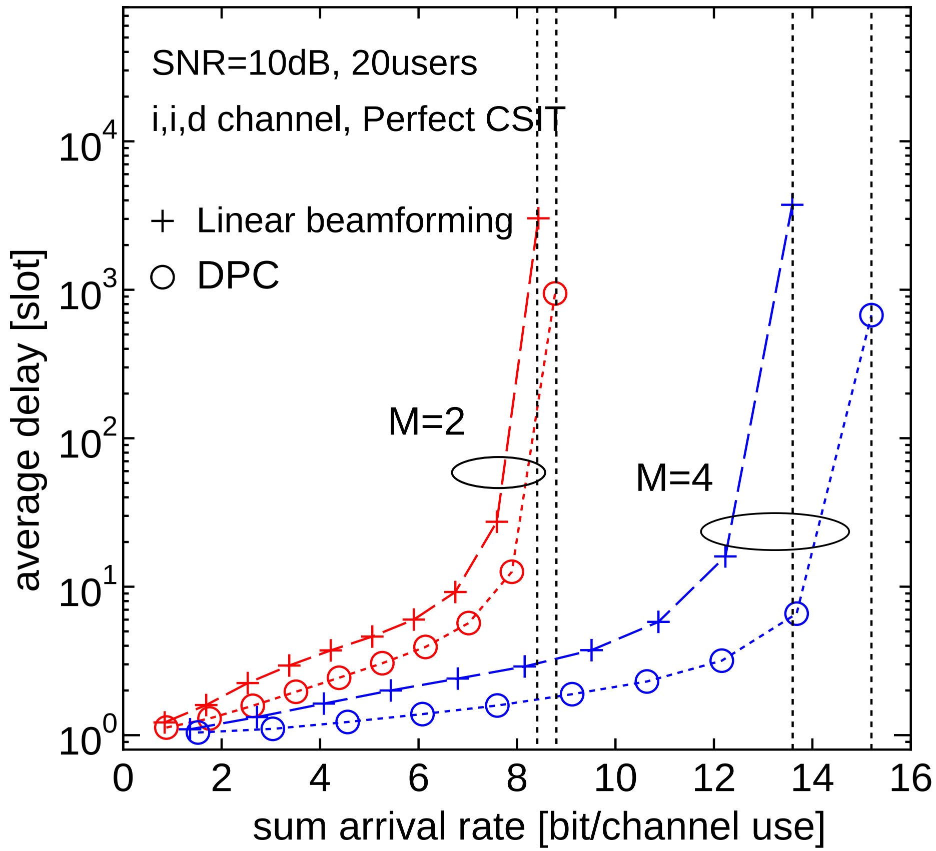
$$\mathcal{K}_0 = \mathcal{K}_0 - \{k_B\}$$

3. Sum rate performance under PFS (9/10)



- Linear beamforming with greedy user selection yields near-DPC performance

3. Average delay performance (10/10)



- Linear beamforming with greedy user selection yields near-DPC performance also in terms of stability

IV. Imperfect CSIT case

4. Imperfect CSIT (1/9)

- In practice, CSIT is obtained via feedback from terminals thus cannot be perfect
- As source of imperfectness we consider :
  - Delayed and noisy feedback
  - Time-varying fading channel
- For a general CSIT model, weighted sum rate maximization is much more complicated
  - the optimal signaling scheme is not known
  - for a fixed $\mathcal{S}$, the rate region $\mathcal{R}_{\mathcal{S}}(t)$ might be non-convex

$$\downarrow$$

Robust schemes with simple heuristics are necessary !!!

4. Linear beamforming with analog feedback (2/9)

- "Analog feedback" : user sends its estimated channel (M complex)
- CSIT : transmitter tracks the channels by Kalman filter

$$\boldsymbol{\alpha}_k(t) = \mathbb{E}[\mathbf{h}_k(t)|\mathbf{z}_k(t-d), \ldots, \mathbf{z}_k(0)]$$

where $\{\mathbf{z}_k(\tau)\}$ is noisy observation with a d-slot delay

- Transmitter has statistical channel knowledge given CSIT

$$\boldsymbol{\Sigma}_k(t) = \mathbb{E}[\mathbf{h}_k(t)\mathbf{h}_k^H(t)|\boldsymbol{\alpha}(t)]$$

- Use approximated SINR

$$\widetilde{\text{SINR}}_k(\boldsymbol{\alpha}) = \frac{\mathbf{g}_k^H(\boldsymbol{\alpha})\boldsymbol{\Sigma}_k(\boldsymbol{\alpha})\mathbf{g}_k(\boldsymbol{\alpha})}{\text{tr}(\mathbf{G}^H\mathbf{G})/P + \sum_{j\neq k}\mathbf{g}_j^H(\boldsymbol{\alpha})\boldsymbol{\Sigma}_k(\boldsymbol{\alpha})\mathbf{g}_j(\boldsymbol{\alpha})}$$

4. Heuristic approach for linear beamforming (3/9)

- Objective function : *approximated* weighted sum rate

$$f_2(\mathbf{G}) = \sum_k w_k \log_2 \left(1 + \frac{\mathbf{g}_k^H \mathbf{\Sigma}_k \mathbf{g}_k}{\mathrm{tr}(\mathbf{G}^H \mathbf{G})/P + \sum_{j \neq k} \mathbf{g}_j^H \mathbf{\Sigma}_k \mathbf{g}_j} \right)$$

Remarks :

– the problem is non-convex !!

– for perfect CSIT $\mathbf{\Sigma}_k = \mathbf{h}_k \mathbf{h}_k^H$, the objective function represents "achievable" weighted sum rate

- By letting the gradient $\nabla f_2(\mathbf{G}) = \mathbf{0}$, we obtain

Iterative beamforming algorithm (KC)

1. Initialize $\mathbf{G}^{(0)} = \boldsymbol{\alpha}^H$
2. At iteration n, update $\mathbf{G}^{(n)}$

$$\mathbf{g}_k^{(n)} = \left(\frac{\mathrm{tr}(\mathbf{D}^{(n-1)})}{P} \mathbf{I}_M + \sum_j D_j^{(n-1)} \mathbf{\Sigma}_j \right)^{-1} \mathbf{\Sigma}_k \mathbf{g}_k^{(n-1)}$$

4. Heuristic approach for linear beamforming (4/9)

- In high SNR regime, the system might be interference-limited by scheduling more than one user
- Under imperfect CSIT, a greedy user selection is mandatory

4. Other practical options - Zero-forcing with greedy search (5/9)

- CSIT : transmitter tracks the channels by Kalman filter

$$\boldsymbol{\alpha}_k(t) = \mathbb{E}[\mathbf{h}_k(t)|\mathbf{z}_k(t-d), \ldots, \mathbf{z}_k(0)]$$

- Mismatched ZF : channel inversion based on $\hat{\mathbf{H}} = [\boldsymbol{\alpha}_1(t), .., \boldsymbol{\alpha}_K(t)]$

$$\mathbf{x} = \hat{\mathbf{H}}(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1}\mathbf{u}$$

- Power allocation :

$$p_k = \left[\frac{\mu w_k}{[(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1}]_{k,k}} - 1\right]_+$$

where $\mu > 0$ is determined to satisfy the total power constraint

- We enhance zero-forcing with a greedy user selection

4. Other practical options - Random Beamforming (6/9)

- Originally proposed by Viswanath-Tse-Laroia02, Sharif-Hassibi04
- The transmit vector is formulated by unitary matrix $\mathbf{\Phi} \in \mathbb{C}^{M \times M}$

$$\mathbf{x} = \mathbf{\Phi}\mathbf{u}$$

  - $\mathbf{\Phi}$ is known a priori to all users
  - $\mathbf{\Phi}$ induces faster fading !

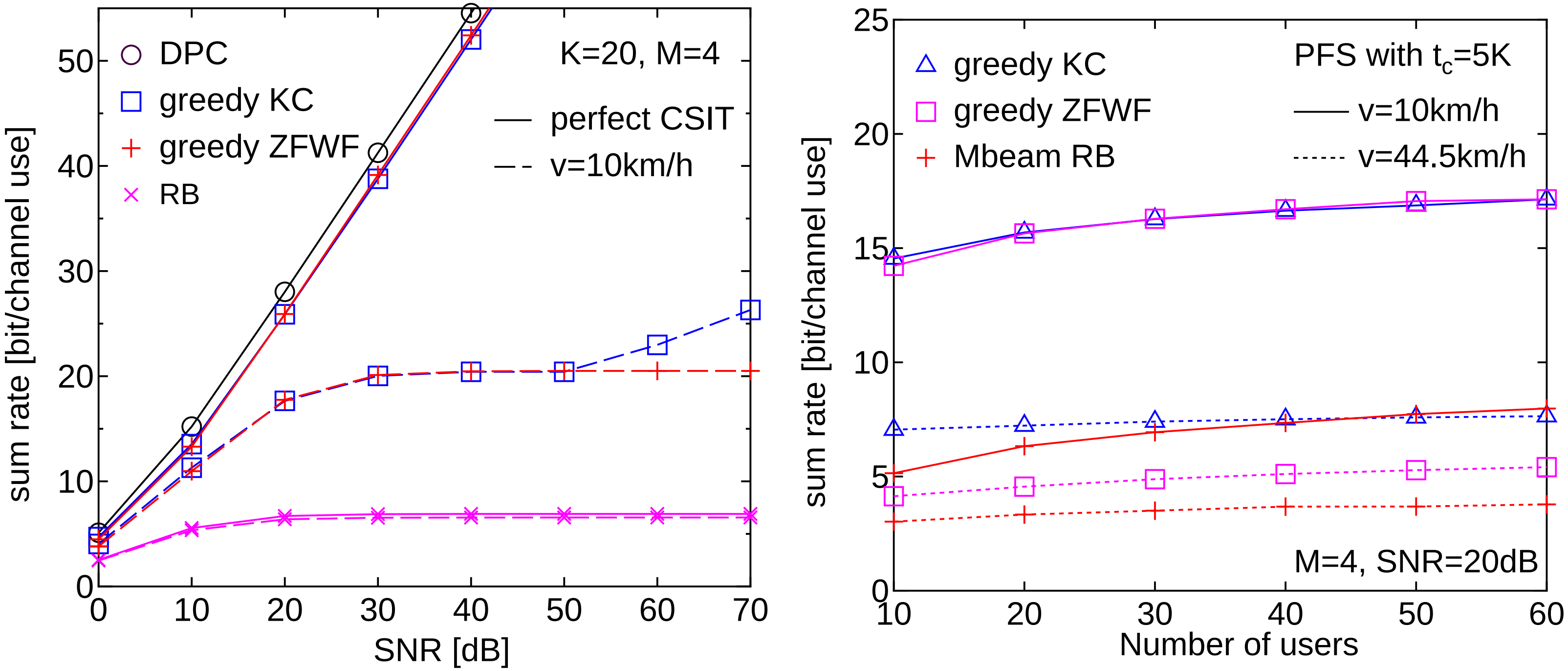
- CSIT : user sends M real values using its MMSE prediction

$$\boldsymbol{\alpha}_k(t) = (|\hat{\mathbf{h}}_k^H(t)\boldsymbol{\phi}_1(t)|^2, \dots, |\hat{\mathbf{h}}_k^H(t)\boldsymbol{\phi}_M(t)|^2)$$

- The scheduler chooses the user for each beam $m = 1, \dots, M$

$$k^\star = \text{arg}\max_k w_k(t) \log\left(1 + \frac{\alpha_{k,m}(t)}{M/P + \sum_{n \neq m} \alpha_{k,n}(t)}\right)$$

4. Numerical Examples- sum rate performance (7/9)

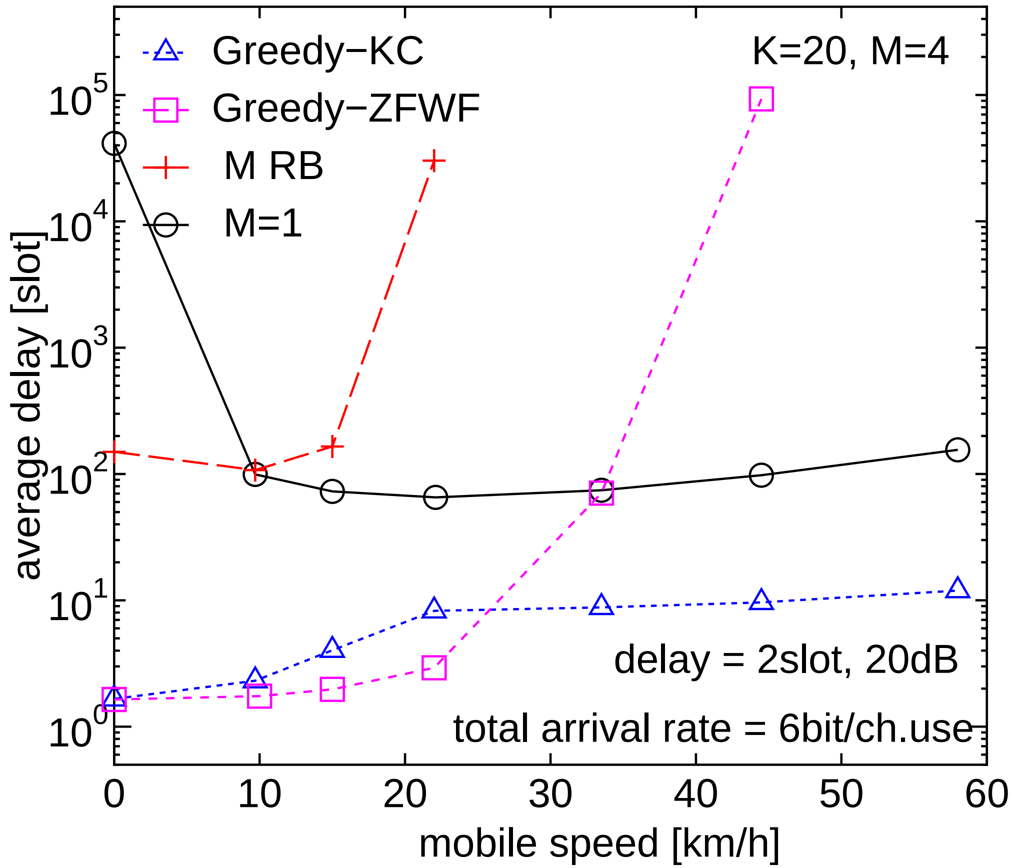


- With a feedback delay, multiplexing gain is lost
- ZF and RB become interference-limited

4. Numerical Examples- average delay vs. mobile speed (8/9)

- Channel model : 1st-order Gauss-Markov model with Jake's autocorrelation
- Feedback link : noisy (AWGN) and delayed by $d = 2$ slots
- Conditional covariance : $\boldsymbol{\Sigma}_k(t) = \boldsymbol{\alpha}_k(t)\boldsymbol{\alpha}_k^H(t) + \sigma_e^2 \mathbf{I}_M$
  σ_e^2 denotes prediction MMSE
- packet arrival [packet/slot] : Poisson λ_k
- packet size [bit/packet] : exponential
- arrival rate : $\lambda_k = 0.3$ [bit/channel] use for all k

4. Numerical Examples- average delay vs. mobile speed (9/9)



- RB is interference-limited, thus becomes unstable
- Greedy-KC is most robust to non-perfect CSIT

V. Conclusions

5. Summary of practical schemes

	Opportunistic beamforming	Zero-forcing greedy search	Proposed greedy-KC
Feedback	SINR	channel vector	channel vector
#feedback/user	M real	M complex	M complex
# users/slot	M	$\approx M$	$1 \leq B \leq M$
slow fading performance (role)	multiplexing +MUD (induce faster fading)	multiplexing + MUD (exact ch.inversion)	multiplexing + MUD (exact beamforming)
fast fading performance (role)	interference-limited (no help)	interference-limited (no help)	multiplexing $\rightarrow 1$ + MUD (TDMA)

5. Conclusions

- Under perfect CSIT, simple linear beamforming with greedy user selection achieves near DPC performance
- Under a fixed feedback delay, multiplexing gain collapses for any signaling scheme
- Greedy-KC is much robust to non-perfect CSIT and outperforms the previously proposed schemes

5. Future works

- A practical precoding in MIMO-BC (a long-standing open problem)
  Is there a better and seamless solution, especially one that converges to space-time coding under no CSIT ? Some powerful tools such as geometric programming might be applied.

- Extension to a multi-carrier system
  With DPC, the weighted sum rate maximization can be solved very efficiently via Lagrangian dual decomposition. Since in the regime of large number of subcarriers even a non-convex problem can be solved by dual decomposition, there might be also an elegant solution with a practical signaling.

- Analog vs. digital (quantized) feedback
  Which scheme can provide better multiplexing gain ? In which condition ? How do mobility, a feedback delay, a type of fading impact the performance ?

References

- M.Kobayashi and G.Caire, "An Iterative Waterfilling Algorithm for Weighted Rate Sum of MIMO-MAC and MIMO-BC", IEEE J. Select. Areas Commun., Special Issue on Nonlinear Optimization, vol. 24, no. 8, August, 2006
- M. Kobayashi and G. Caire, "Joint Beamforming and Scheduling for a Multi-Antenna Downlink with Imperfect Transmitter Channel Knowledge", submitted to IEEE J. Select. Areas Commun., Special Issue on Optimisation of MIMO Transceivers for Realistic Communications Networks, June, 2006
- M.Kobayashi, D.Gesbert, and G.Caire, "Transmit Diversity vs. Opportunistic Beamforming in Data Packet Mobile Downlink Transmission"', to appear IEEE Trans. on Communications, December, 2006