# Coding with state information. Application to cooperative communication in wireless networks

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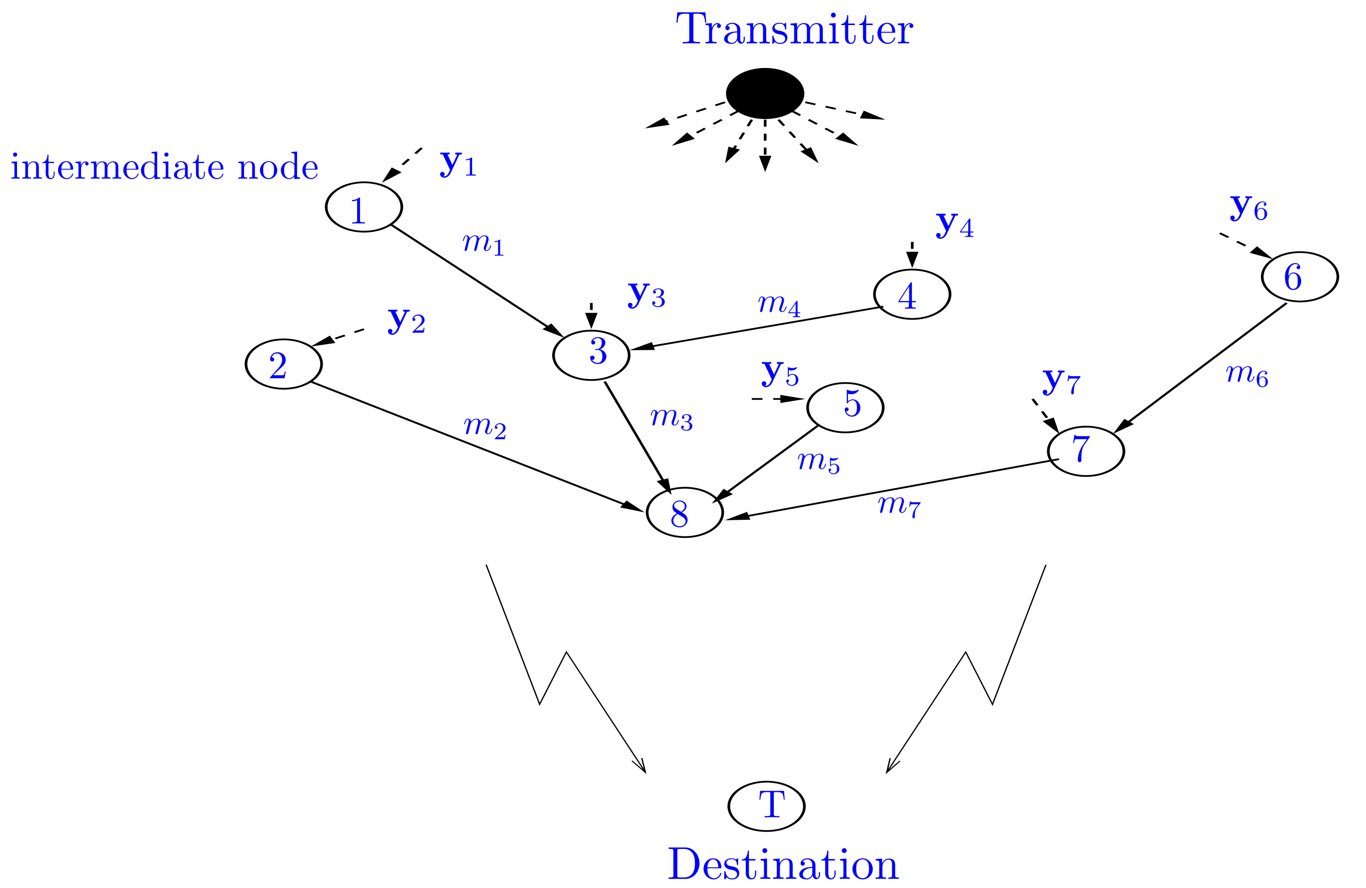
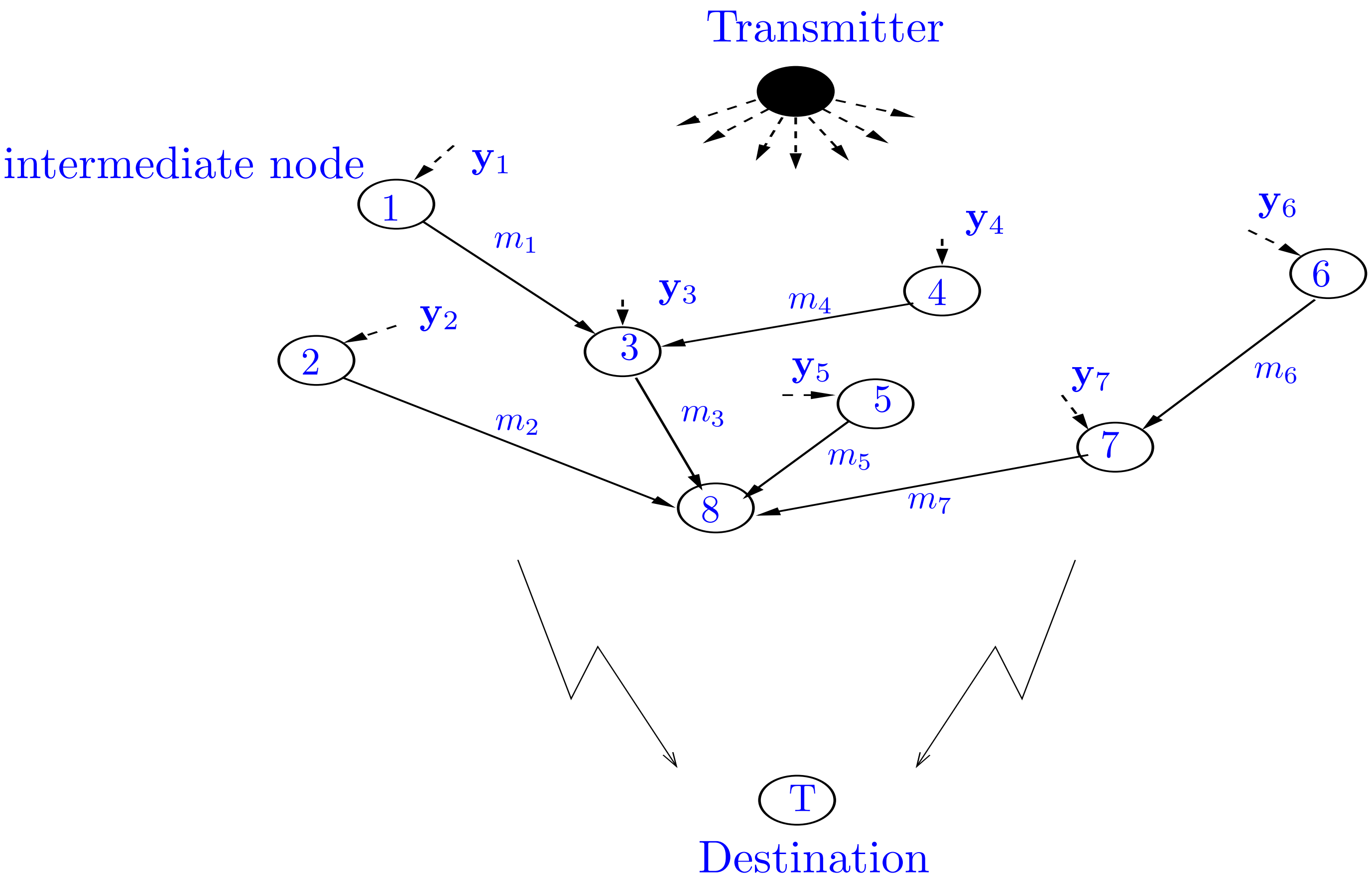
# Cooperation in wireless networks: A review



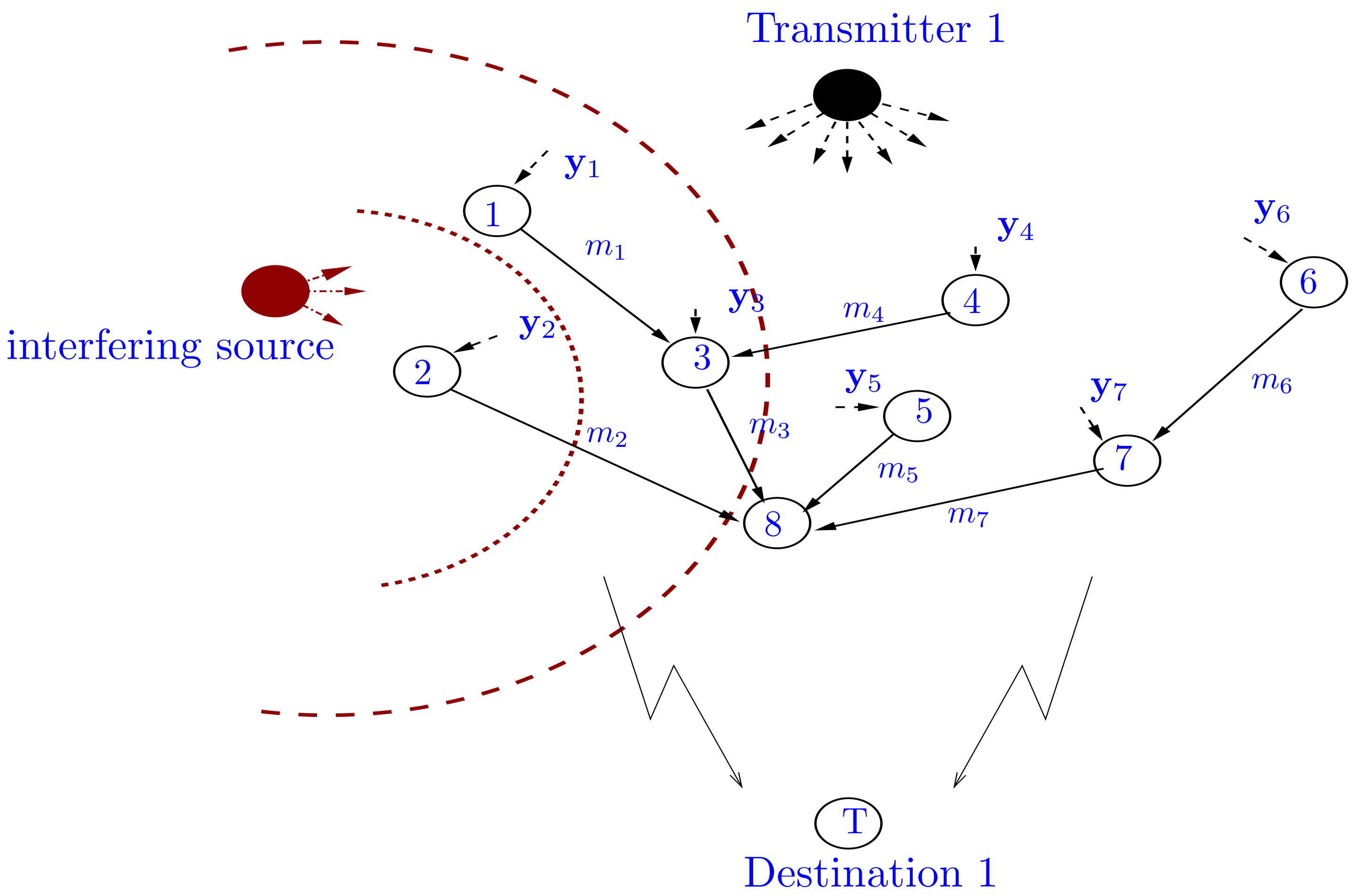

**C**ooperation means assitance + coordination

## Cooperation in wireless networks: Some challenging questions




1. What is the best transmission rate ?
2. How it can be achieved ?
3. What is the best "strategy" for each node ?

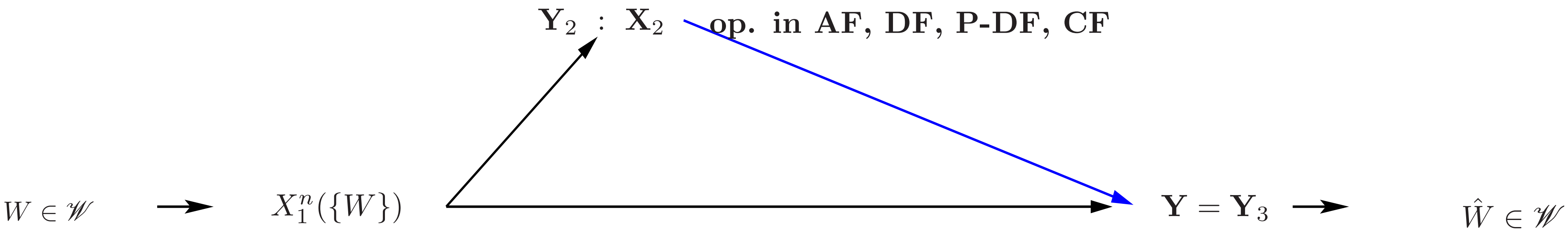
# Cooperation in wireless networks: Some challenging problems (contd.)




❶ Can cooperation cancel the interference ? (t/T: unsolved)

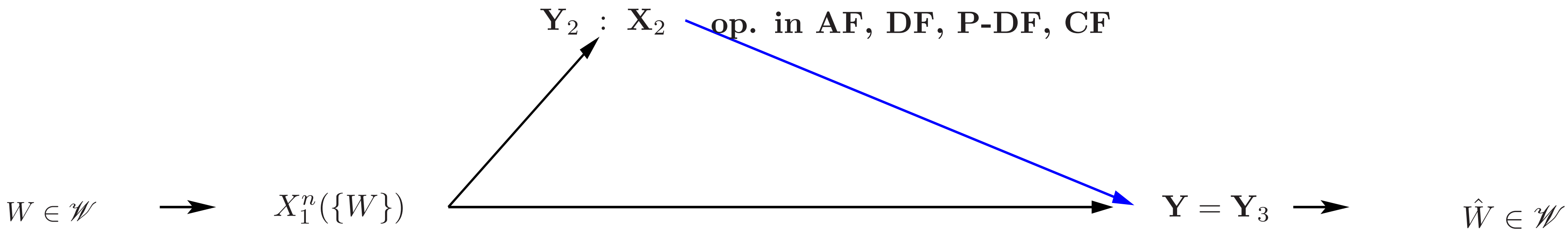
❷ All the intermediate nodes, but the destination, know the interference ? (T-1/T: this talk)

# Relaying and relaying-broadcasting in wireless networks

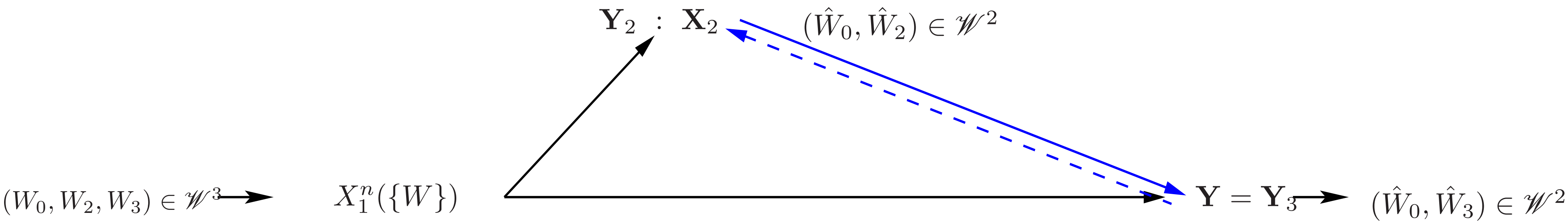



**[Van der Meulen 1971, Cover 1979]**

# Relaying and relaying-broadcasting in wireless networks

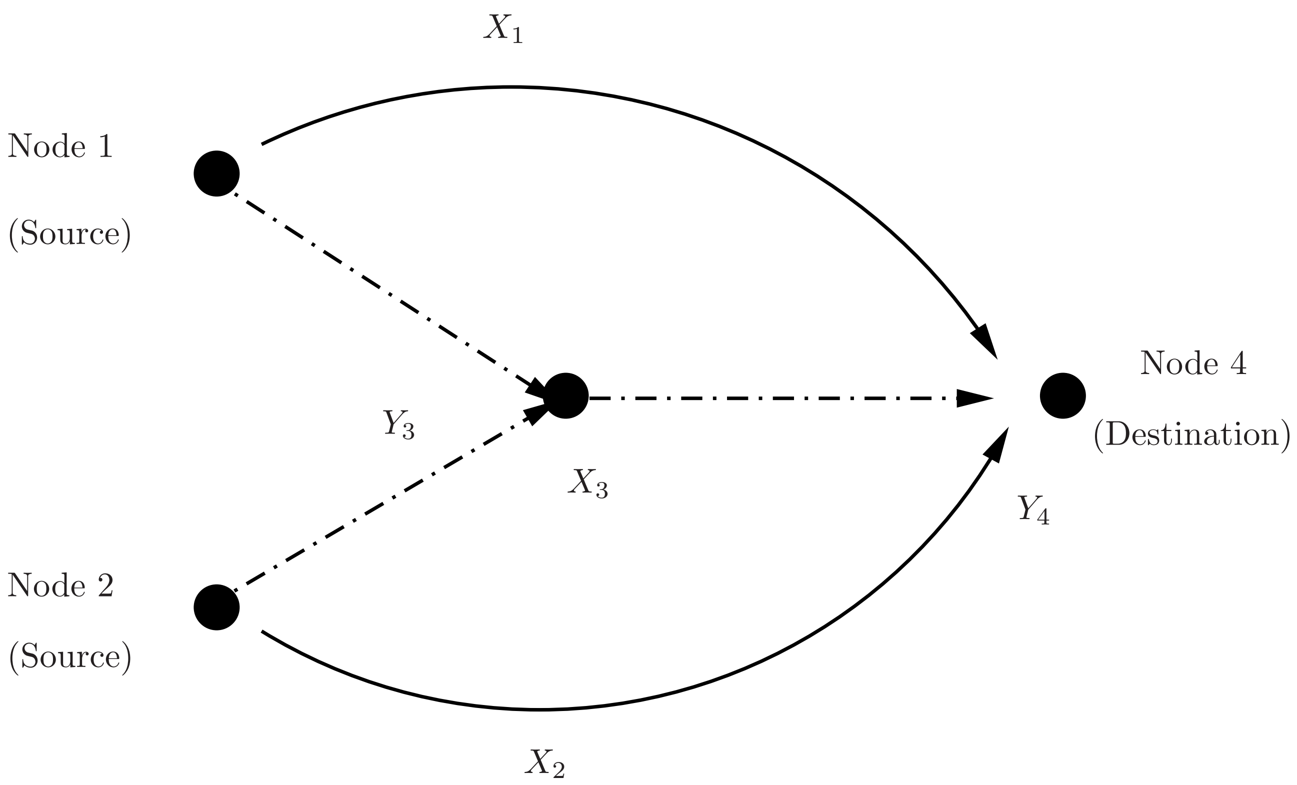



[Van der Meulen 1971, Cover 1979]



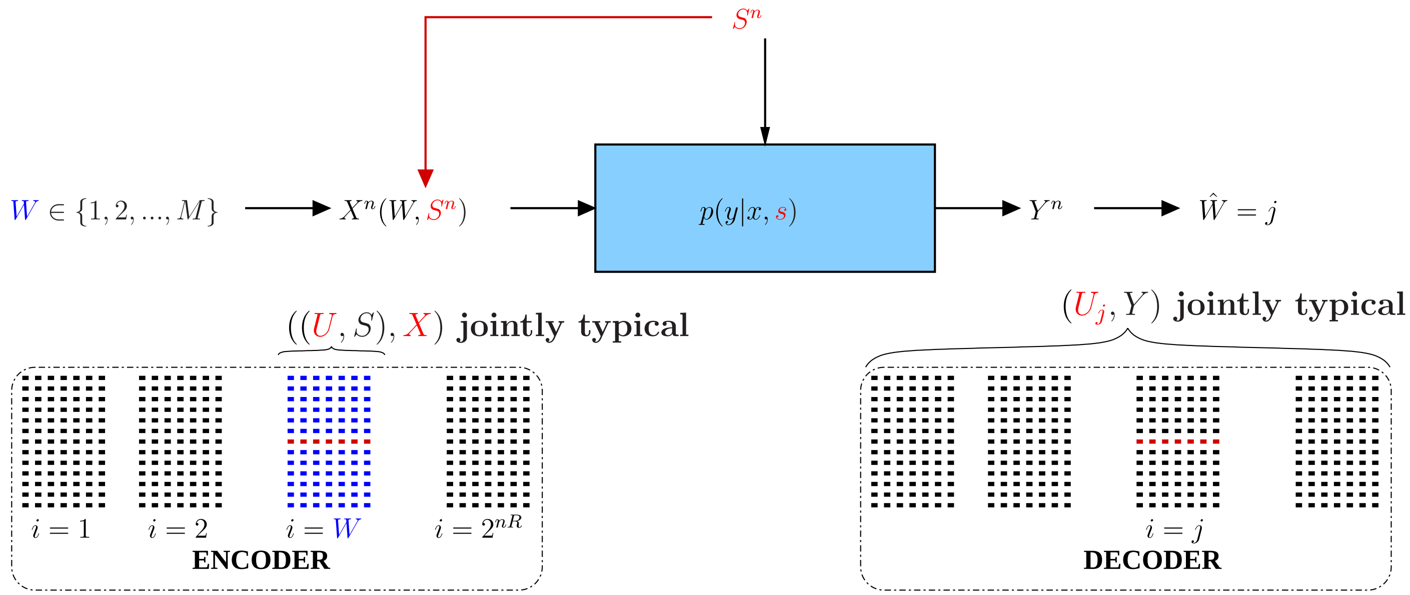

[Liang and Veeravalli 2005, Kramer 2006]

# The multiple access relay channel (MARC)




**[Erkip 1998, G. Wornell 2004, Kramer 2006]**

# Non-causal CSI at the Encoder: Gelfand-Pinsker Problem




**Theorem** [Gelfand-Pinsker 1980]

$$\mathcal{C}_{10}^{nc} = \max_{p(u|s)p(x|u,s)} \{I(U;Y) - I(U;S)\} \leq \mathcal{C}_{11}^{nc} \triangleq \max_{p(x|s)} I(X;Y|S)$$

**Achievability** (Random Binning).

Encoder: ? $\exists\, U \in \mathcal{U}_w \mid (U, S)$ jointly typical, (ii) ? $\exists\, X \mid (U, X, S)$ jointly typical.

Decoder: ? $\exists\, U \in \mathcal{U} \triangleq \cup\, \mathcal{U}_w \mid (U, Y)$ jointly typical, (ii) declare $\hat{W} = j \mid \mathcal{U}_j \ni U$.

# The IID Gaussian case: Dirty Paper Coding (DPC)

IID Gaussian case :

1. $X \sim \mathcal{N}(0, P)$ independent of $S$
2. $U = X + \alpha S$ with $\alpha = \frac{P}{P+N}$

$$\mathcal{C}_{10}^{nc} = \frac{1}{2}\log(1 + \frac{P}{N}) = \mathcal{C}_{11}^{nc}$$

**Max. M. Costa "Writing on Dirty Paper", 1983**

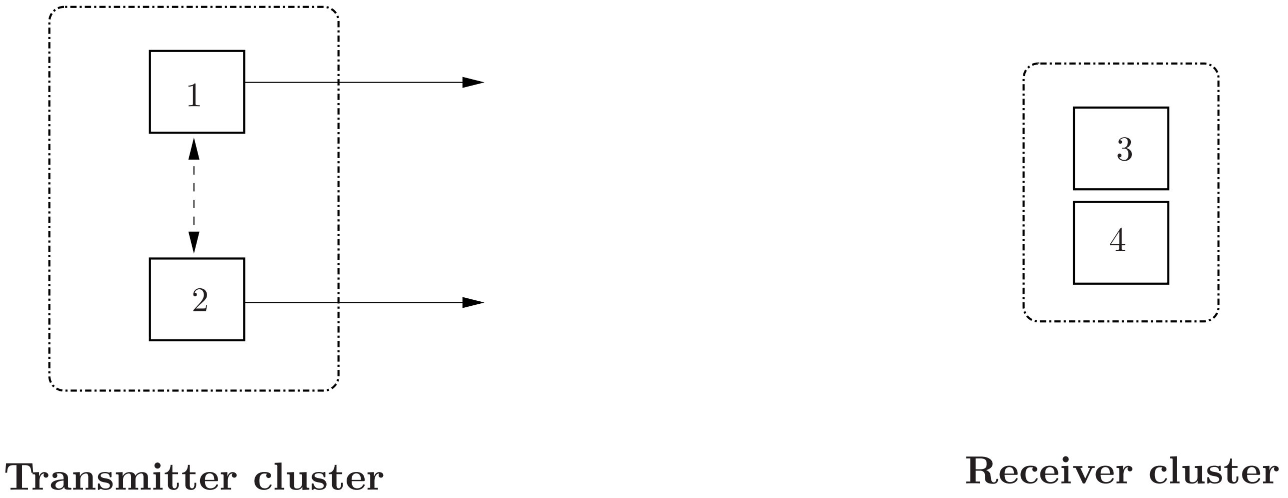
In practice:

Quantize the state (THP, Lattice coding, TCQ)

# Presentation outline

- **Dirty paper coding cooperation**
  1. for relaying (RC).
  2. for relaying-broadcasting (RBC).
  3. the multiple-node RBC.
- **Coding for a side-informed RBC network**
  1. The four-node relay case
  2. The multi-relay case
- **The effect of a partial knowledge of the SI**
  1. on relaying (CF)
  2. on DPC cooperation
- **Concluding remarks**

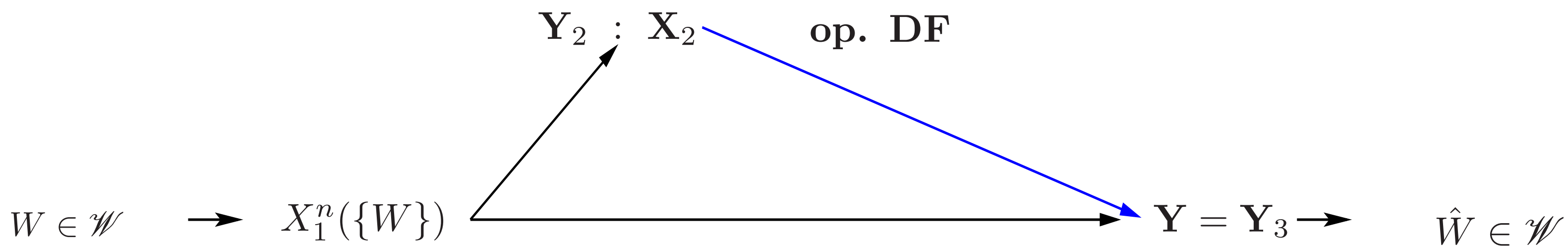
# DPC for relaying: a motivating example




**Transform the channel into a 2-antenna BC**

**DPC achieves the capacity of this 2-antenna BC**

# The degraded relay channel




**Assumptions:**

- Physically degraded GRC $\{\mathscr{X}_1, p(y_2, y_3|x_1, x_2), \mathscr{Y}_2 \times \mathscr{Y}_3\}$: $X_1 \ominus (Y_2, X_2) \ominus Y_3$
- Encoders $X_1 : \{1, 2, \cdots, 2^{nR}\} \longrightarrow \mathscr{X}_1^n$: $\mathbb{E}[X_1^2] \le P_1$; $X_{2i}(Y_2^{i-1}) : \mathscr{Y}_2^n \longrightarrow \mathscr{X}_2^n$: $\mathbb{E}[X_2^2] \le P_2$.

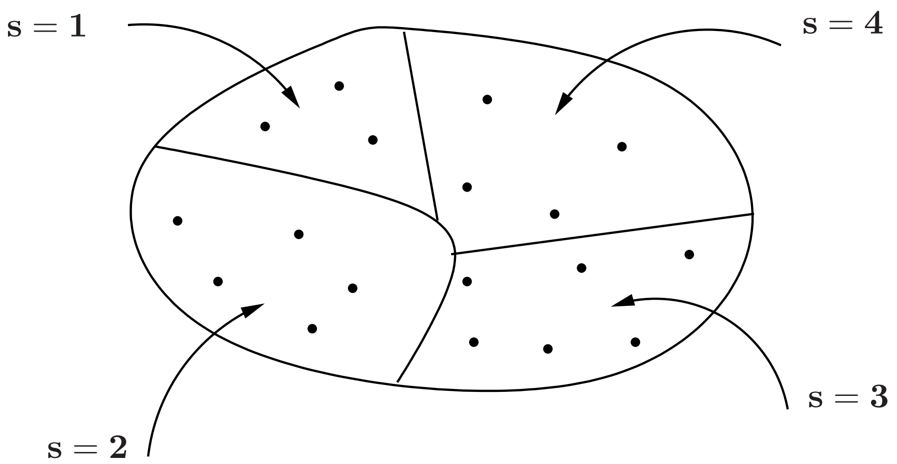
$$\mathscr{C}_{\mathrm{DF}} = \max_{p(x_1,x_2)} \min\{I(X_1;Y_2|X_2), I(X_1X_2;Y)\}$$

$$= \max_{0\le\alpha\le1} \min\left\{C\left(\alpha|h_{12}|^2P_1\right), C\left(|h_{13}|^2P_1 + |h_{23}|^2P_2 + 2h_{13}h_{23}\sqrt{\bar{\alpha}P_1P_2}\right)\right\}$$

❶ regular enc. + sliding-window/backward dec. [**King** 1978, **Carleil (MAC-GF)** 1982]

❷ **irregular enc. + successive dec.** [**Cover** 1979]

# The degraded relay channel: Block Markov coding




➢ Block $i$: $\left\{x_1(w_i|\mathbf{s_i} \ni w_{i-1})\right\}$, $\left\{x_2(\mathbf{s}_i \ni w_{i-1})\right\}$

no need for full decoding!!

➢ We write

$$\begin{cases} W = \underbrace{W_r}_{\text{over } X_1^{(r)} \text{at } R_r} + \underbrace{W_d}_{\text{over } X_1^{(d)} \text{at } R_d} : R = R_r + R_d. \\ X = \underbrace{X_1^{(r)}}_{\textbf{cooperatively sent}} + \underbrace{X_1^{(d)}}_{\textbf{interference for } W_r} . \end{cases}$$

idea: increase $R_r$ (by DPC), without decreasing $R_d$

# Dirty paper coding for relaying

➢ Decoding at the receiver

$$\begin{cases} ❶ \quad \text{decode } W_r : Y_3 = h_{13}X_1^{(r)} + h_{23}X_2 + \underbrace{h_{13}X_1^{(d)} + Z_3}_{\textbf{noise}} \\ ❷ \quad \text{decode } W_d : Y_2' = Y_2 - h_{12}X_1^{(r)} \end{cases}$$

➢ Decoding at the relay

$$Y_2 = h_{12}X_1^{(r)} + \underbrace{h_{12}X_1^{(d)}}_{\textbf{known at } X_1} + Z_2$$

➢ Coding at the transmitter **( use DPC )**

$$\begin{cases} X_1^{(r)} = h_{12}^{-1}U_1^{(r)} + \alpha X_1^{(d)} \\ U_1^{(r)} \sim \mathcal{N}(\alpha h_{12}X_1^{(d)}, \bar{\alpha}|h_{12}|^2 P_1) \end{cases}$$

# Dirty paper coding for relaying (contd.)

- Decoding is successful iff

**1**
$$R_r \leq I(U_1^{(r)}; Y_2|X_2) - I(U_1^{(r)}; X_1^{(d)}|X_2)$$

$$Y_3 = \underbrace{U_1^{(r)} + X_2}_{\text{input 1 for MAC}} + \underbrace{(1-\alpha)X_1^{(d)}}_{\text{input 2 for MAC}} + Z_3$$

**2**
$$\begin{cases} R_r \leq I(U_1^{(r)} X_2; Y_3) \\ R_d \leq I(X_1^{(d)}; Y_3|U_1^{(r)} X_2) \end{cases}$$

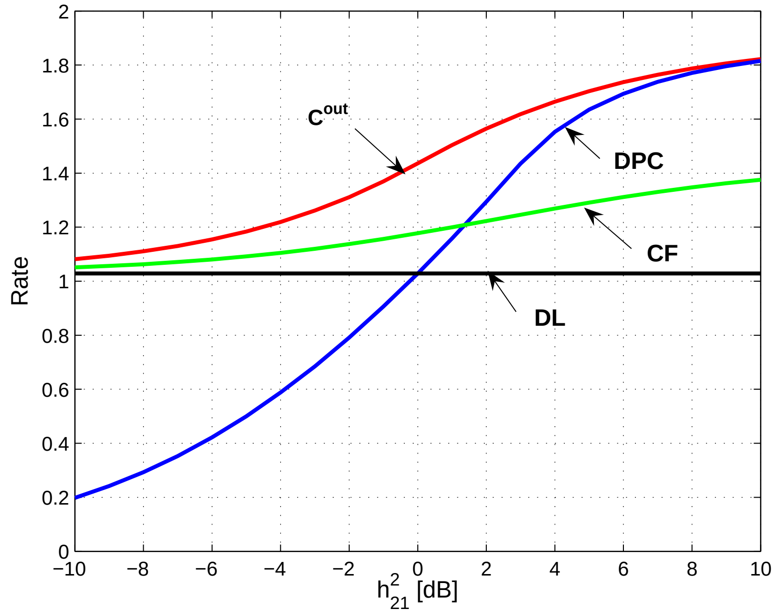
- Achievable rate

$$R \leq \min\left\{ \min\left\{ I(U_1^{(r)} X_2; Y_3), \underbrace{I(U_1^{(r)}; Y_2|X_2) - I(U_1^{(r)}; X_1^{(d)}|X_2)}_{\text{broadcast bottleneck at } X_2} \right\}, I(X_1^{(d)}; Y_3|U_1^{(r)} X_2) \right\}$$

$$R \leq \min\left\{ \min\left\{ I(X_1^{(d)}; Y_3), \underbrace{I(U_1^{(r)}; Y_2|X_2) - I(U_1^{(r)}; X_1^{(d)}|X_2)}_{\text{broadcast bottleneck at } X_2} \right\}, I(U_1^{(r)} X_2; Y_3|X_1^{(d)}) \right\}$$

$$R_{\text{DPC}}(\text{degraded GRC}) = R_{\text{DF}}(\text{degraded GRC}) = C(\text{degraded GRC})$$

# Dirty paper coding for relaying (contd.)



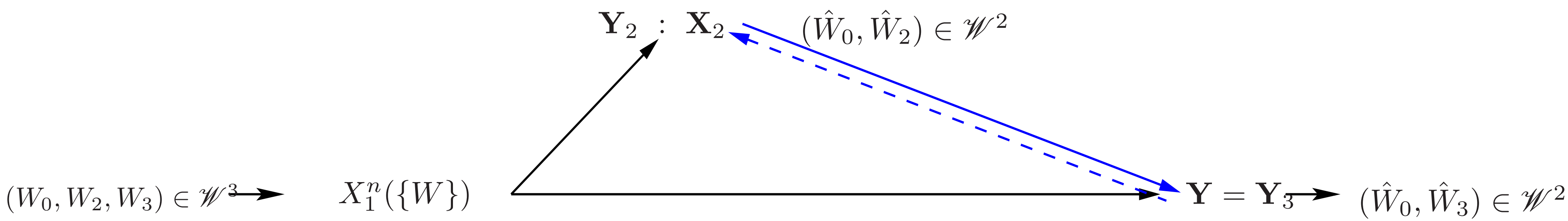

➢ Advantages over BM, SW-dec., Bck-dec.

✓ easily extendable to the MIMO case

✓ implementable in practice, e.g., LDPC + THP (e.g., Lattice code)

✓ can perform very close to capacity

# Cooperation in relaying-broadcasting: the D-AWGN RBC




**Assumptions:**

- Physically degraded-AWGN GRBC $\{\mathscr{X}_1, p(y_2, y_3|x_1, x_2), \mathscr{Y}_2 \times \mathscr{Y}_3\}$: $X_1 \ominus (Y_2, X_2) \ominus Y_3$
- Encoders $X_1 : \mathcal{W}^3 \longrightarrow \mathscr{X}_1^n$: $\mathbb{E}[X_1^2] \leq P_1$; $X_{2i}(Y_2^{i-1}) : \mathscr{Y}_2^n \longrightarrow \mathscr{X}_2^n$: $\mathbb{E}[X_2^2] \leq P_2$.

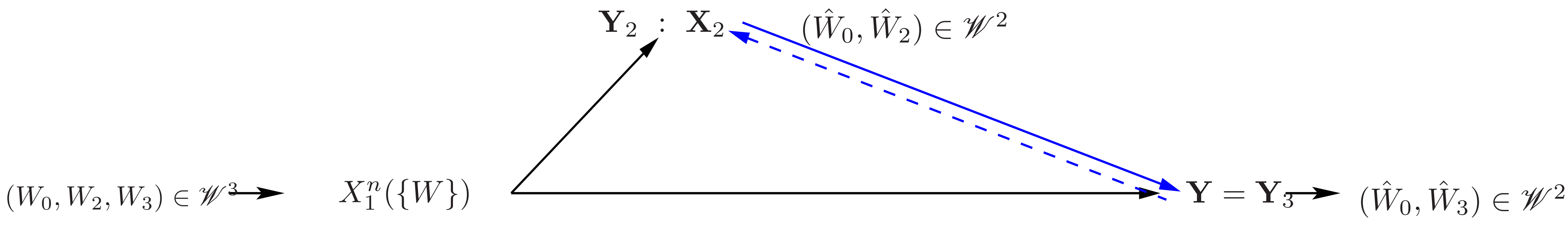
➢ $$\mathscr{C} = \bigcup_{0 \leq \alpha \leq 1} \Bigg\{ (R_0, R_2, R_3) \ :$$

$$R_2 < C\Big(\frac{\alpha P_1}{N_2}\Big)$$

$$R_0 + R_3 < \max_{\beta} \min \Bigg\{ C\Big(\frac{\beta \bar{\alpha} P_1}{\alpha P_1 + N_2}\Big), C\Big(\frac{P_2 + \bar{\alpha} P_1 + 2\sqrt{\bar{\beta}\bar{\alpha} P_1 P_2}}{\alpha P_1 + N_3}\Big) \Bigg\} \Bigg\}.$$

[Liang and Veeravalli 2005]

# Cooperation in relaying-broadcasting: the D-AWGN RBC




- We write

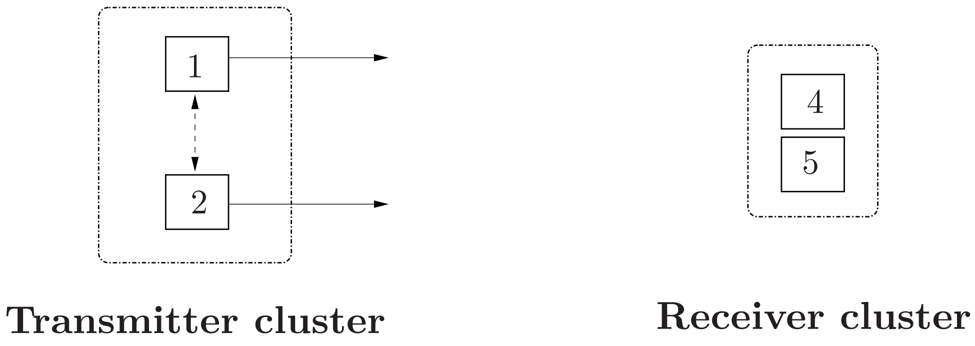
$$\begin{cases} X_1 = \underbrace{X_1^{(2)}}_{\text{transmit } W_2:\ \alpha P_1} + \underbrace{X_1^{(3)}}_{\text{transmit } W_3 \text{ through } X_2:\ \bar{\alpha} P_1} . \\ W_3 = \underbrace{W_{3,r} + W_{3,d}}_{\text{use DPC for relaying}} \; : R_3 = R_{3,r} + R_{3,d}. \end{cases}$$

- Achievable rate-region

$$R_2 < C\Big(\frac{\alpha P_1}{N_2}\Big)$$

$$R_0 + R_3 < \max_{\beta} \min \Big\{ C\big(\frac{\beta \bar{\alpha} P_1}{\alpha P_1 + N_2}\big), C\Big(\frac{P_2 + \bar{\alpha} P_1 + 2\sqrt{\bar{\beta} \bar{\alpha} P_1 P_2}}{\alpha P_1 + N_3}\Big) \Big\} \Big\}.$$

# DPC-based transmitter cooperation: example




1. use $\frac{P_t}{2}$ to exchange messages (orthogonal cooperation channel)

$$R_t = \frac{1}{2}\log_2(1 + \frac{P_t}{2d^\alpha})$$

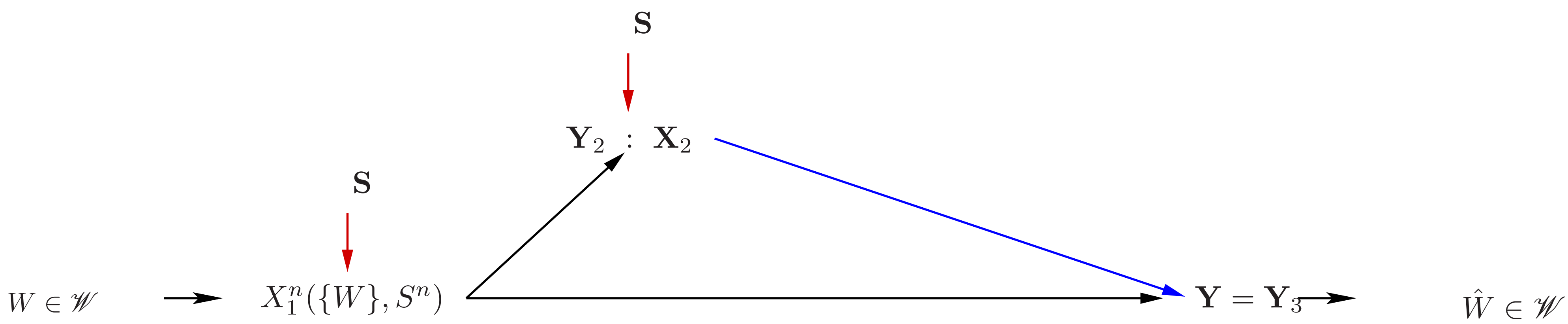
2. joint encoding of $(W_1, W_2)$ ($\equiv$ 2-antenna BC)

$$R_{\text{DPC}} = \max_{P_1:\,0 \le P_1 \le P - P_t} \log|I + H_1^T P_1 H_1 + H_2^T (P - P_t - P_1) H_2|$$

$$\begin{aligned} R_{\text{sum}} &= \max_{P_t:\,0 \le P_t} \min\,(2R_t, R_{\text{DPC}}) \\ &\ge R_{\text{TD}} = \log(1 + \frac{P}{(1-d)^\alpha}) \\ &\ge R_{\text{TD-BC}} = \log(1 + \frac{P}{(1-d)^\alpha}) \end{aligned}$$

# Relaying and Relaying-broadcasting with state information

# Three-node RC with state information




**Assumptions:**

- RC with state $\{\mathscr{X}_1, p(y_2, y_3|x_1, x_2, s), \mathscr{Y}_2 \times \mathscr{Y}_3\}$: $(X_1, S) \ominus (Y_2, X_2, S) \ominus Y_3$
- Encoders $X_1(W, S^n) : (W \times \mathscr{S}^n) \longrightarrow \mathscr{X}_1^n$; $X_{2i}(Y_2^{i-1}, S^n) : (\mathscr{Y}_2^n \times \mathscr{S}^n) \longrightarrow \mathscr{X}_2^n$.

➢ If the RC is **degraded**

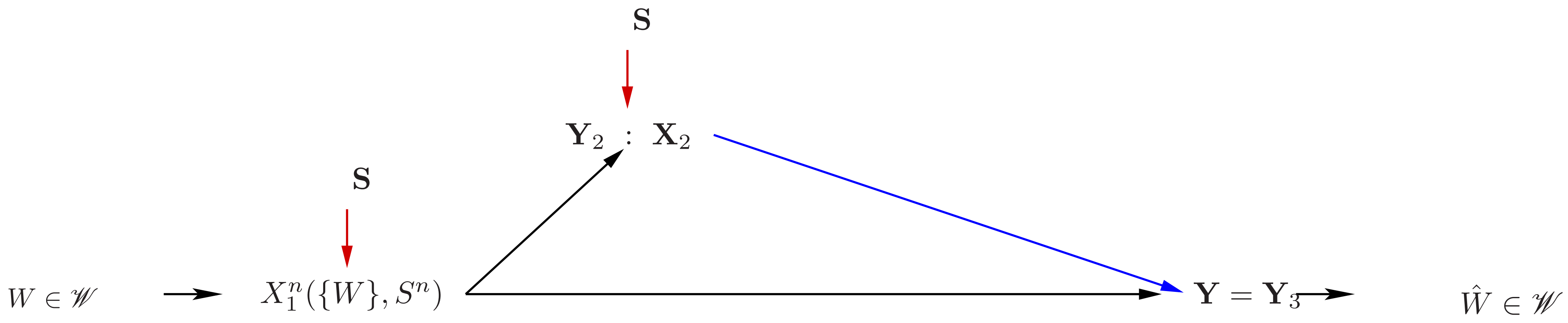
$$R_{\text{conj}} = \max_{p(u_1,u_2,x_1,x_2|s)} \min \left\{ I(U_1; Y_2|SX_2), I(U_1U_2; Y) - I(U_1U_2; S) \right\}$$

**[conjectured by Kim, ISIT 2004]**

➢ if $S$ is **additive** and the DRC is **Gaussian**

$$R_{\text{conj}} \text{ (with state)} = \mathscr{C}_{\text{DGRC}} \text{ (with no state)}$$

# Three-node RC with state information: an achievable region



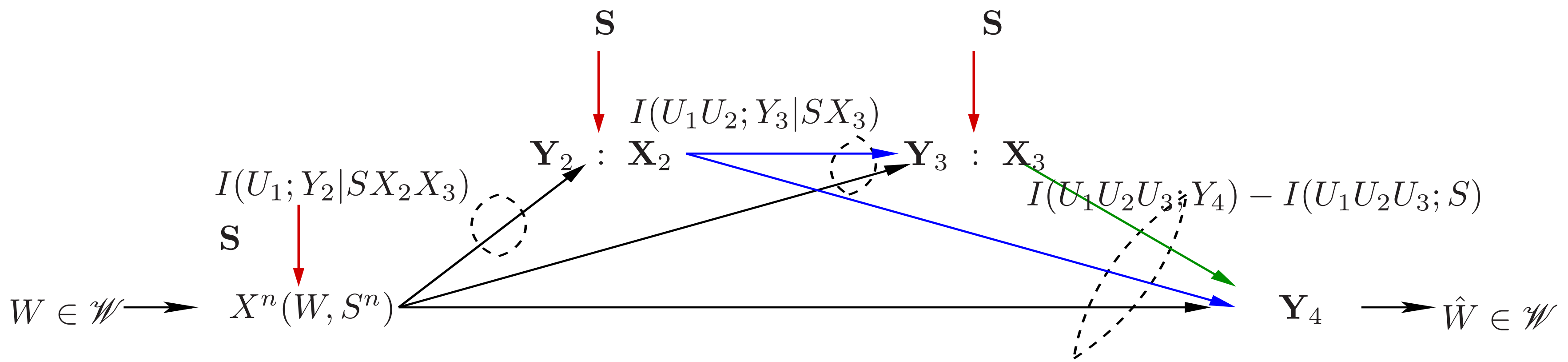

- The following rate is achievable (combine SW-dec. and binning)

$$R = \max_{p(u_1,u_2,x_1,x_2|s)} \min \Big\{ \underbrace{I(U_1;Y_2|SU_2)}_{\text{instead of } I(U_1;Y_2|SX_2)}, I(U_1U_2;Y) - I(U_1U_2;S) \Big\} \leq R_{\text{conj}}$$

- Capacity for DGRC

$$R \text{ (with state)} = \mathscr{C}_{\text{DGRC}} \text{ (with no state)}$$

# RC with state information: the multiple-node case

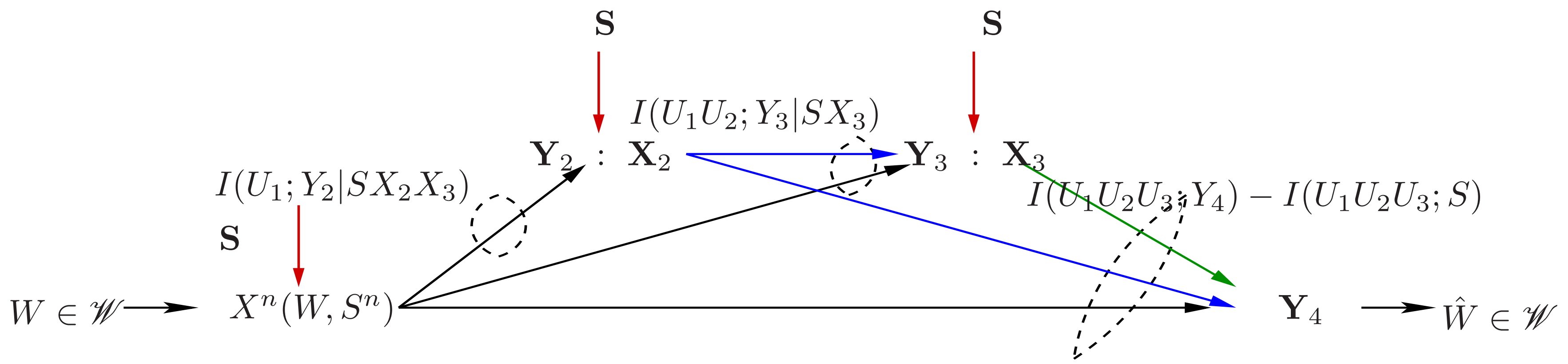



➢ The following rate is achievable for a $T$-node RC

$$R = \max_{\pi(\cdot)} \max_{p(\cdot|\cdot)} \min_{1\le t\le T-2} \left\{ \underbrace{I\Big(U_{\pi(1:t)};Y_{\pi(t+1)}|SU_{\pi(t+1:T-1)}\Big)}_{\text{info. transfer to settle up coop.}}, \underbrace{I\Big(U_{\pi(1:T-1)};Y_T\Big) - I\Big(U_{\pi(1:T-1)};S\Big)}_{\text{Gelfand-Pinsker rate } (U_1U_2U_3,S)\to Y_T} \right\}$$

**(interference cancellation is possible) iff (coop. is well enough)**

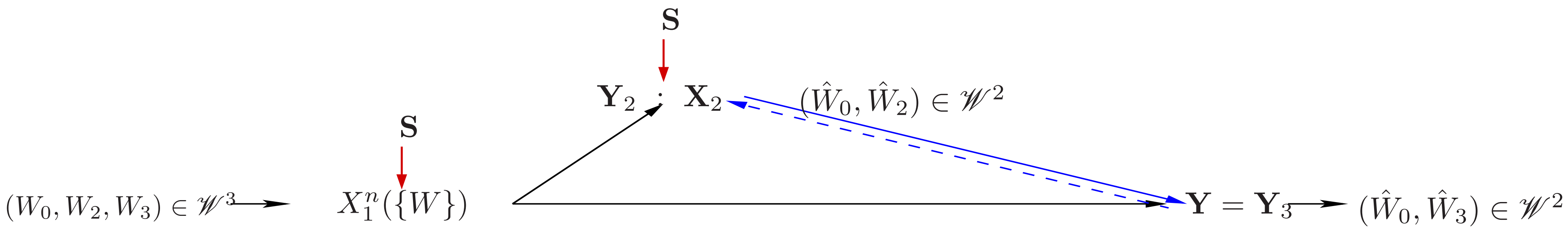
# RC with state information: the multiple-node case (contd.)




$$C_{\mathrm{DGRC}}(S \text{ known only at the transmitter and the relays }) = \max_{\beta_{ij}} \min \Big\{ f(1), \cdots, f(T-1) \Big\}$$
$$= C_{\mathrm{DGRC}}(S \text{ known everywhere})$$

$$\text{where } f(t) = C\left( \sum_{j=1}^{t} \left( \sum_{i=1}^{j} |h_{i(t+1)}| \sqrt{\beta_{ij} P_i} \right)^2 \right), t = 1, \cdots, T-1$$

# Three-node RBC with state information: an achievable region



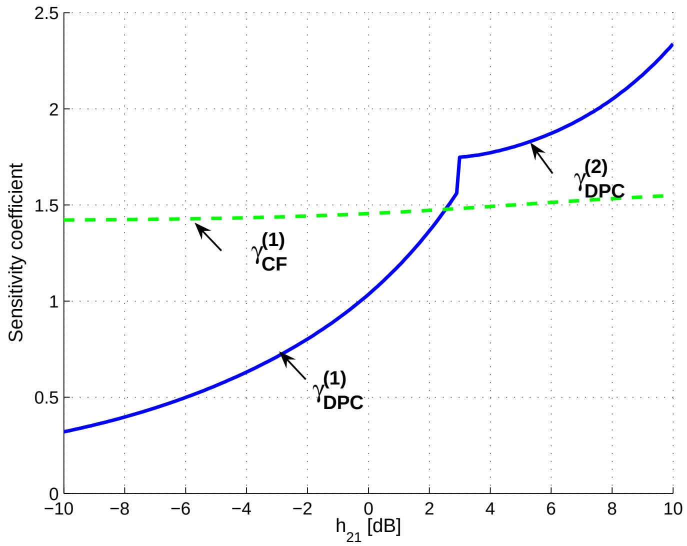

➢ The following rate is achievable (combine superposition coding, SW-dec. and binning)

$$R_2 < I(X_1; Y_2|SU_1X_2)$$

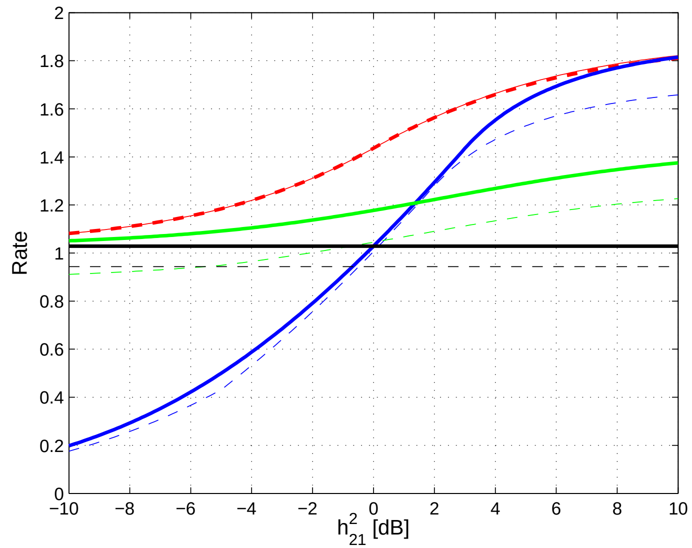
$$R_0 + R_3 < \min\left\{I(U_1; Y_2|SU_2), I(U_1U_2; Y) - I(U_1U_2; S)\right\}$$

$$R \text{ (with state)} = \mathscr{C}_{\mathrm{DGRBC}} \text{ (with no state)}$$

# DPC-cooperation: is that that good ?




(a) sensitivity coeff.




(b) rates under estimation error

**DPC-cooperation is highly sensitive to CSI !!**

**The improvement may depend on the bandwidth cost for the orthogonal channel.**

# Conclusions and Perspectives

We have shown that:

❶ Coding with side information (DPC) provides an insightful understanding of cooperation in wireless networks.
❷ The efficiency of DPC-cooperation is even more pronounced when this collaboration should also handle interference cancellation.
❸ DPC is mainly efficient when full transmitter cooperation is possible.
❹ In this case, it improves upon classic techniques for cooperation.
❺ Under certain conditions (D-GRC SI, D-AWGN partially-cooperative/fully-cooperative RC SI), it suffices that only $T - 1/T$ nodes know the interference (but not the destination) to have a reliable communication.
❻ The results have potential use is emphasizing the effect of channel estimation error in relaying.

Thank You for your Attention !!