On Interference Alignment over Asynchronous Wireless Networks

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Degrees of Freedom in Interference Channels

- The total number of degrees of freedom (DOF) of a K-user *synchronous* interference channel (IC) is shown to be upper bounded by $K/2$.[1]
- This upper bound is achieved by a technique called Interference Alignment. The key idea is to design signals such that they overlap at the receivers where they interfere while remain distinct at the receivers where they are desired [Maddah-Motahari-Khandani, August 2008].
- Interference alignment has first been proposed for time/frequency/space varying *synchronous* fading channels. [Cadambe-Jafar, August 2008]

[1] The DOF is defined as $\lim_{SNR \longrightarrow \infty} C_{sum} / \log \text{SNR}$.

Interference Alignment Over Constant Synchronous Interference Channel

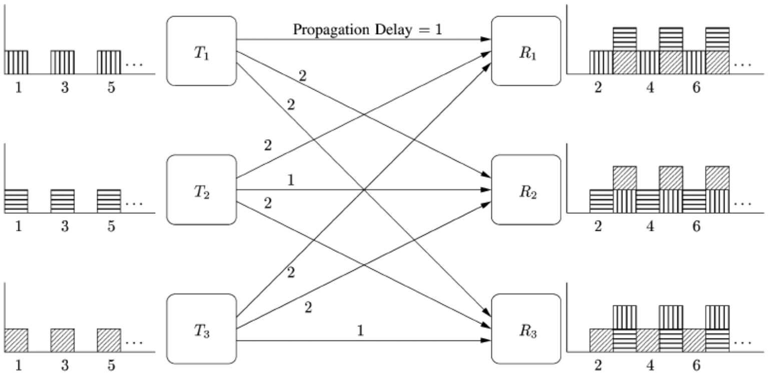
- The proposed alignment schemes for time varying channels requires the full CSI of all links at all nodes.
- Moreover, in practice, communication mediums are mostly block fading or quasi-static fading channels. Thus, alignment schemes for such channels are of practical interest.
- Alignment schemes for quasi-static scenarios:
  - *Signal level interference alignment based on lattices [Etkin-Ordentlich, June 2008], [Jafarian-Vishwanath, 2009]*
  - *Real interference alignment [Motahari-Gharan-Khandani, August 2009]*
- Some unrealistic assumptions: symmetric channel coefficients, rational/irrational channel coefficients over some links, infinite SNR, or infinite quantization precision.

Interference Channel is Inherently Asynchronous.

- Asynchronism *inherently* exists in many communication networks due to the effects of multi-path and propagation delay.
- If there is more than one receiver node, the system cannot be synchronized even if an infrastructure service provider is used.
- The validity of the previous alignment schemes strongly depends on the synchronization among the nodes.

- Our proposed alignment scheme for quasi-static channels
  - *does not make any assumption on fading coefficients.*
  - *does not need infinite quantization precision.*
  - *can be implemented in any range of SNR.*

Toy Example, [Cadambe-Jafar, 2007]

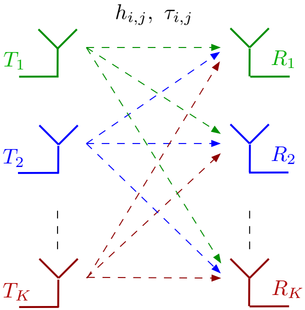


Outline

Asynchronous Interference Channel (AIC)

Channel parameters:

- $h_{i,j}$: fading coefficients
- $\tau_{i,j}$: relative propagation delays



We assume that $0 \leq \tau_{i,j} < T_s$. Let

$$\tau_{q,j}^{[i]} \triangleq \tau_{i,q} - \tau_{i,j}.$$

$\tau_{q,j}^{[i]}$ is a continuous random variable over $[-T_s, T_s]\ \forall i,j,q \in \{1,2,\ldots K\}$.

System Model

- Transmitted signal model by the j-th transmitter

$$x_j(t) = \sum_k x_j(k)\psi(t - kT_s),$$

$x_j(k)$ is the k-th transmitted symbol of the j-th user.
$\psi(t)$ is a unit energy shaping waveform.

- Received signal model at the i-th receiver

$$y_i(t) = \sum_{j=1}^{K} h_{i,j}x_j(t - \tau_{i,j}) + z_i(t).$$

Main Result

Theorem

The total number of DOF of the underlying K-user asynchronous interference channel is $K/2$.

- Proof of converse (upper bound on the DOF)?
- DOF-achieving signaling scheme?

Proof of Converse

Let $\psi(t)$ be the shaping waveform used by the transmitters. By taking the Fourier Transform (FT) of both sides of the received signal model at the i-th receiver node, we obtain

$$Y_i(f) = \sum_{j=1}^{K} h'_{i,j}(f) X_j(f) + Z_i(f), \quad -W \leq f \leq W,$$

where $h'_{i,j}(f) = h_{i,j}\Psi(f)e^{-\xi 2\pi f \tau_{i,j}}$. $\Psi(f)$ is the FT of $\psi(t)$ and $X_j(f)$ is the 2π-periodic Discrete-Time-Fourier-Transform (DTFT) of the transmitted sequence by the j-th transmitter.

For every value of f, the system is modeled as a synchronous K-user IC. Hence, the total DOF $\leq K/2$.

Signaling Scheme: At the Transmitters

At the j-th transmitter

- $\underline{x}_j = [x_j(0), x_j(1), \ldots, x_j(N-1)]^T$ is the vector of information symbols.
- $\psi(t)$ is the shaping waveform spanned over p symbol intervals.

$\underline{x}_j$ is supported by cyclic prefix and cyclic suffix (CPS) symbols each of length $p+1$.

- $\underline{x}_j \xrightarrow{CPS} \underline{x}_j^{cps} =$
$[x_j(N-p-1), \ldots, x_j(N-1), x_j(0), x_j(1), \ldots, x_j(N-1), x_j(0), \ldots, x_j(p)]^T$

$\underline{x}_j^{cps}$ of length $\ell = N + 2(p+1)$ is transmitted over the channel.

$$x_j(t) = \sum_{k=0}^{\ell-1} x_j^{cps}(k)\psi(t-kT_s).$$

Signaling Scheme: At the Receivers

The received signal at each receiver is first passed through a matched filter adjusted on the desired link and then is sampled at $t = kT_s + \tau_{i,i}$, $k = 0, 1, \ldots, \ell - 1$. After discarding the CPS symbols, we obtain

$$\underline{y}_i = \sum_{j=1}^{K} h_{i,j} \mathbf{\Gamma}_{i,j} \underline{x}_j + \underline{z}_i, \quad i \in \{1, 2, \ldots, K\},$$

where

$$\begin{aligned} \underline{x}_j &= [x_j(0), x_j(1), \ldots, x_j(N-1)]^T, \\ \underline{y}_i &= [y_i^{cps}(p+1), y_i^{cps}(p+2), \ldots, y_i^{cps}(N+p)]^T, \\ \underline{z}_i &= [z_i(p+1), z_i(p+2), \ldots, z_i(N+p)]^T. \end{aligned}$$

and

$$z_i(k) = \int_{kT_s+\tau_{i,i}}^{(k+p)T_s+\tau_{i,i}} z_i(t)\psi^*(t - kT_s - \tau_{i,i})dt.$$

Matrix $\Gamma_{i,j}$

$$\mathbf{\Gamma}_{i,j} = \begin{bmatrix} \gamma_{i,j}(0) & \cdots & \gamma_{i,j}(-p) & 0 & \ldots & 0 & \gamma_{i,j}(p) & \ldots & \gamma_{i,j}(1) \\ \gamma_{i,j}(1) & \cdots & \gamma_{i,j}(-p+1) & 0 & \ldots & 0 & \gamma_{i,j}(p) & \ldots & \gamma_{i,j}(2) \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & \gamma_{i,j}(p) & \cdots & \gamma_{i,j}(1) & \gamma_{i,j}(0) & \cdots & \gamma_{i,j}(-p) \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \gamma_{i,j}(-1) & \cdots & \gamma_{i,j}(-p) & 0 & \cdots & 0 & \gamma_{i,j}(p) & \cdots & \gamma_{i,j}(0) \end{bmatrix}$$

is the circular convolution matrix of the sequence
$\underline{\hat{\gamma}}_{i,j} \triangleq [\gamma_{i,j}(0), \gamma_{i,j}(1), \ldots, \gamma_{i,j}(p), 0, \ldots, 0, \gamma_{i,j}(-p), \gamma_{i,j}(-p+1), \ldots, \gamma_{i,j}(-1)]^T$,
where

$$\gamma_{i,j}(q) = \int_0^{pT_s} \psi(t - qT_s - \tau_{j,i}^{[i]})\psi^*(t)dt.$$

- Note that the use of CPS symbols is to make $\mathbf{\Gamma}_{i,j}$ a circulant matrix.

Matrix $\Gamma_{i,j}$, Cont'd

Since $\mathbf{\Gamma}_{i,j}$ is circulant, it can be decomposed on the DFT basis. i.e.,

$$\mathbf{\Gamma}_{i,j} = \mathbf{U}^{\dagger} \mathbf{\Lambda}_{i,j} \mathbf{U},$$

where $\mathbf{U}$ is the Discrete Fourier Transform (DFT) matrix of dimension N and $\mathbf{\Lambda}_{i,j}$ is a diagonal matrix containing the DFT of the vector $\underline{\hat{\gamma}}_{i,j}$ on its main diagonal.

Proposition

$\mathbf{\Gamma}_{i,j}$ *is a full rank matrix* $\forall\, i,j \in \{1, 2, \ldots, K\}$. *In addition, its eigenvalues (the diagonal entries of* $\mathbf{\Lambda}_{i,j}$*) are bounded.*

Shift Property of $\mathbf{\Lambda}_{i,j}$

It is proved that if the shaping waveform has a sub-linear decaying rate in time (i.e., $|\psi(t)| \leq a/|t/T_s|^{\eta}$, $\eta > 1$), $\mathbf{\Lambda}_{i,j}$ can be written as follows.

$$\mathbf{\Lambda}_{i,j} = \mathbf{\Lambda}_0 \mathbf{E}\left(\hat{\tau}_{i,j}^{[i]}\right) + \epsilon,$$

where $|\epsilon|$ is bounded and goes to zero if $p \to \infty$. Here, $\hat{\tau}_{i,j}^{[i]} = \frac{\tau_{i,j}^{[i]}}{T_s}$, $\mathbf{\Lambda}_0$ is defined similar to $\mathbf{\Lambda}_{i,j}$ when $\hat{\tau}_{i,j}^{[i]} = 0$, and

$$\mathbf{E}(\hat{\tau}_{i,j}^{[i]}) = \mathtt{diag}\{1, e^{-\xi \frac{2\pi}{N} \hat{\tau}_{i,j}^{[i]}}, \ldots, e^{-\xi \frac{2\pi(N-1)}{N} \hat{\tau}_{i,j}^{[i]}}\}.$$

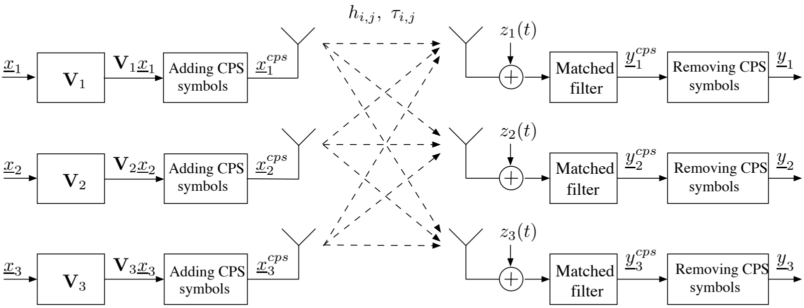
Prior Works

The idea of using propagation delay to align the interfering signals has been reported before in the following works.

- Can 100 speakers talk for 30 minutes each in one room within one hour and with zero interference to each other's audience? (Cadambe-Jafar, 2007).
- Degrees of freedom of wireless networks - what a difference delay makes, (Cadambe-Jafar, 2007).
- Interference Alignment for Line-of-Sight Channels, (Grokop-Tse-Yates, 2008).

- Our work is in the line of vector interference alignment algorithm proposed in [Cadambe-Jafar, August 2008], however, for constant fading channels. The main idea is to convert the quasi-static links into time varying channels using the inherent asynchronism among the nodes.

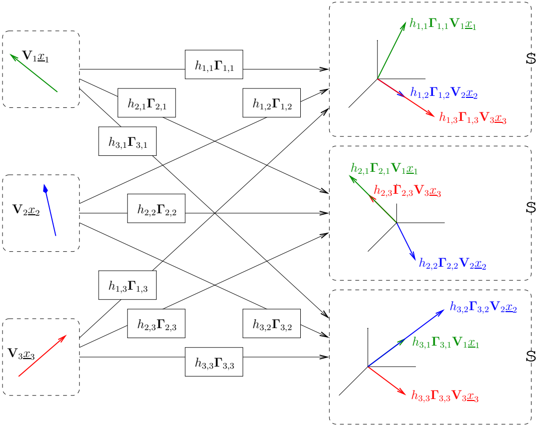
Alignment Scheme in a Three-User Interference Channel



$\underline{x}_1$: of length $(n+1)$, $\underline{x}_2, \underline{x}_3$: of length n,

$\mathbf{V}_1$: of size $N \times (n+1)$, $\mathbf{V}_2, \mathbf{V}_3$: of size $N \times n$, $N = 2n+1$.

Alignment Scheme in a Three-User Interference Channel



$$\mathtt{span}\left[h_{1,1}\mathbf{\Gamma}_{1,1}\mathbf{V}_1,\ h_{1,3}\mathbf{\Gamma}_{1,3}\mathbf{V}_3\right] = \mathbb{C}^N,$$

$$\mathtt{span}\left[h_{2,2}\mathbf{\Gamma}_{2,2}\mathbf{V}_2,\ h_{2,1}\mathbf{\Gamma}_{2,1}\mathbf{V}_1\right] = \mathbb{C}^N,$$

$$\mathtt{span}\left[h_{3,3}\mathbf{\Gamma}_{3,3}\mathbf{V}_3,\ h_{3,1}\mathbf{\Gamma}_{3,1}\mathbf{V}_1\right] = \mathbb{C}^N,$$

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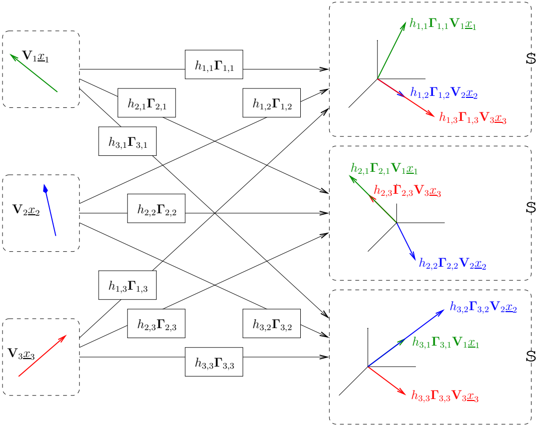

$$\mathtt{span}\left[h_{1,1}\mathbf{\Gamma}_{1,1}\mathbf{V}_1,\ h_{1,3}\mathbf{\Gamma}_{1,3}\mathbf{V}_3\right] = \mathbb{C}^N,$$

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$$\mathtt{span}\left[h_{3,3}\mathbf{\Gamma}_{3,3}\mathbf{V}_3,\ h_{3,1}\mathbf{\Gamma}_{3,1}\mathbf{V}_1\right] = \mathbb{C}^N,$$

A Simple choice of generator matrices [Cadambe-Jafar 2008]

$$\begin{aligned}\mathbf{\Gamma}_{1,2}\mathbf{V}_2 &= \mathbf{\Gamma}_{1,3}\mathbf{V}_3,\\ \mathbf{\Gamma}_{2,3}\mathbf{V}_3 &\prec \mathbf{\Gamma}_{2,1}\mathbf{V}_1,\\ \mathbf{\Gamma}_{3,2}\mathbf{V}_2 &\prec \mathbf{\Gamma}_{3,1}\mathbf{V}_1.\end{aligned} \quad \longrightarrow \quad \begin{aligned}\mathbf{B} &= \mathbf{TC},\\ \mathbf{B} &\prec \mathbf{A},\\ \mathbf{C} &\prec \mathbf{A},\end{aligned} \quad \text{where} \quad \begin{aligned}\mathbf{A} &= \mathbf{V}_1,\\ \mathbf{B} &= \mathbf{\Gamma}_{2,1}^{-1}\mathbf{\Gamma}_{2,3}\mathbf{V}_3,\\ \mathbf{C} &= \mathbf{\Gamma}_{3,1}^{-1}\mathbf{\Gamma}_{3,2}\mathbf{V}_2,\\ \mathbf{T} &= \mathbf{\Gamma}_{2,1}^{-1}\mathbf{\Gamma}_{2,3}\mathbf{\Gamma}_{1,3}^{-1}\mathbf{\Gamma}_{1,2}\mathbf{\Gamma}_{3,2}^{-1}\mathbf{\Gamma}_{3,1}.\end{aligned}$$

By considering $\Gamma_{i,j} = \mathbf{U}^{\dagger}\mathbf{\Lambda}_{i,j}\mathbf{U} = \Gamma_{i,j} = \mathbf{U}^{\dagger}\mathbf{\Lambda}_0\mathbf{E}(\hat{\tau}_{i,j}^{[i]})\mathbf{U}$, we get

$$\begin{aligned}\mathbf{A} &= \mathbf{V}_1,\\ \mathbf{B} &= \mathbf{U}^{\dagger}\mathbf{E}(\hat{\tau}_{1,3}^{[2]})\mathbf{U}\mathbf{V}_3,\\ \mathbf{C} &= \mathbf{U}^{\dagger}\mathbf{E}(\hat{\tau}_{1,2}^{[3]})\mathbf{U}\mathbf{V}_2,\\ \mathbf{T} &= \mathbf{U}^{\dagger}\mathbf{E}(\hat{\tau}_t)\mathbf{U},\end{aligned}$$

where $\hat{\tau}_t = \hat{\tau}_{1,3}^{[2]} + \hat{\tau}_{3,2}^{[1]} + \hat{\tau}_{2,1}^{[3]}$.

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where $\hat{\tau}_t = \hat{\tau}_{1,3}^{[2]} + \hat{\tau}_{3,2}^{[1]} + \hat{\tau}_{2,1}^{[3]}$.

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where $\hat{\tau}_t = \hat{\tau}_{1,3}^{[2]} + \hat{\tau}_{3,2}^{[1]} + \hat{\tau}_{2,1}^{[3]}$.

A Proper Choice of Precoding Matrices

$\mathbf{A}, \mathbf{B}, \mathbf{C}$ are chosen as follows.

$$\mathbf{A} = [\underline{w}\ \mathbf{T}\underline{w}\ \ldots\ \mathbf{T}^n\underline{w}],$$
$$\mathbf{B} = [\mathbf{T}\underline{w}\ \mathbf{T}^2\underline{w}\ \ldots\ \mathbf{T}^n\underline{w}],$$
$$\mathbf{C} = [\underline{w}\ \mathbf{T}\underline{w}\ \ldots\ \mathbf{T}^{n-1}\underline{w}].$$

$\underline{w}$ is a vector of length N such that $\underline{w}' = \mathbf{U}\underline{w}$ contains non-zero entries. The precoding matrices $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$ are obtained accordingly.

Proposition

Matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and accordingly the precoding matrices $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$ are almost surly full column rank.

Sketch of the proof:

$$\mathbf{A} = [\underline{w}, \mathbf{T}\underline{w}, \ldots, \mathbf{T}^n\underline{w}] = \mathbf{U}^\dagger \mathbf{W} \hat{\mathbf{T}}$$

$$= \mathbf{U}^\dagger \mathbf{W} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \phi & \phi^2 & \cdots & \phi^n \\ 1 & \phi^2 & \phi^4 & \cdots & \phi^{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \phi^{N-1} & \phi^{2(N-1)} & \cdots & \phi^{n(N-1)} \end{bmatrix}$$

where $\mathbf{W} = \mathtt{diag}\{\underline{w}'\}$, $\phi = e^{-\xi \frac{2\pi}{N} \hat{\tau}_t}$, and $\hat{\tau}_t = \hat{\tau}_{3,1}^{[2]} + \hat{\tau}_{2,3}^{[1]} + \hat{\tau}_{1,2}^{[3]}$, $\hat{\tau}_t \in (-3, 3)$.
$\hat{\mathbf{T}}$ is a Vandermonde matrix.

The proof is similar for **B** and **C**.

Proposition

The images of the precoding matrices at each receiver node span the whole vector space of dimension N.

Sketch of the proof: Let $\mathbf{F} \triangleq \mathbf{\Gamma}_{1,1}^{-1}\mathbf{\Gamma}_{1,3}\mathbf{\Gamma}_{2,3}^{-1}\mathbf{\Gamma}_{2,1} = \mathbf{U}^{\dagger}\mathbf{E}\left(\hat{\tau}_f\right)\mathbf{U}$ where $\hat{\tau}_f = \hat{\tau}_{3,1}^{[1]} + \hat{\tau}_{1,3}^{[2]}$.

$$[\mathbf{A},\ \mathbf{FB}] = \left[\underline{w},\ \mathbf{T}\underline{w},\ \ldots,\ \mathbf{T}^n\underline{w},\ \mathbf{F}\underline{w},\ \mathbf{FT}\underline{w},\ \ldots,\ \mathbf{FT}^{n-1}\underline{w}\right] = \mathbf{U}^{\dagger}\mathbf{W}\tilde{\mathbf{T}} = \mathbf{U}^{\dagger}\mathbf{W}\times$$

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ 1 & \phi & \cdots & \phi^n & \theta & \theta\phi & \cdots & \theta\phi^{(n-1)} \\ 1 & \phi^2 & \cdots & \phi^{2n} & \theta^2 & \theta^2\phi^2 & \cdots & \theta^2\phi^{2(n-1)} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \phi^{N-1} & \cdots & \phi^{n(N-1)} & \theta^{N-1} & \theta^{N-1}\phi^{N-1} & \cdots & \theta^{N-1}\phi^{(N-1)(n-1)} \end{bmatrix}$$

$\phi = e^{-\xi\frac{2\pi}{N}\hat{\tau}_t}, \theta = e^{-\xi\frac{2\pi}{N}\hat{\tau}_f}$.

Sketch of the Proof

$\tilde{\mathbf{T}}$ is a Vandermonde matrix. $\det \tilde{\mathbf{T}}$ is zero only when

$$\theta^{\alpha_1}\phi^{\beta_1} = \theta^{\alpha_2}\phi^{\beta_2} \Rightarrow (\alpha_1 - \alpha_2)\hat{\tau}_f + (\beta_1 - \beta_2)\hat{\tau}_t = kN, \quad k \in \mathbb{Z},$$

where $\alpha_1, \alpha_2 \in \{0, 1\}$ and $\{\beta_1, \beta_2\} \in \{0, 1, \ldots, n\}$.
Since $\hat{\tau}_f$, $\hat{\tau}_t$ are independent continuous random variables, the probability that they satisfy the above equation is zero. ■
As a sufficient condition, if $\{\hat{\tau}_f, \hat{\tau}_t, N\}$ are algebraically independent over the integers, the above equation does not hold.

Main Result for 3-User IC

Theorem

The proposed interference alignment scheme achieves the total DOF equal to $3/2$ *over the constant three-user asynchronous interference channel almost surly.*

- $3n+1$ independent information symbols are transmitted interference free over $2n+1+2(p+1)$ symbol intervals.
- The efficiency of the transmission is $\frac{3n+1}{2n+1+2(p+1)} \rightarrow \frac{3}{2}$ as $n \rightarrow \infty$ regardless of the type of the shaping waveforms.
- If all transmitters use a long enough truncated version of the Root-Square-Raised-Cosine waveform, the total DOF of the channel equal to 3/2 is achieved.

A similar alignment scheme is proposed for the K-user asynchronous IC to achieve the total $K/2$ DOF.

Discussion

- The existence of CPS symbols avoids achieving the same total DOF of the corresponding time varying channel equal to $\frac{3n+1}{2n+1}$ at finite length codewords.
- One can use OFDM processing at the transmitters and the receivers to simplify the math. However, inserting CPS symbols at the beginning and at the end of each frame is necessary to validate our proofs (to make $\mathbf{\Gamma}_{i,j}$ matrices circulant).
- The scheme does not apply to fully synchronous channel scenarios by inserting artificial delays among users. Note that $\mathbf{F} = \mathbf{U}^{\dagger}\mathbf{E}\left(\hat{\tau}_f\right)\mathbf{U} = \mathbf{I}_N$ $(\hat{\tau}_f = \hat{\tau}_{3,1}^{[1]} + \hat{\tau}_{1,3}^{[2]})$ if the users are synchronous or artificial delays are inserted.
- It is proved that the DOF region of the underlying channel is the same as that of the corresponding three-user synchronous network with varying fading coefficients.
- Similar results are obtain when the shaping waveform has infinite time support.

Conclusion

- A novel vector interference alignment scheme was proposed for the constant three-user interference channel.
- The transmitters do not need to know the CSI of the links; however, the asynchronous delays of the links need to be known to the entire network.
- Asynchronous delays do not necessarily need to be irrational numbers. Therefore, in difference with the real interference alignment technique, a finite number of quantization levels is required to implement the scheme in any range of SNR.
- Asynchronous delays do not necessarily need to be less than a symbol interval.
- The proposed scheme is generalized to use in a K-user interference channel, $K > 3$ with no major modifications.
- The proposed scheme can be applied to other interference scenarios such as X-channel and Z-channel with single antenna or multiple antenna nodes to achieve the total DOF under quasi-static assumption.

Thanks for your attention.