Data-Selective Estimation Algorithms with Applications

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Outline

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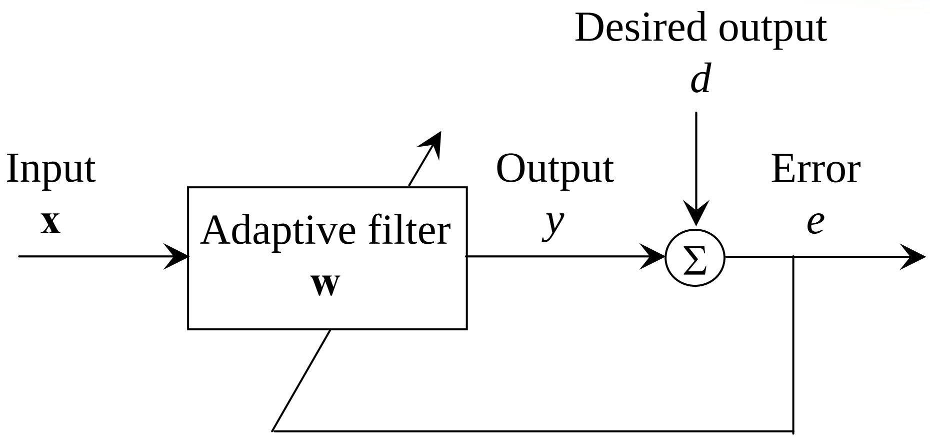
3. Set-membership filtering
4. Set-membership binormalized data-reusing algorithms
5. Set-membership affine projection algorithm
6. Partial-update adaptive filters with data-selective updating
7. Conclusions

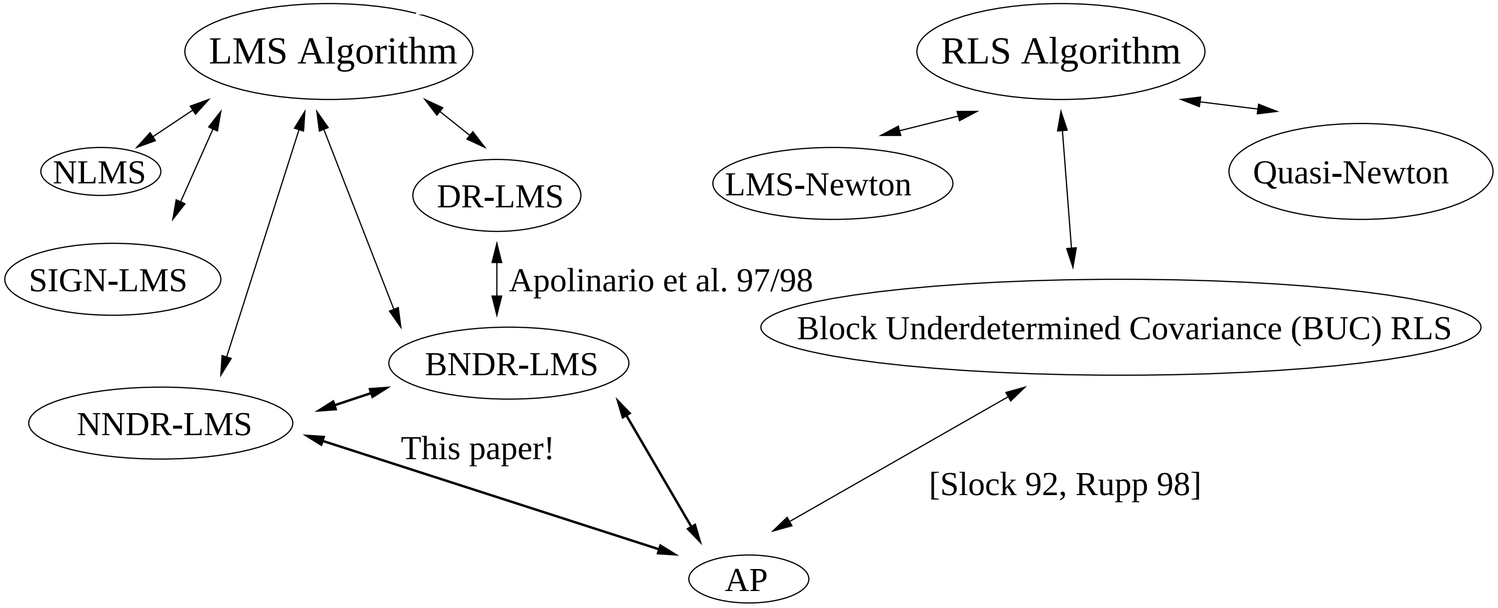
Introduction



- Adjust the adaptive filter such that the output error is minimized using some optimality criterion
- Basic design goals
  - fast convergence
  - low misadjustment
  - low computational complexity

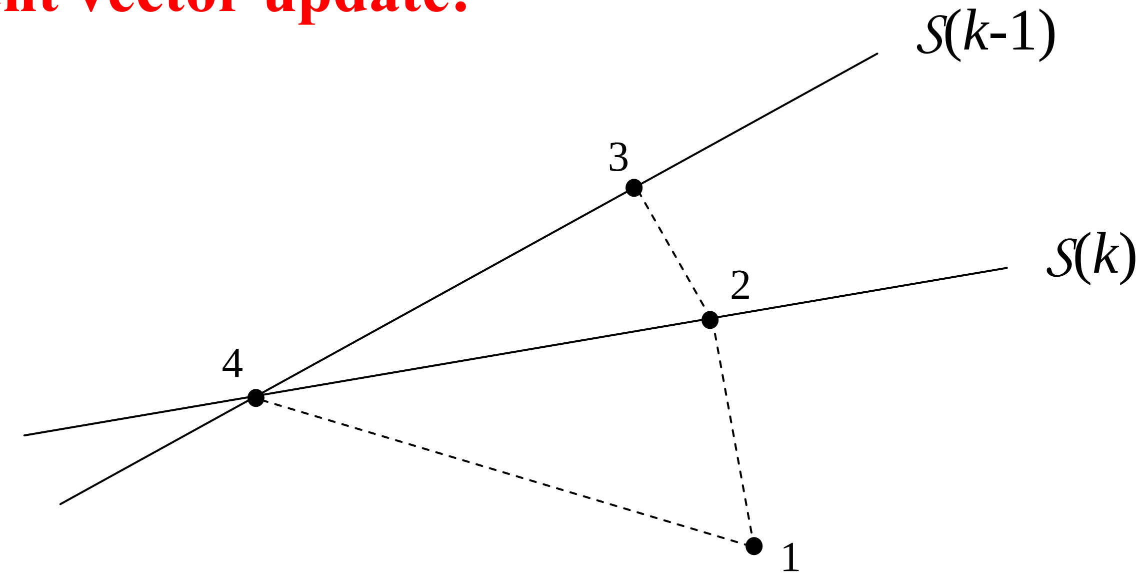
Usually conflicting requirements

Relationship between algorithms



The algorithms in R^2

Coefficient vector update:



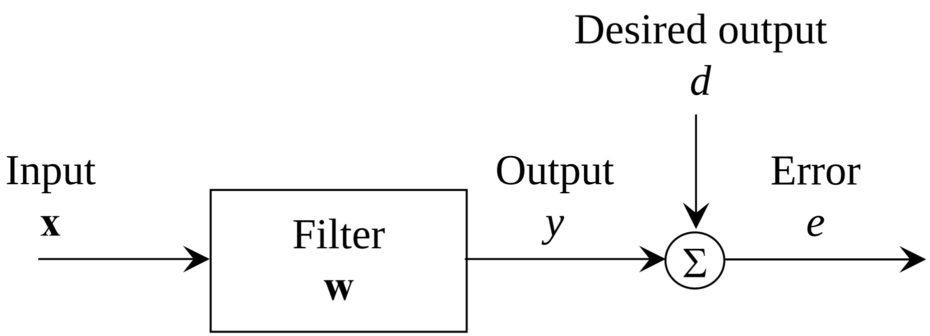
Position 1. $\mathbf{w}(k)$;

Position 2. $\mathbf{w}_{NLMS}(k+1)$ and first step towards $\mathbf{w}_{NNDR\text{-}LMS}(k+1)$;

Position 3. $\mathbf{w}_{NNDR\text{-}LMS}(k+1)$;

Position 4. $\mathbf{w}_{BNRLMS}\,(k+1)$ and $\mathbf{w}_{AP}(k+1)$;

Set-membership filtering (SMF)



- The filter is designed to achieve a specified bound γ on the magnitude of the estimation error $e = d - \mathbf{w}^T\mathbf{x}$
- Any coefficient vector resulting in an error with magnitude less than γ is a valid estimate

... Set-membership filtering (SMF)

- *Constraint $\mathcal{H}(k)$ set:* input-desired output pairs at time k

$$\mathcal{H}(k) = \{\mathbf{w} : |d(k) - \mathbf{w}^{\mathrm{T}}\mathbf{x}(k)| \leq \gamma\},$$

- *Exact membership set $\psi(k)$*: input-desired output pairs up to time k

$$\psi(k) = \bigcap_{i=1}^{k} \mathcal{H}(i)$$

Problem: find a solution **w** in the memerbship set $\psi(k)$

Adaptive SM solutions

Problem:

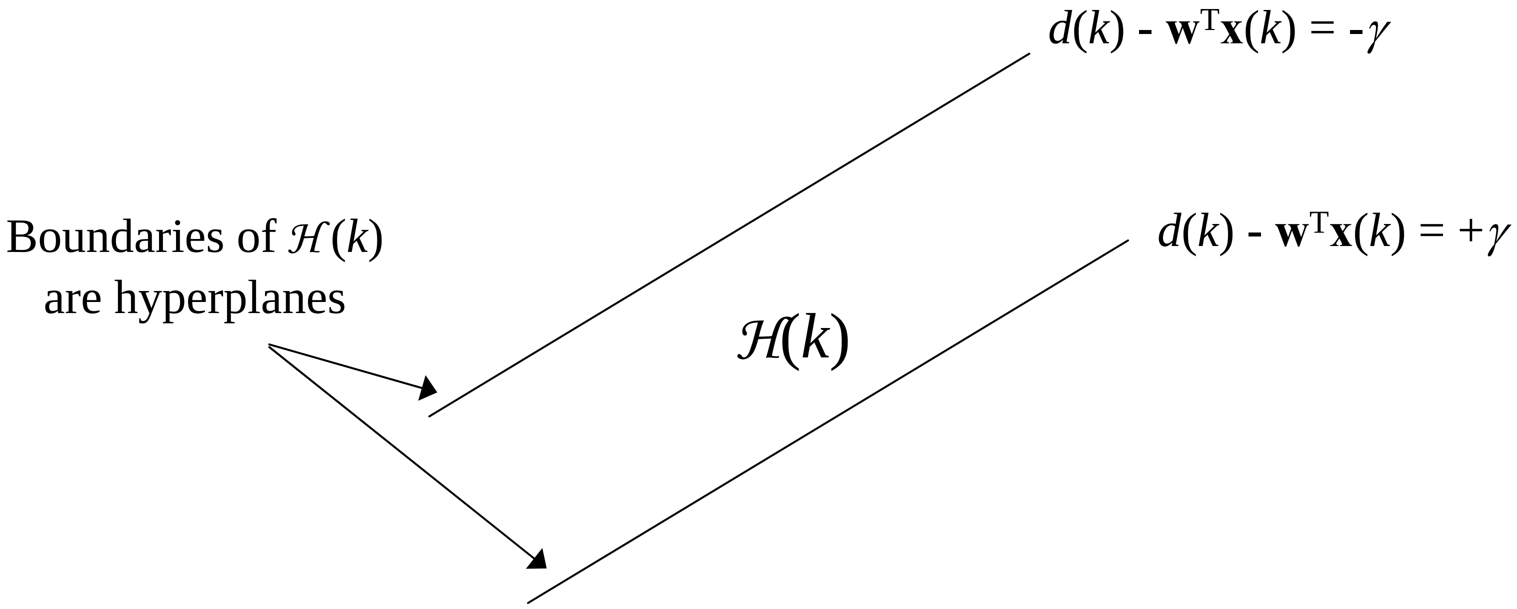
The set $\psi(k)$ is not easily computed

Solution:

- Try to outer bound $\psi(k)$ ellipsoids $\Rightarrow$ OBE algorithms
  - "Similar" to the RLS algorithm but with selective updating
- Develop algorithms that only work with a fixed number of past constraint sets $\mathcal{H}(i)$
  - Set-membership NLMS (SM-NLMS) [Gollamudi *et. al*. 1998] uses the information provided by $\mathcal{H}(k) = \{\mathbf{w} : |d(k) - \mathbf{w}^T\mathbf{x}(k)| \leq \gamma\}$

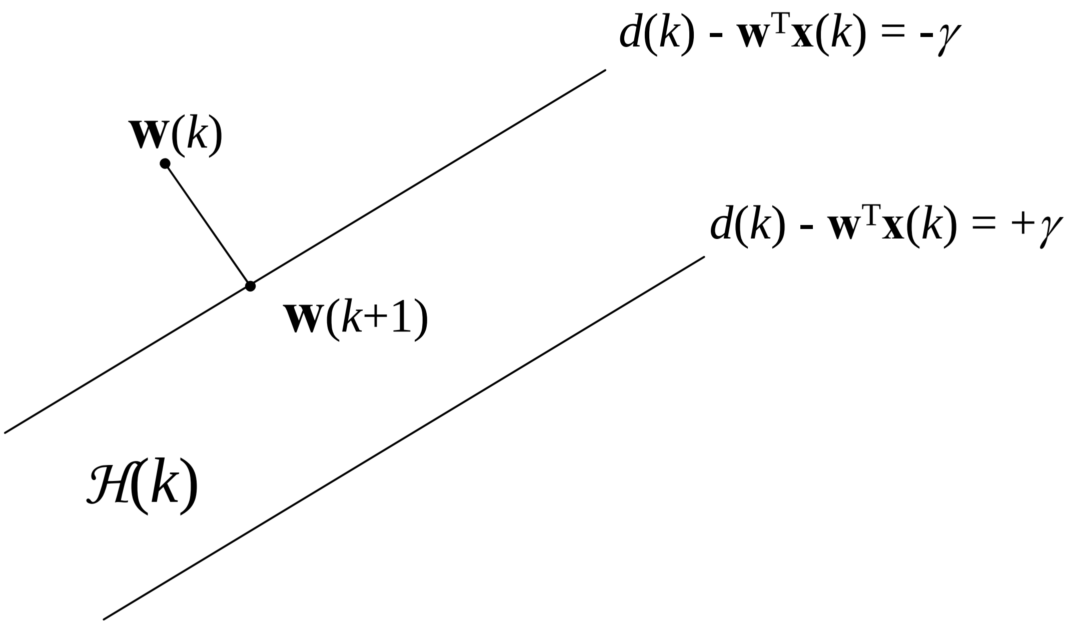
... Adaptive SM solutions

Graphical view of $\mathcal{H}(k) = \{\mathbf{w} : |d(k) - \mathbf{w}^T\mathbf{x}(k)| \le \gamma\}$



Set-membership NLMS in R^2

Utilize $\mathcal{H}(k)$ in the update [Gollamudi et. al. in SPL May 98]:



- If $\mathbf{w}(k) \in \mathcal{H}(k)$, i.e., $|e(k)| \leq \gamma \Rightarrow$ no update
- Otherwise choose $\mathbf{w}(k+1)$ as in figure

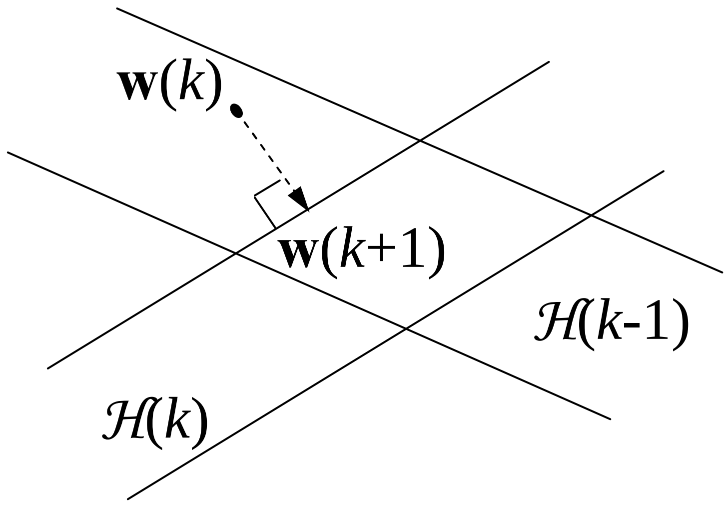
SM-BNDRLMS algorithms

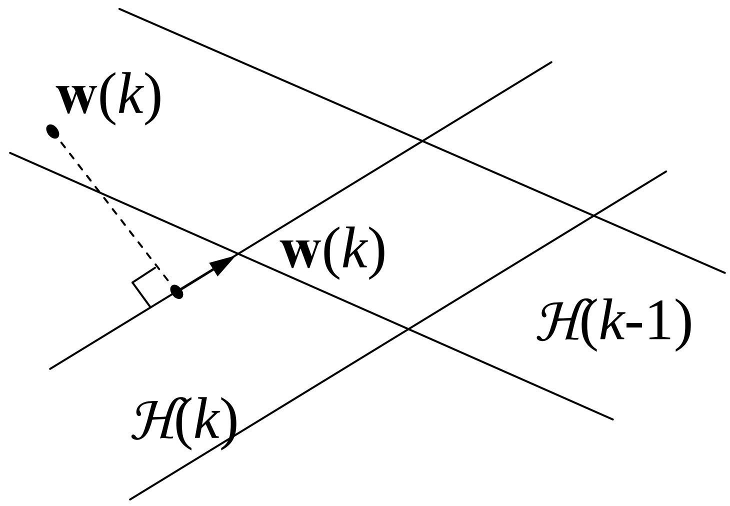
Objective:

Improve performance for correlated input. Derive data-selective algorithms with low computational complexity per update by reusing the constraint sets at time instants k and k-1, i.e., $\mathbf{w}(k+1) \in \mathcal{H}(k)$ and $\mathbf{w}(k+1) \in \mathcal{H}(k-1)$.

- The two algorithms to be presented here
  - SM-BNDRLMS-I minimizes the Euclidean distance between two updates $\|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2$ such that $\mathbf{w}(k) \in \mathcal{H}(k) \cap \mathcal{H}(k-1)$
  - SM-BNDRLMS-II chooses a different update strategy with a lower computational complexity per update

SM-BNDRLMS-I algorithm in R^2

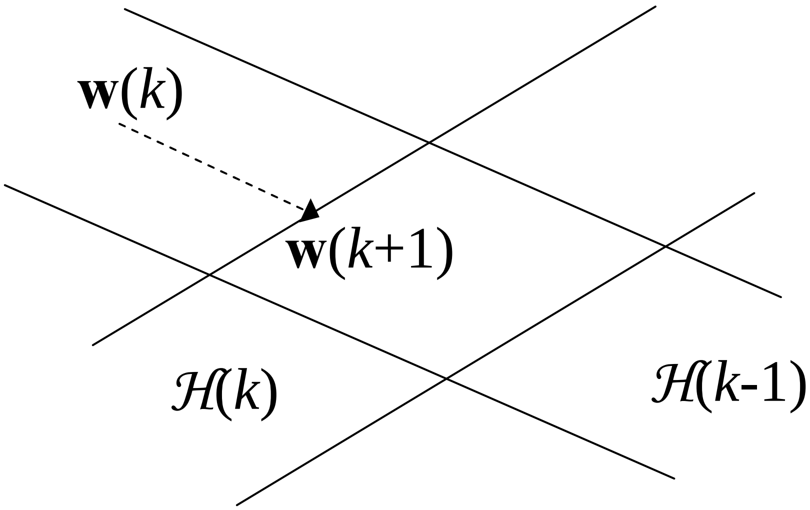




- If $\mathbf{w}_n \in \mathcal{H}(k)$, i.e., $|e(k)| \leq \gamma \Rightarrow$ no update
- Otherwise project onto closest boundary of $\mathcal{H}(k)$

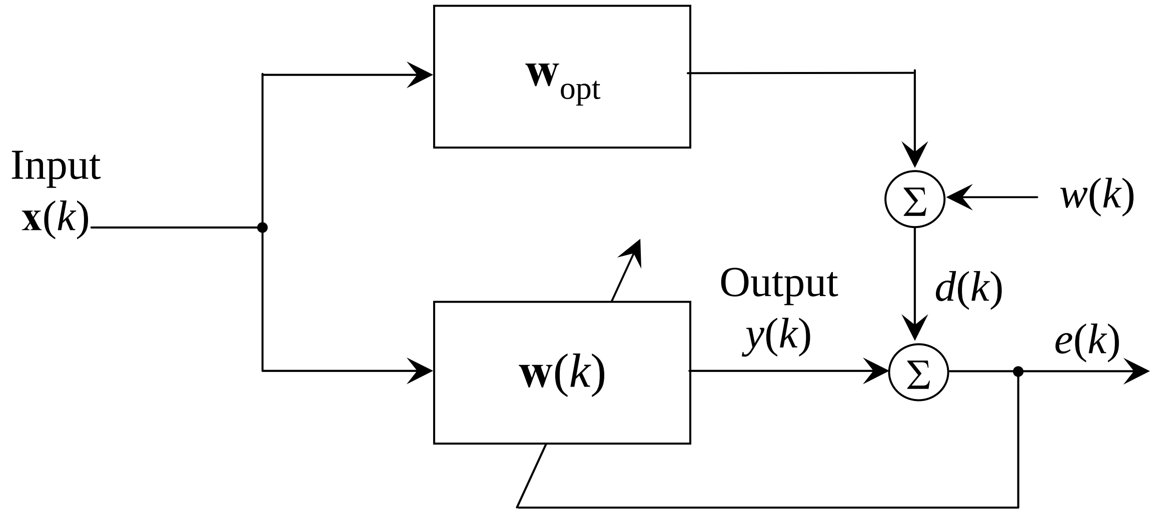
- If first projection outside $\mathcal{H}(k\text{-}1)$ $\Rightarrow$ perform second step s.t. solution at intersection

SM-BNDRLMS-II algorithm in R^2



- If $\mathbf{w}(k) \in \mathcal{H}(k)$, i.e., $|e(k)| \leq \gamma \Rightarrow$ no update
- Otherwise choose $\mathbf{w}(k+1)$ such that the *a posteriori* error at iteration k-1 [$\varepsilon(k-1) = d(k-1) - \mathbf{w}^T(k)\,\mathbf{x}(k-1)$] is kept constant.

System identification example:



Order of plant: $p = 10$

Input signal: colored noise, lowpass filtered

Threshold used with the algorithms: $\gamma = \sigma_w \cdot \sqrt{5}$

… Simulations

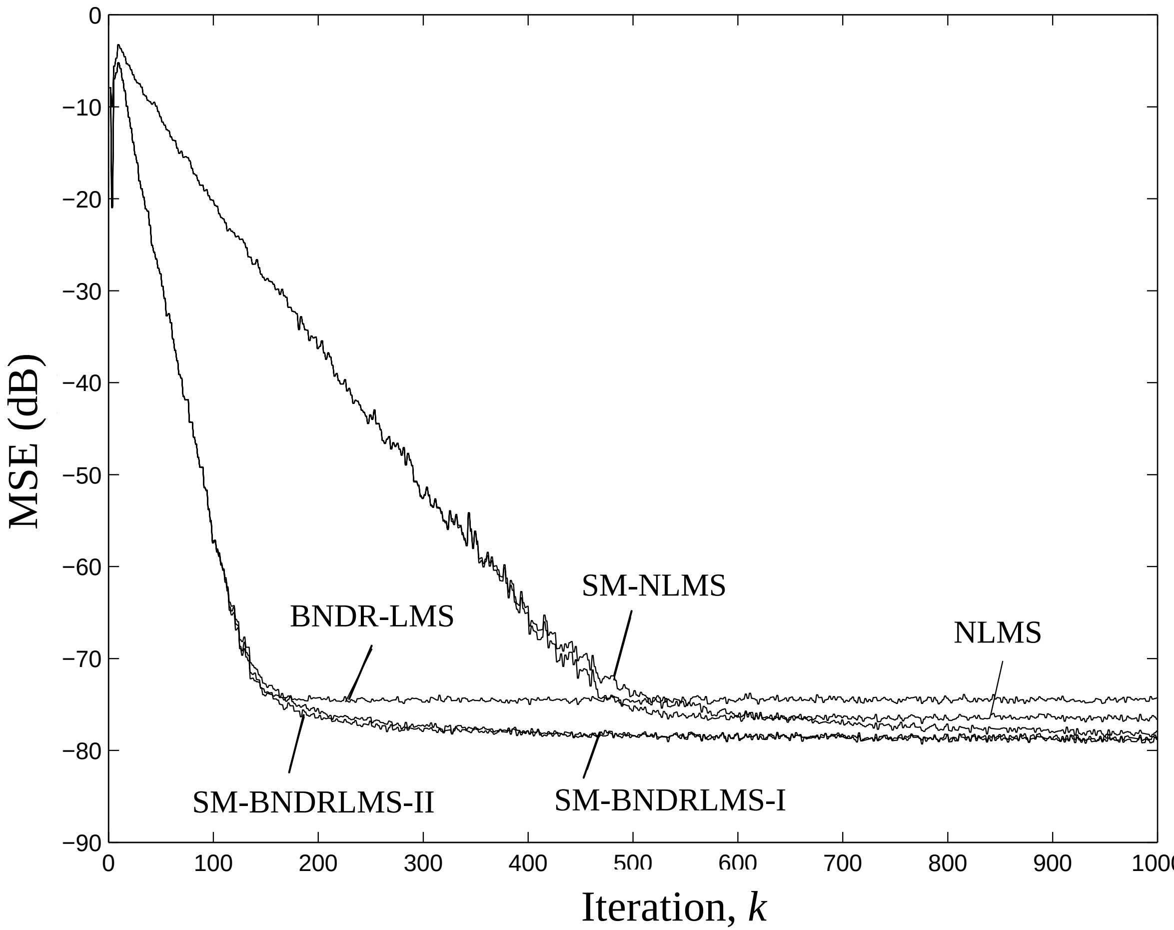
SNR = 80 dB

Number of updates:

SM-NLMS: 436/1000

SM-BNDRLMS-I: 185/1000

SM-BNDRLMS-II: 180/1000



... Simulations

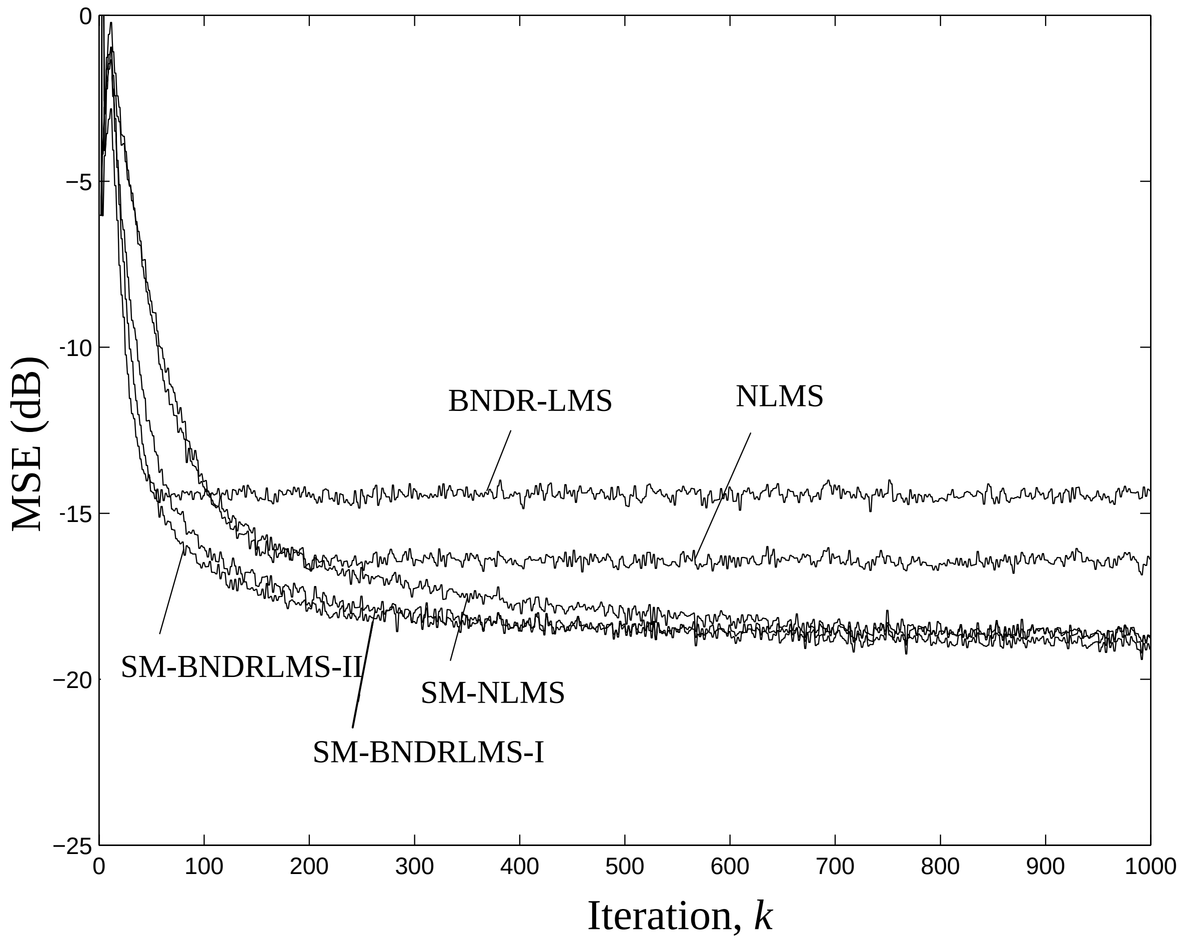
SNR = 20 dB

Number of updates:

SM-NLMS: 129

SM-BNDRLMS-I: 100

SM-BNDRLMS-II: 95



Set-Membership Affine Projection Algorithm

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SM-AP algorithm

- The exact membership set suggests more than one reuse, express $\psi(k)$ as

$$\psi(k) = \bigcap_{i=1}^{k-P} \mathcal{H}(i) \bigcap_{j=k-P+1}^{k} \mathcal{H}(i) = \psi^{k-P}(k) \bigcap \psi^{P}(k)$$

information provided by $\mathcal{H}(k)$, $\mathcal{H}(k\text{-}1)$, …, $\mathcal{H}(k\text{-}P\text{+}1)$

Objective:

Derive an algorithm whose coefficient update belongs to the last P constraint sets, i.e., $\mathbf{w}(k+1) \in \psi^{P}(k)$

... SM-AP algorithm

or more compact

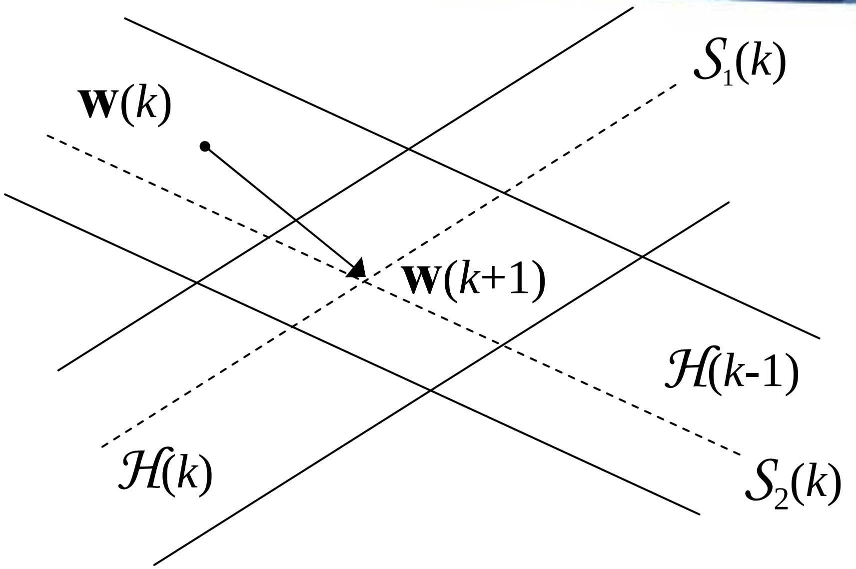
$$\min \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 \quad \text{subjected to:}$$
$$\mathbf{d}(k) - \mathbf{X}^{\mathrm{T}}(k)\mathbf{w}(k+1) = \mathbf{g}(k)$$

where

$$\mathbf{d}(k) = \begin{bmatrix} d(k) & d(k-1) & \cdots & d(k-P+1) \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{g}(k) = \begin{bmatrix} g_1(k) & g_2(k) & \cdots & g_P(k) \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}(k) & \mathbf{x}(k-1) & \cdots & \mathbf{x}(k-P+1) \end{bmatrix}$$
$$\mathbf{x}(k) = \begin{bmatrix} x(k) & x(k-1) & \cdots & x(k-N+1) \end{bmatrix}^{\mathrm{T}}$$

... SM-AP algorithm

Graphical illustration:



Solution:

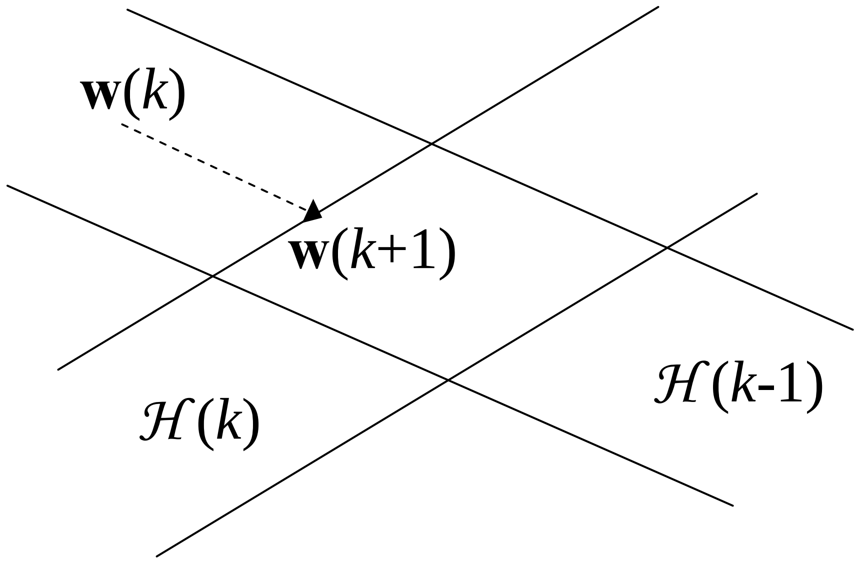
- Filter update: $\mathbf{w}(k+1) = \mathbf{w}(k) + \mathbf{X}(k)\mathbf{t}(k)$

  where

$$\mathbf{X}^{\mathrm{T}}(k)\mathbf{X}(k)\mathbf{t}(k) = \Big(\underbrace{\mathbf{d}(k) - \mathbf{X}^{\mathrm{T}}(k)\mathbf{w}(k)}_{\mathbf{e}(k)} - \mathbf{g}(k)\Big)$$

... SM-AP algorithm in R^2

For $P = 2$ we have the SM-BNDRLMS-II algorithm:



Simulations

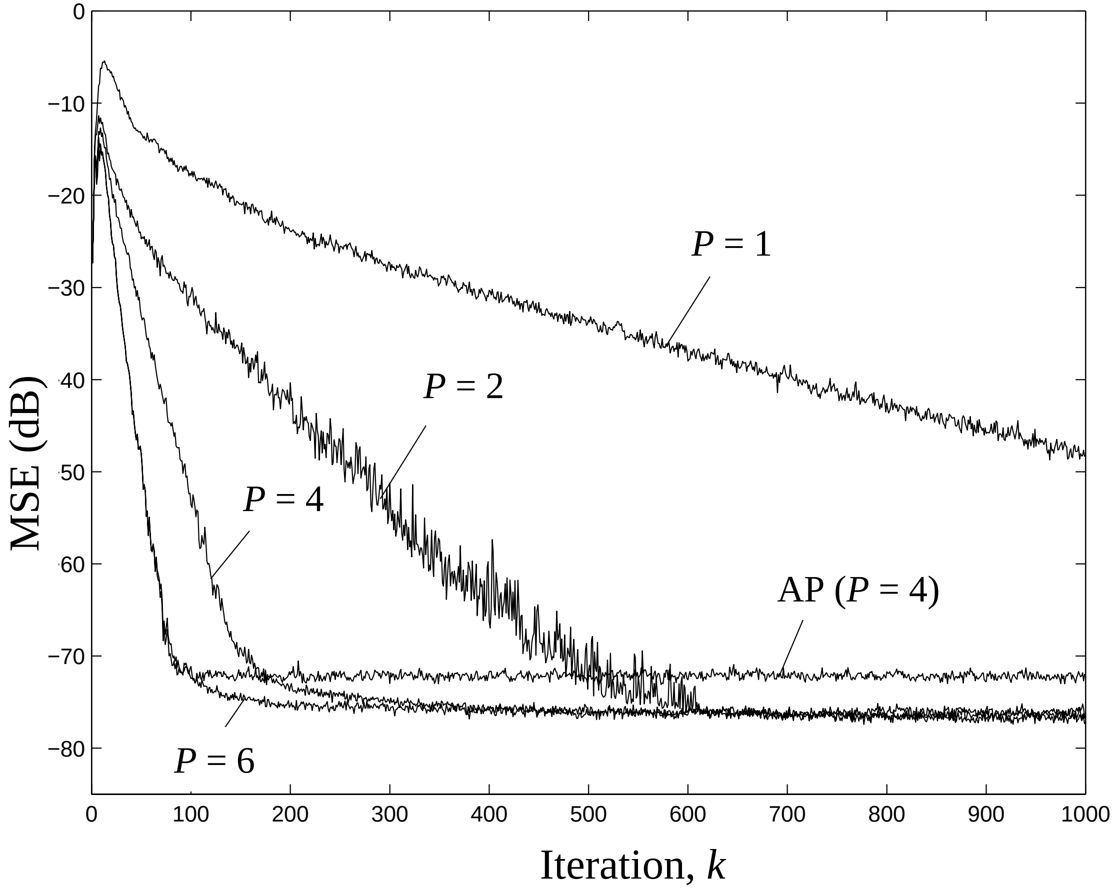
Number of updates:

P=1 (SM-NLMS): 967

P=2 (SM-BNDRLMS): 386

P=4: 263

P=6: 214



Partial-Update NLMS algorithms with data-selective updating

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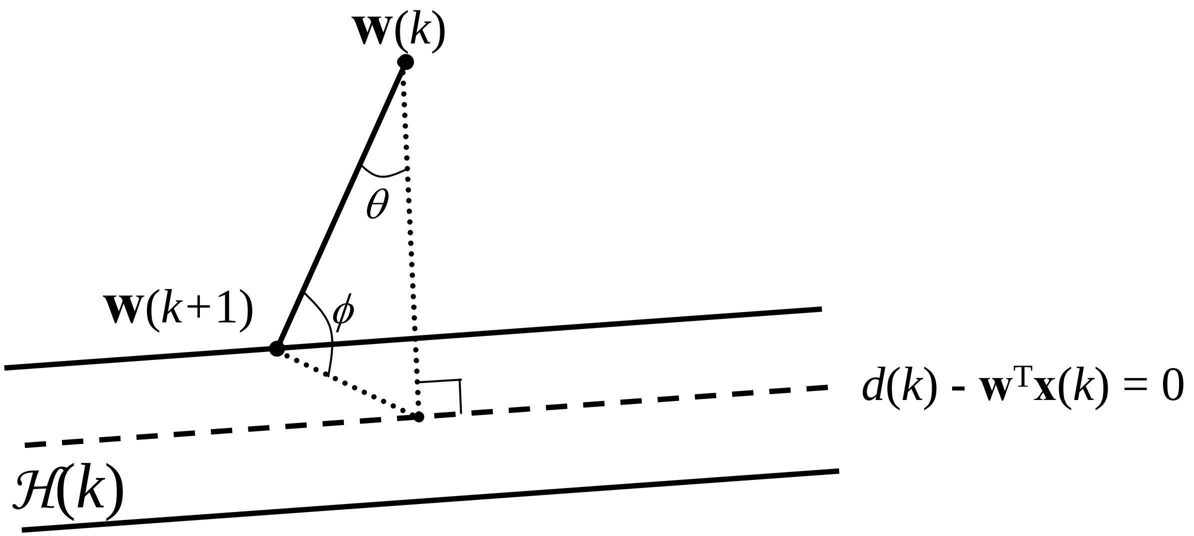
SM-PU-NLMS in R^2

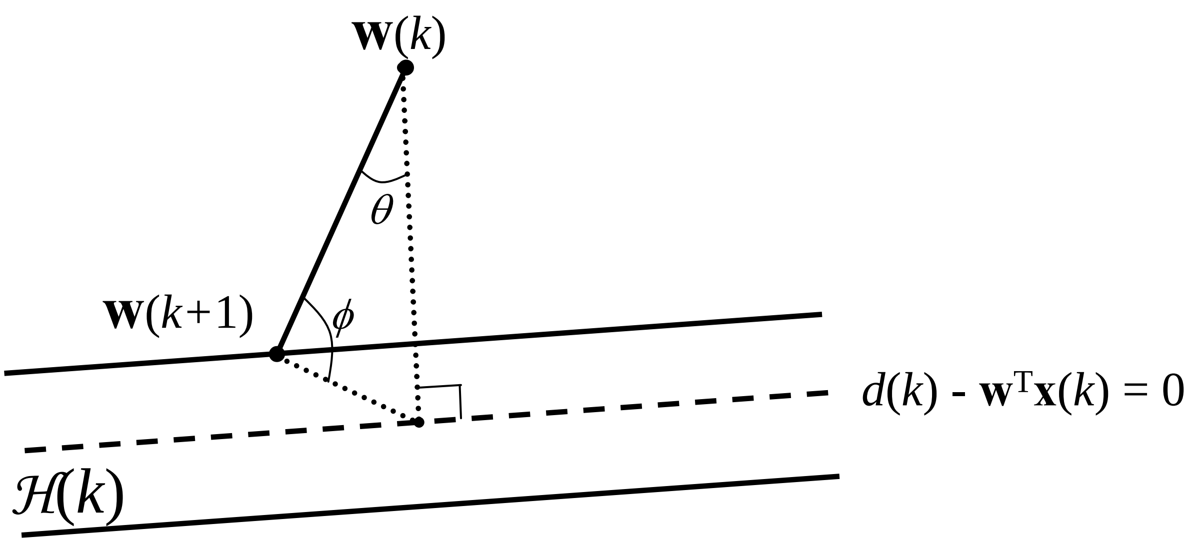
$N = 2$ and $L=1$:



The angle θ is due to the partial updating, and increases when L is decreases. For $L=N$, $\theta = 0$, and we have the conventional SM-NLMS algorithm.

Proposition 1 in R^2

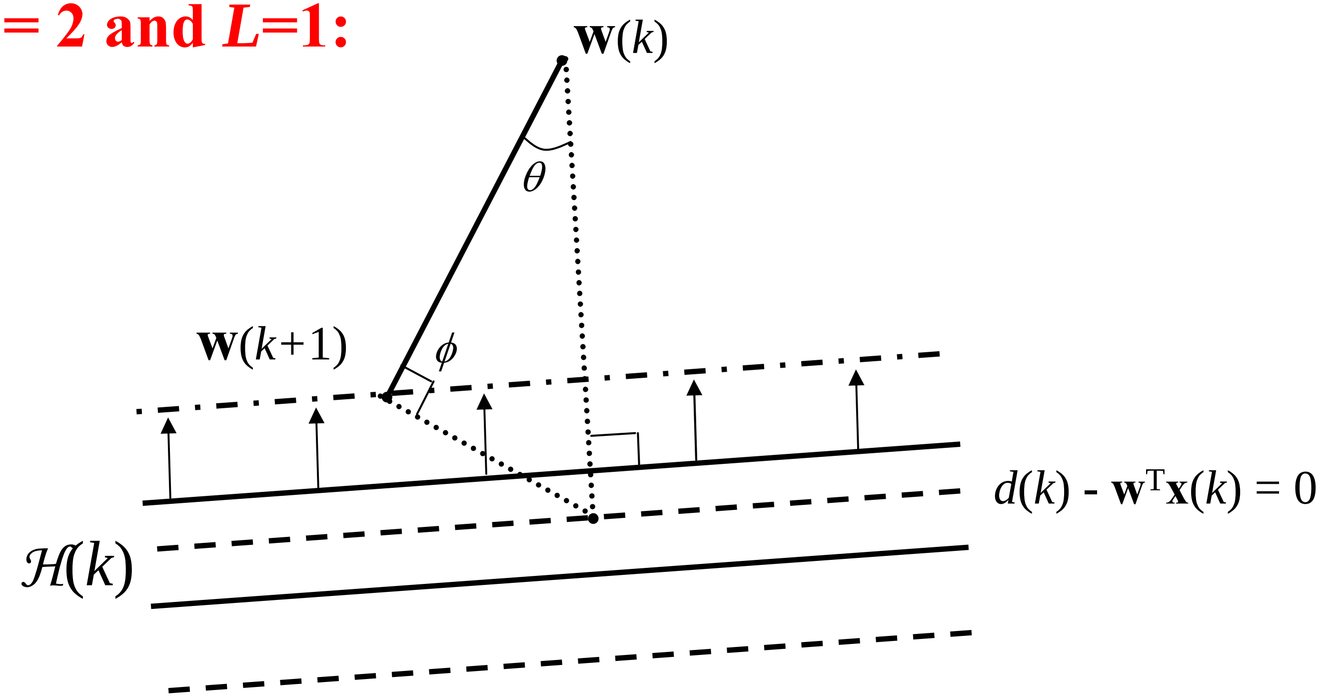
$N = 2$ and $L=1$:



The angle ϕ is in Proposition 1 always greater or equal to $\pi/2$.

Proposition 2 in R^2

$N+1 = 2$ and $L=1$:



SM-PU-NLMS with temporary expansion of the constraint set. The angle ϕ is in Proposition 2 equal to $\pi/2$ whenever an expansion occur.

MSE versus *k* (*L* is fixed):

SNR = 60 dB, colored input

- Filter length: $N = 51$
- Order of update:

  L=5, L=10, L=25
- Threshold: $\gamma = \sigma_w \cdot \sqrt{5}$

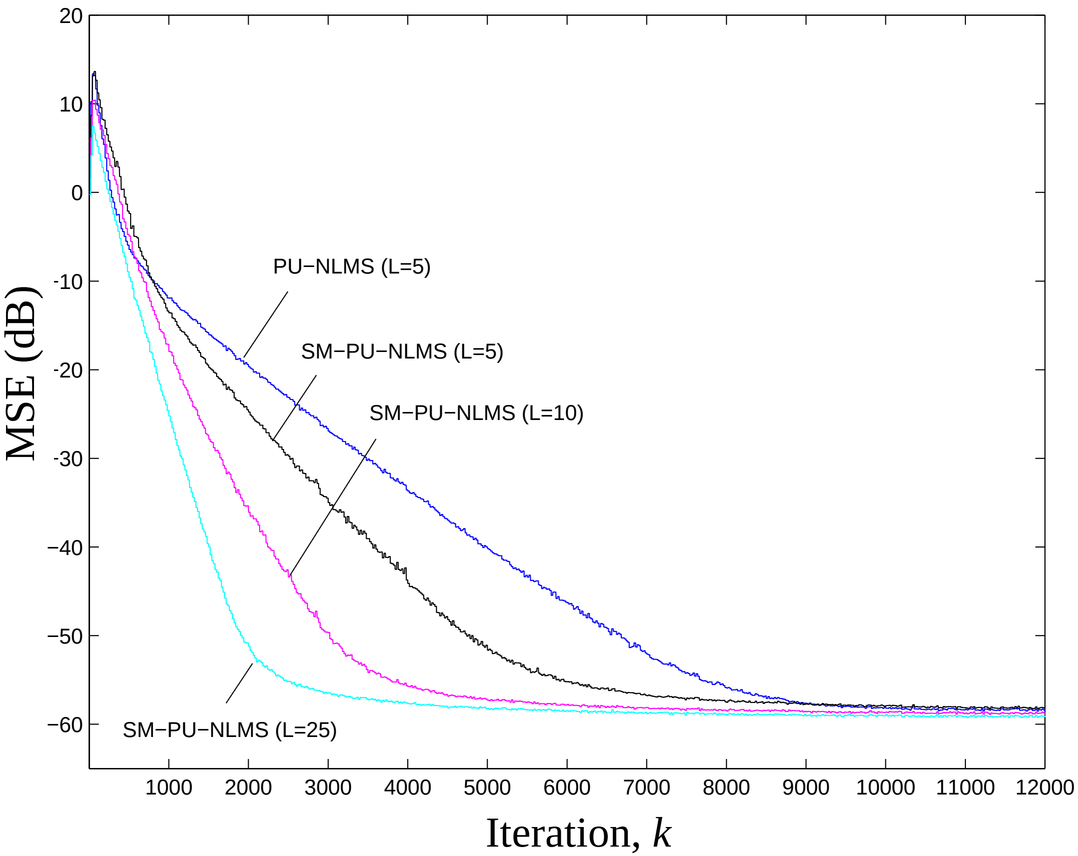
Number of updates:

PU-NLMS: 12000

SM-PU-NLMS (L=5): 4950

SM-PU-NLMS (L=10): 3340

SM-PU-NLMS (L=25): 2420



MSE versus *k* (*L* varies according to Proposition 2):

SNR = 60 dB, colored input

- Filter length: $N = 51$
- Maximum order of update:

  L_{max}=5, L_{max}=10, L_{max}=25
- Threshold: $\gamma = \sigma_w \cdot \sqrt{5}$

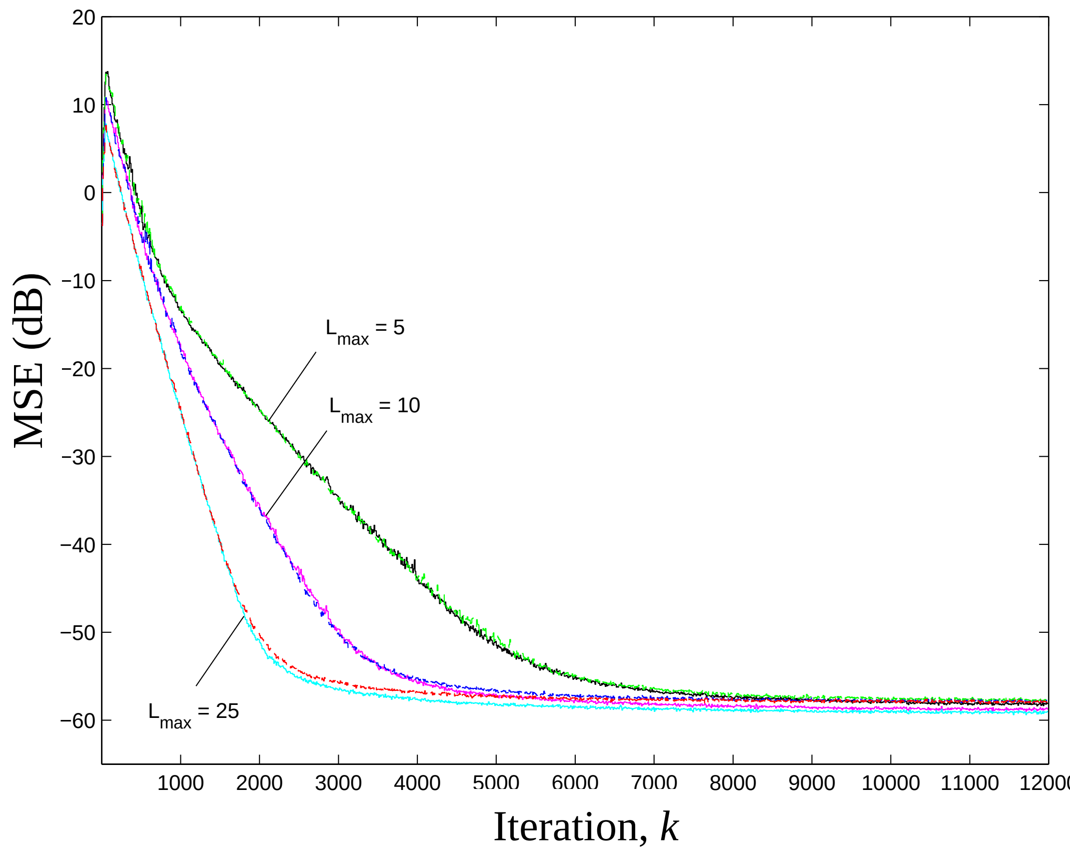
Number of updates:

PU-NLMS: 12000

SM-PU-NLMS (L_{max}=5): 5070

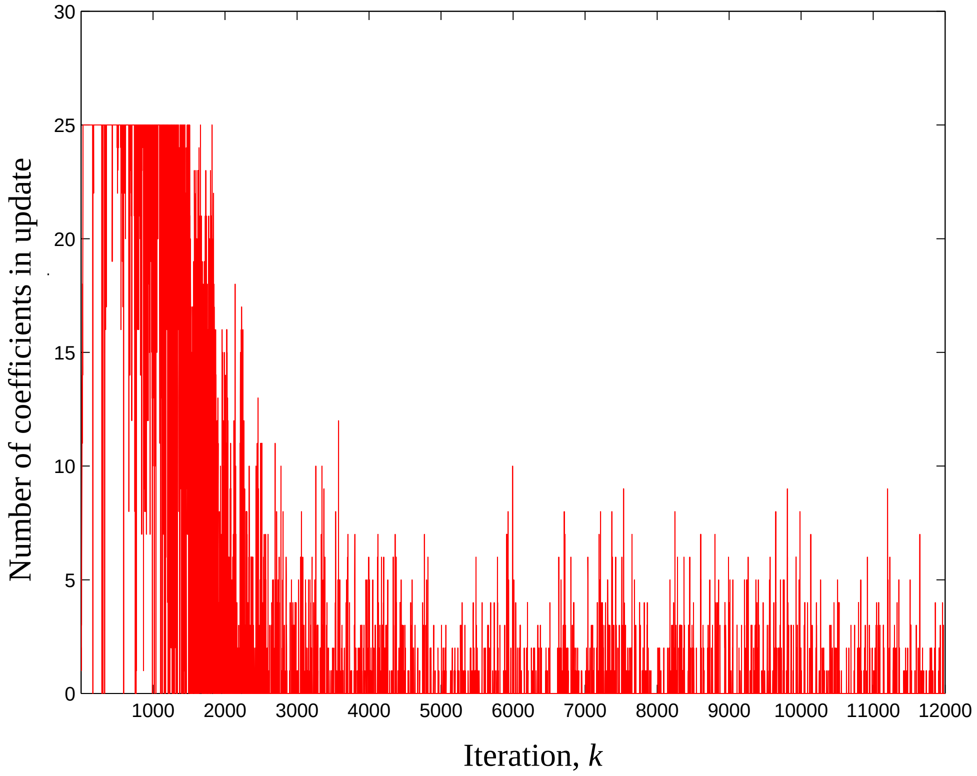
SM-PU-NLMS (L_{max}=10): 3640

SM-PU-NLMS (L_{max}=25): 2840



… Simulations

Number of coefficients in the update L versus k for $L \leq L_{max} = 25$



L close to L_{max} during the initial adaptation whereas later on this value is decreases.

Applications in Communications

- Downlink DS-CDMA, K users, N chips per symb, and L_p is the span of paths in chips.

- Discrete-time received signal with dimension $M \times 1$

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{C}_k \mathbf{h}_k(i) + \boldsymbol{\eta}_k(i) + \mathbf{n}(i)$$

where $M = N + L_p - 1$, $\mathbf{n}(i) = [n_1(i) \ \ldots \ n_M(i)]^T$ is the vector with complex additive gaussian noise.

Applications in Communications

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- Symbol for user k: $b_k(i) \in \{\pm 1 + j0\}$.

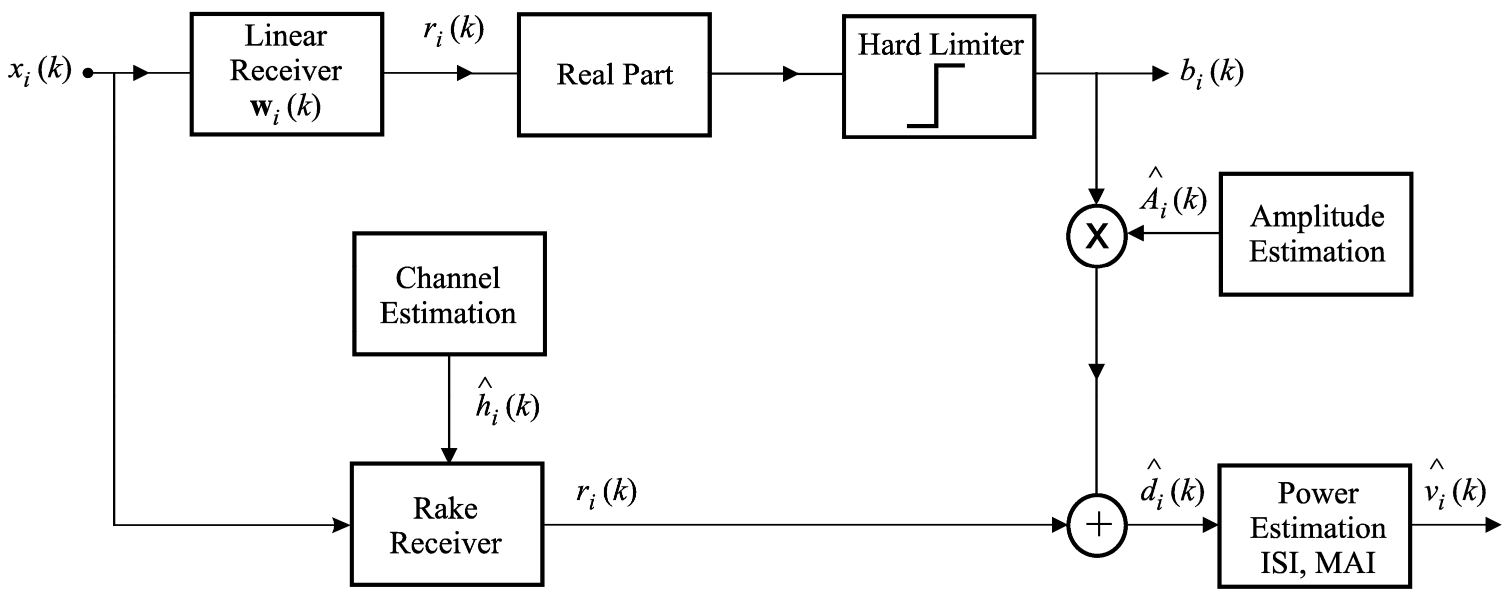
- Amplitude for user k is A_k.

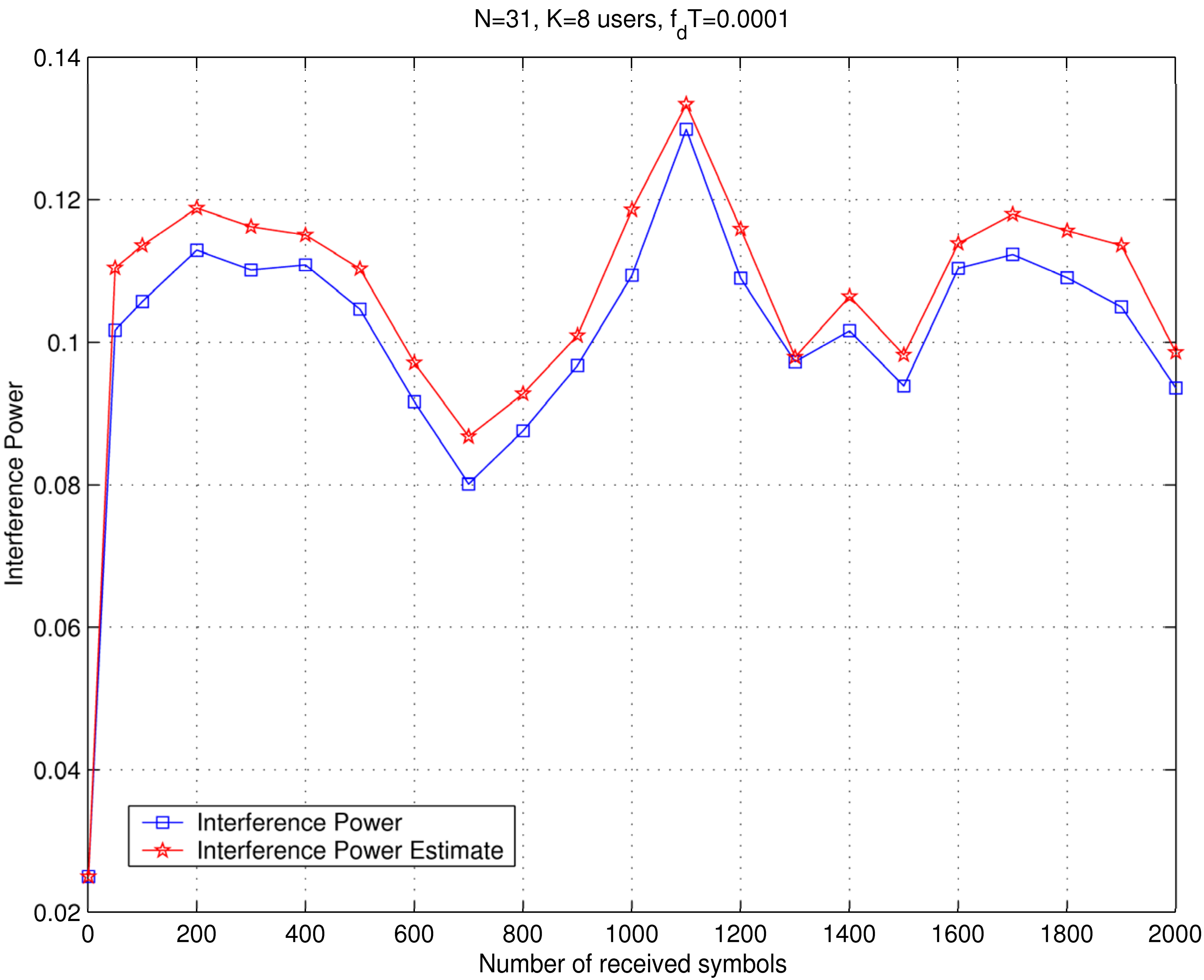
Applications in Communications

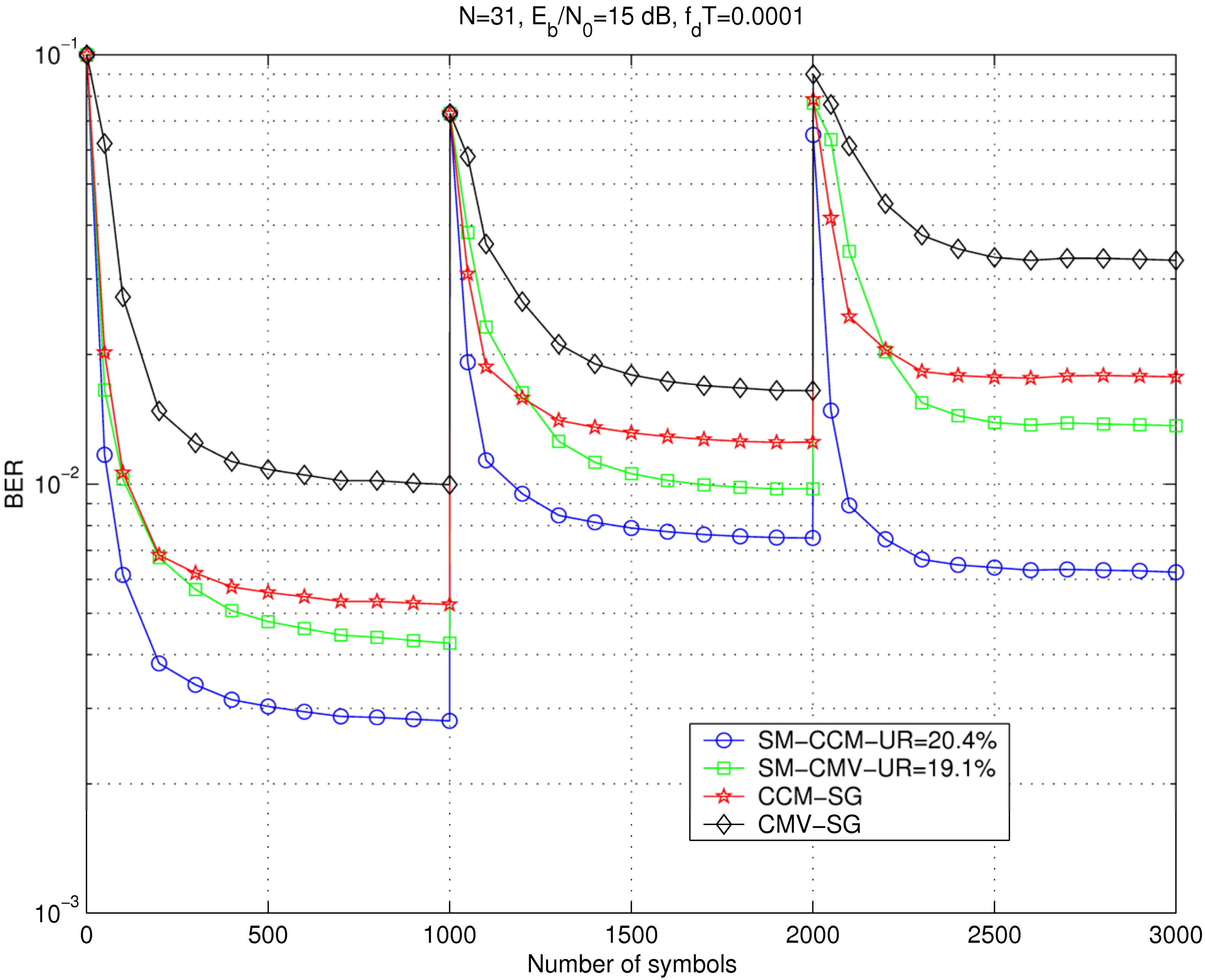
- Channel vector (equal for all users): $\mathbf{h}(i) = [h_0(i) \ldots h_{L_p-1}(i)]^T$.

- $\mathbf{C}_k$ is a conv. matrix $M \times L_p$ and $\mathbf{s}_k = [c_k(1) \ldots c_k(N)]^T$ is the signature sequence of user k.

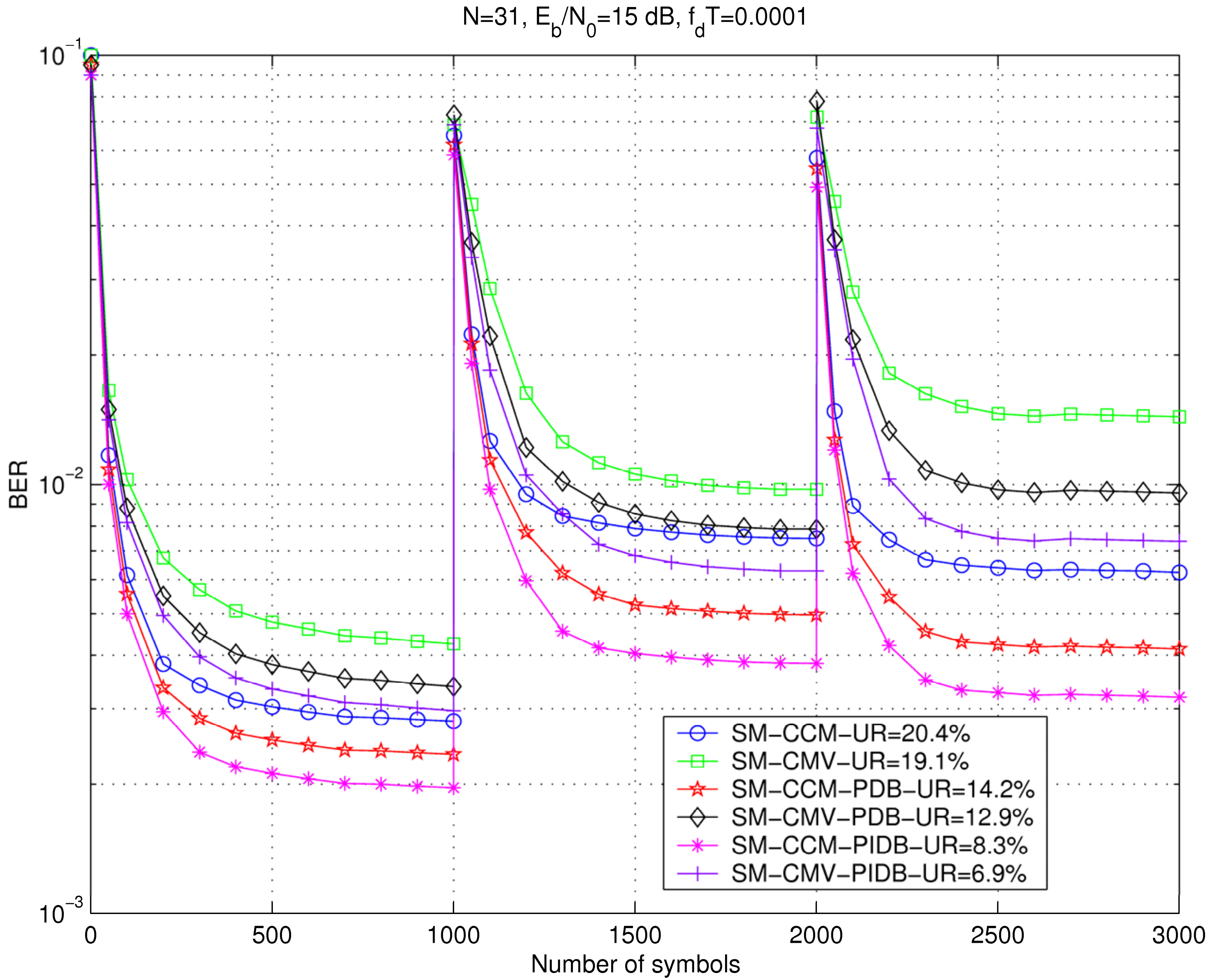
Applications in Communications

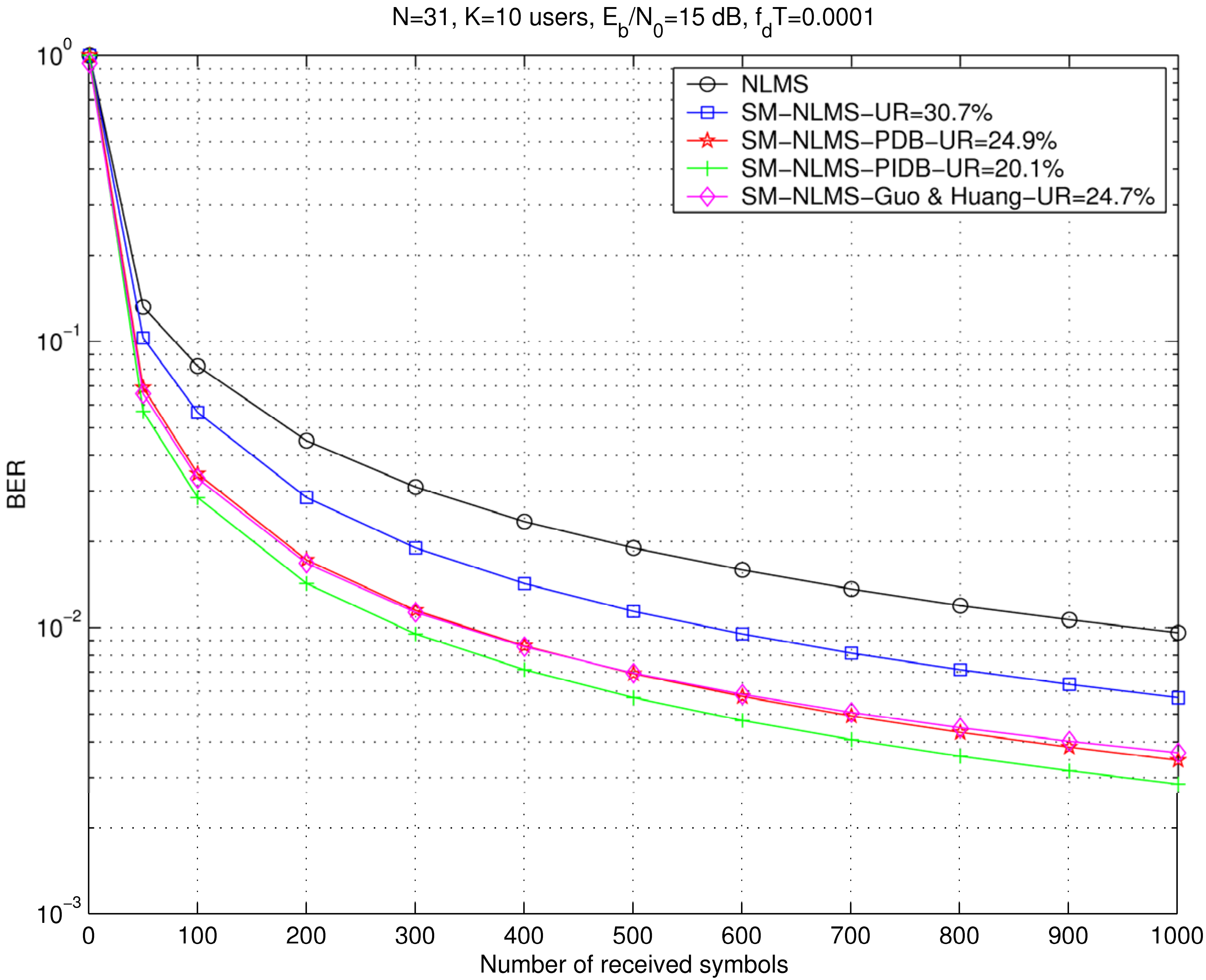



N=31, K=8 users, f_dT=0.0001
Interference Power
Number of received symbols
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Interference Power
Interference Power Estimate



N=31, E_b/N_0=15 dB, f_dT=0.0001
BER
Number of symbols
10^{-1}
10^{-2}
10^{-3}
0
500
1000
1500
2000
2500
3000
SM−CCM−UR=20.4%
SM−CMV−UR=19.1%
CCM−SG
CMV−SG


N=31, E_b/N_0=15 dB, f_dT=0.0001
BER
Number of symbols
10^-1
10^-2
10^-3
0
500
1000
1500
2000
2500
3000
SM−CCM−UR=20.4%
SM−CMV−UR=19.1%
SM−CCM−PDB−UR=14.2%
SM−CMV−PDB−UR=12.9%
SM−CCM−PIDB−UR=8.3%
SM−CMV−PIDB−UR=6.9%



N=31, K=10 users, E_b/N_0=15 dB, f_dT=0.0001
NLMS
SM−NLMS−UR=30.7%
SM−NLMS−PDB−UR=24.9%
SM−NLMS−PIDB−UR=20.1%
SM−NLMS−Guo & Huang−UR=24.7%
BER
10^0
10^{-1}
10^{-2}
10^{-3}
0
100
200
300
400
500
600
700
800
900
1000
Number of received symbols

Conclusions

- The results of the experiments show that the new SM algorithms are capable of significantly outperforming previously reported techniques at a reduced complexity.

- The new algorithms can save a considerable larger number of filter updates as compared to the existing techniques, while they still can provide better performance.

Where to Find !!!