Towards Efficient and Fair Resource Allocation in Wireless Networks

Sławomir Stańczak[1,2]
joint work with
Holger Boche[1,2], Marcin Wiczanowski[1] and Angela Feistel[2]

[1]Fraunhofer German-Sino Lab for Mobile Communications (MCI)
Berlin, Germany
[2]Heinrich Hertz Chair for Mobile Communications
Faculty of EECS
Technical University of Berlin

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Outline

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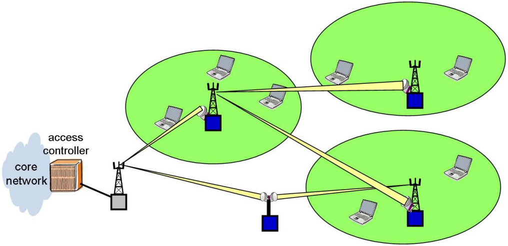
1. Introduction
2. Physical-Layer Abstraction by Interference Functions
   - Generalizations and Connections to Game Theory
3. User-centric Approaches
4. Network-centric approaches
   - Distributed Power Control Algorithms
   - Incorporating QoS requirements

Wireless Networks

- Goal: Study resource allocation and interference management
- Focus: High data rates, low or moderate channel dynamics
  - Energy supply is not a bottleneck.
  - Wireless mesh networks, cellular networks



Wireless Channel Characteristics

- Radio propagation channel is unreliable.
  - channel fading, path loss, channel conditions are time-varying ...
- Power and bandwidth are limited.
- Wireless spectrum is a shared medium.
  - Network cannot be regarded as a collection of point-to-point links.
  - The performance is generally maximized by tolerating interference in a controlled way.

➠ Link capacities are elastic.

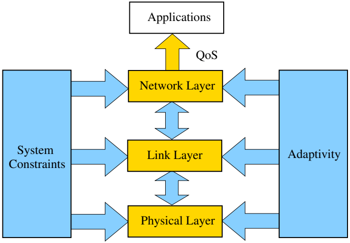
Resource allocation and interference management are necessary.

Wireless Network Resources and Mechanisms

- Wireless resources: power, time, frequency, space, codes, routes...
- Mechanisms for resource allocation and interference management
  - Multiple antenna techniques
  - MAC: power control and scheduling
  - routing
  - ...
- Cross-layer protocols

Cross-Layer Approaches

- Exploit interdependencies between different layers.
  - channel-aware MAC (e.g. HSDPA,HSUPA)
  - MAC+routing+congestion control



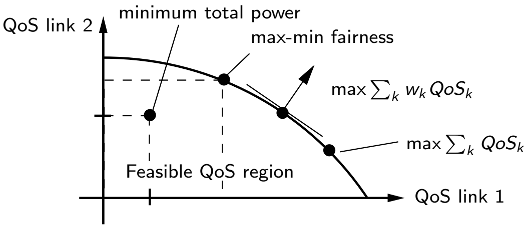
- It may be computationally prohibitive to be of any use in practice.

Applications

- Voice transmission
  - Inelastic traffic: QoS requirements need to be satisfied permanently.
- Data applications (WWW browsing,e-mail,ftp)
  - Low QoS levels are temporarily acceptable.
  - Elastic traffic: Applications modify their data rates according to available resources in communication networks.

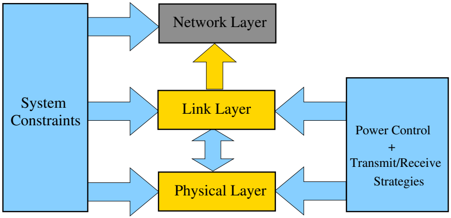
Quality of Service

- User-centric approaches (inelastic applications):
  - Satisfy strict QoS requirements of applications permanently.
- Network-centric approaches (elastic applications):
  - Maximize the aggregate utility as perceived by the network operator.
  - Address the issue of fairness.



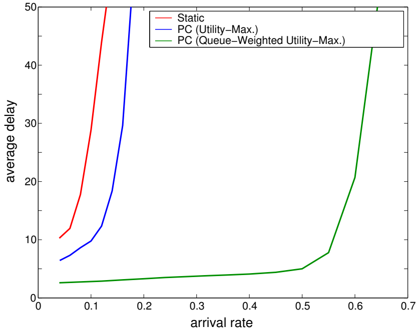
The focus of this talk

- Power control with some aspects of the physical-layer design.

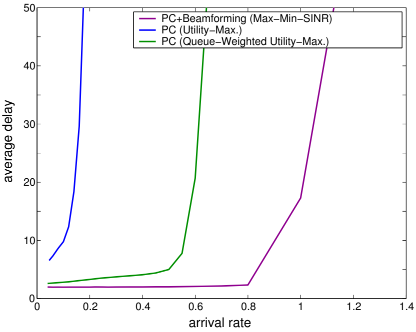


- Single-hop communication with $K > 1$ logical links (users)
- Concurrent transmission (works with any scheduling protocol).
- Each user is decoded (single-user decoding).
- Combination with routing and network coding strategies possible.

Appetizer



Static
PC (Utility−Max.)
PC (Queue−Weighted Utility−Max.)
average delay
arrival rate
0
10
20
30
40
50
0.1
0.2
0.3
0.4
0.5
0.6
0.7

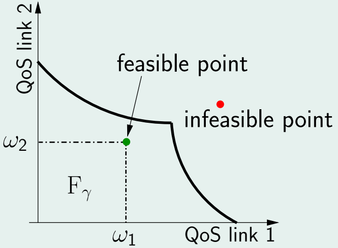


PC+Beamforming (Max−Min−SINR)
PC (Utility−Max.)
PC (Queue−Weighted Utility−Max.)
average delay
arrival rate
0
10
20
30
40
50
0.2
0.4
0.6
0.8
1
1.2
1.4

Outline

Feasible QoS Region: F_γ

Given a channel, F_γ is the set of all QoS values that can be achieved by means of power control with all links being active concurrently.



- Assumption: $\omega_k \uparrow \Leftrightarrow$ QoS $\uparrow$
  - Downward-comprehensive
  - Upper-bounded
- may be non-convex.
- depends on power constraints P.

- F_γ depends on the physical-layer realization: Key properties of many multiuser systems are captured by interference functions.

Signal-to-Interference(+noise) Ratio (SIR)

Strictly monotonic QoS-SIR map: $\gamma : \mathbb{R} \rightarrow \mathbb{R}_+$

For any $\boldsymbol{\omega} \in \mathrm{F}_\gamma$, there is a power vector $\mathbf{p} \in \mathrm{P}$ such that

$$\gamma(\omega_k) = \mathrm{SIR}_k(\mathbf{p}) = \frac{p_k}{I_k(\mathbf{p})} \quad \begin{array}{l} \leftarrow \text{transmit power} \\ \leftarrow \text{interference function} \end{array}$$

- e.g. Gaussian capacity (in nats/channel use): $\gamma(x) = e^x - 1, x \geq 0$.

Axiomatic Interference Function

Standard Interference Functions (SIF), Yates'95

A1 Positivity: $I_k(\mathbf{p}) > 0$ for all $\mathbf{p} \geq 0$.

A2 Scalability: $I_k(\mu\mathbf{p}) < \mu I_k(\mathbf{p})$ for any $\mathbf{p} \geq 0$ and for all $\mu > 1$.

A3 Monotonicity: $I_k(\mathbf{p}^{(1)}) \geq I_k(\mathbf{p}^{(2)})$ if $\mathbf{p}^{(1)} \geq \mathbf{p}^{(2)}$.

- It models many practical interference scenarios.

Interference Function: Examples

Linear interference function

$$I_k(\mathbf{p}) = (\mathbf{V}\mathbf{p} + \mathbf{z})_k$$

- Matched-filter receiver
- SIC receiver

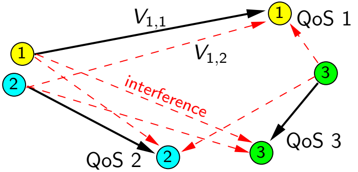
Minimum interference function

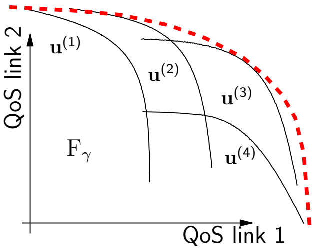
$$I_k(\mathbf{p}) = \min_{u_k \in \mathrm{U}_k} (\mathbf{V}(\mathbf{u})\mathbf{p} + \mathbf{z}(\mathbf{u}))_k$$

- MMSE receiver

$\mathbf{V} = (v_{k,l}) = (\frac{V_{k,l}}{V_{k,k}})$

$\mathbf{V} \geq 0$: non-symmetric, traceless





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General Interference Functions

Interference Function (Boche&Schubert 04)

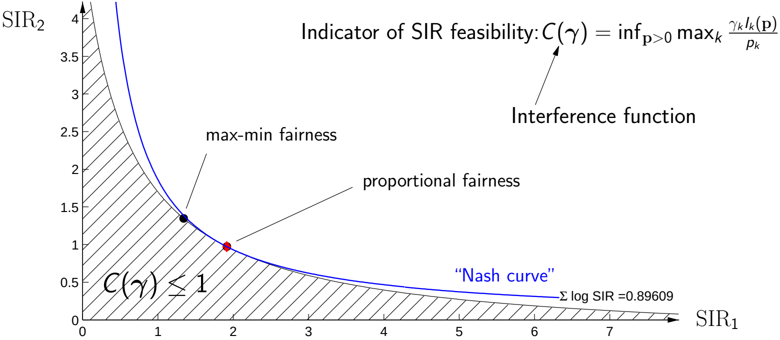
A1 Non-negativeness: $I_k(\mathbf{p}) \geq 0$ for all $\mathbf{p} > 0$.

A2 Homogeneity: $I_k(\mu\mathbf{p}) = \mu I_k(\mathbf{p})$ for all $\mathbf{p} > 0$ and for all $\mu > 0$.

A3 Monotonicity: $I_k(\mathbf{p}^{(1)}) \geq I_k(\mathbf{p}^{(2)})$ if $\mathbf{p}^{(1)} \geq \mathbf{p}^{(2)} > 0$.

- The framework includes the noiseless case.
- It generalizes the concept of standard interference functions.

Application to Cooperative Game Theory



If $\mathrm{U} = \{\omega : C(e^{\omega}) \leq 1\}$ is strictly convex, then the Nash axioms WPO, IIA, SYM, STC characterize a single-valued solution $\mathbf{u}^*$ to $\max_{\mathbf{u}\in\mathrm{U}} \sum_k \log u_k$.

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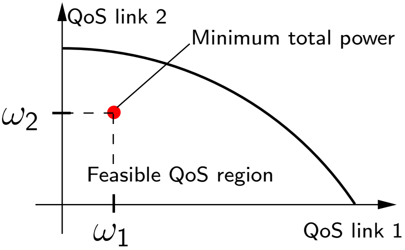
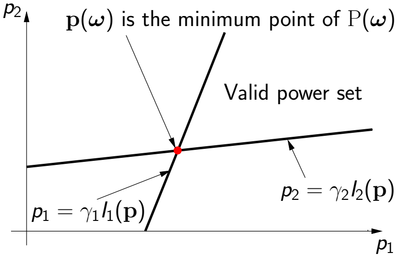
Problem Statement

Problem (QoS-based power control under SIFs)

$$\mathbf{p}(\boldsymbol{\omega}) = \arg\min_{\mathbf{p}\in \mathrm{P}(\boldsymbol{\omega})} \mathbf{w}^T\mathbf{p} \qquad \mathbf{w} > 0$$

$$\mathrm{P}(\boldsymbol{\omega}) := \left\{\mathbf{p} \in \mathbb{R}_+^K : \forall_k\, \mathrm{SIR}_k(\mathbf{p}) \geq \gamma(\omega_k)\right\}.$$





Zander'92, Foschini'94, Yates'95, Ulukus'98, Bambos'00, Boche&Schubert ...

Fixed-Point Iteration, Yates'95

If $\boldsymbol{\omega}$ is feasible, then the concurrent iterations

$$\forall_k \; \bar{p}_k(n+1) = \gamma(\omega_k)\, I_k(\bar{\mathbf{p}}(n))$$

converge to $\mathbf{p}(\boldsymbol{\omega})$.

- Component-wise increasing (decreasing) if $\bar{\mathbf{p}}(0) = \mathbf{0}$ ($\bar{\mathbf{p}}(0) \in \mathrm{P}(\boldsymbol{\omega})$).

Amenable to distributed implementation, scalable, works for any SIF

Matrix-Based Algorithm, Boche&Schubert'05

Alternating optimization of $\mathbf{u}(n)$ and $\mathbf{p}(n)$: $\mathbf{p}(0) \in \mathrm{P}(\mathbf{\Gamma})$.

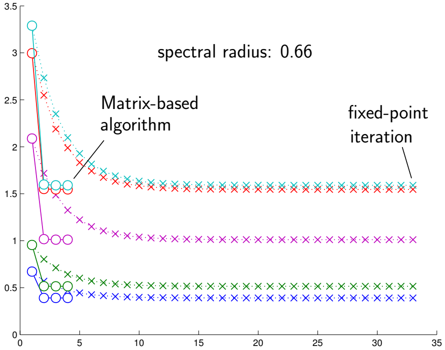
(i) $\forall_k \; u_k(n) = \arg\min_{u_k \in \mathrm{U}_k} \big(\mathbf{V}(\mathbf{u})\mathbf{p}(n) + \mathbf{z}(\mathbf{u})\big)_k$

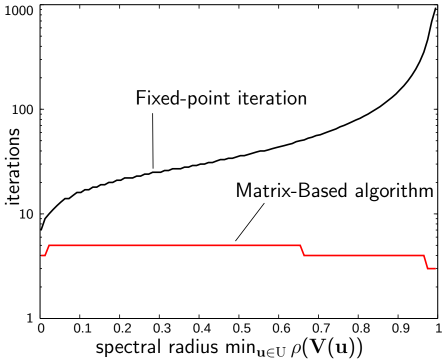
(ii) $\mathbf{p}(n+1) = \big(\mathbf{I} - \mathbf{\Gamma}(\omega)\mathbf{V}\big(\mathbf{u}(n)\big)\big)^{-1}\mathbf{\Gamma}(\omega)\mathbf{z}\big(\mathbf{u}(n)\big)$

- Strictly monotonic convergence to $\mathbf{p}(\omega)$

It requires centralized implementation but provides
super-linear or even quadratic convergence independent of the load.

Observed Convergence



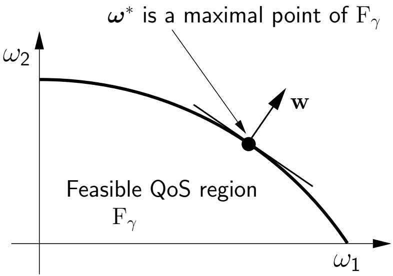


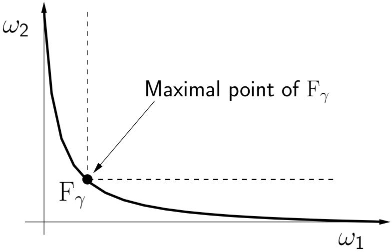
Outline

Problem Statement

Problem (Utility-based power control)

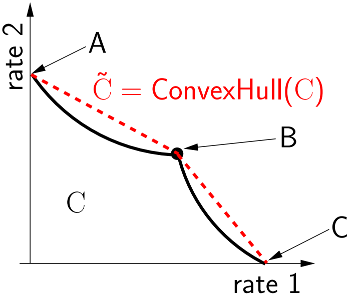
$$\boldsymbol{\omega}^* = \arg\max_{\boldsymbol{\omega}\in \mathrm{F}_\gamma} \mathbf{w}^T\boldsymbol{\omega} \qquad \mathbf{w} > 0\,.$$

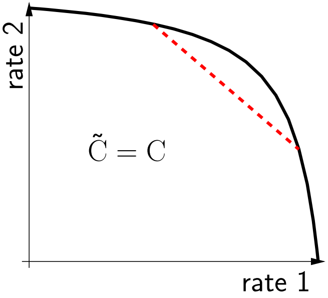




Goodman'00, Saraydar'02, Xiao'03, Neely'03, Chiang'04, Huang'05,Tassiulas'05 ...

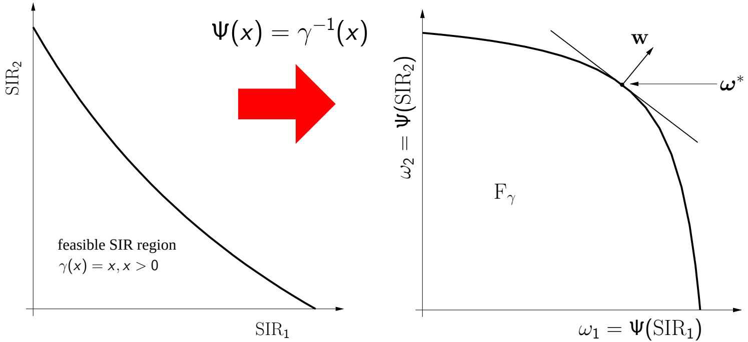
Scheduling vs. Concurrent Transmission





- C is not convex: Joint power control and scheduling (time sharing) is necessary to achieve all boundary points of $\tilde{\mathrm{C}}$.
- C is convex: All boundary points of $\tilde{\mathrm{C}}$ can be achieved by means of power control without having to resort to scheduling.

Convexity of Feasible QoS Region



- Find a class of strictly increasing and concave utility functions Ψ so that F_γ is a convex set.

Convexity of Feasible QoS Region: Sufficient Conditions

Theorem (Convexity under a Linear Interference Function)

If γ with $\gamma(\Psi(x)) = x, x \geq 0$, is log-convex, then the feasible QoS region is a convex set, regardless of the type of power constraints

- Observation: γ is log-convex if and only if $\Psi_e(x) := \Psi(e^x)$ is concave.

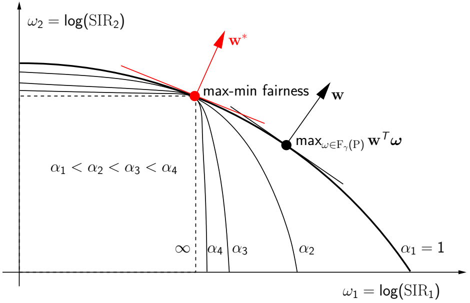
Examples of Function Classes

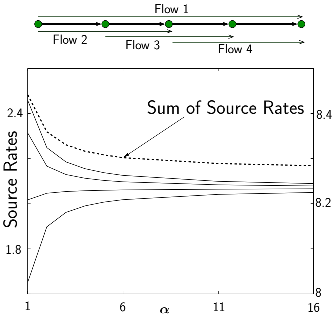
$$\Psi_\alpha(x) = \begin{cases} \frac{x^{1-\alpha}}{1-\alpha} & \alpha > 1 \\ \log(x) & \alpha = 1 \end{cases} \quad \tilde{\Psi}_\alpha(x) = \begin{cases} \log x & \alpha = 1 \\ \log \frac{x}{1+x} & \alpha = 2 \\ \log \frac{x}{1+x} + \sum_{j=1}^{\alpha-2} \frac{1}{j(1+x)^j} & \alpha > 2 \end{cases}$$

Max-Min Rate Allocation

Arbitrarily Close Approximation

Let $\omega_k^* = \Psi_\alpha(\mathrm{SIR}_k^*)$ and let $\nu_k^* = \log(1 + \mathrm{SIR}_k^*)$. Then, $\boldsymbol{\nu}^*$ converges to the max-min rate allocation as $\alpha \to \infty$.





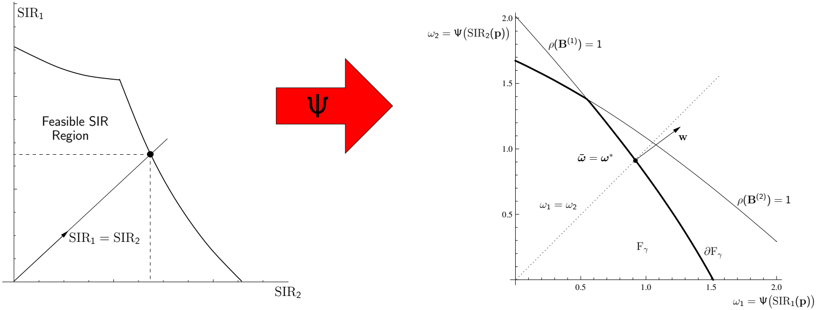
Efficiency of the Max-Min SIR Power Allocation

Theorem

If $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ are positive right and left eigenvectors of $\mathbf{B}^{(k_0)} = \mathbf{V} + \frac{1}{P_{k_0}} \mathbf{z} \mathbf{e}_{k_0}^T$, then

(i) *$\mathbf{p}$ is max-min fair power allocation if and only if $\mathbf{p} = \bar{\mathbf{p}}$.*

(ii) *$\boldsymbol{\omega}^*$ is max-min fair $\bar{\boldsymbol{\omega}}$ if and only if $\mathbf{w} = \mathbf{w}^* = \bar{\mathbf{q}} \circ \bar{\mathbf{p}} > 0$.*



Other Related Results: Linear Interference Functions

- Log-convexity of $\gamma(x)$ seems to be necessary for the feasible QoS region to be convex in general.
- If $\mathbf{V}$ is positive semidefinite, then the feasible QoS region under no power constraints is convex if $\gamma(x)$ is convex.
  - The feasible rate region is convex.
- Infeasible SIR region is not convex in general.
  - The convex hull of the feasible SIR region is not a convex polytope.
- Strict convexity conditions.

Outline

Utility-Based Power Control

Equivalent minimization problem: $\psi(x) = -\Psi(x)$

$$\mathbf{p}^* = \arg\min_{\mathbf{p}\in\mathrm{P}} F(\mathbf{p}) = \arg\min_{\mathbf{p}\in\mathrm{P}} \sum_k w_k \psi(\mathrm{SIR}_k(\mathbf{p})) \, .$$

- Positivity of minimizers: $\mathbf{p}^* > 0$
- Even if $\psi(e^x)$ is convex, the problem is not convex in general.

Convex Statement of the Problem

Theorem

If $I_k(e^{\mathbf{s}})$ is log-convex and $\psi(e^x)$ convex, the following problem is convex:

$$\mathbf{s}^* = \arg\min_{\mathbf{s}\in\mathrm{S}} F_e(\mathbf{s}) \qquad \begin{cases} \mathbf{s} := \log \mathbf{p}, \mathbf{p} > 0 \\ \mathrm{S} := \{\log \mathbf{x} : \mathbf{x} \in \mathrm{P}_+\} \\ F_e(\mathbf{s}) = F(e^{\mathbf{s}}) \end{cases}$$

- $I_k(e^{\mathbf{s}}) = \sum_l v_{k,l} e^{s_l} + z_k$ is log-convex (Hoelder inequality).
- Log-convexity is given in the worst-case design.

Gradient-Projection Algorithm

- Let $\tau > 0$ be constant step size (small enough), and let

$$\mathbf{s}(n+1) = \Pi_{\mathrm{S}}\Big[\mathbf{s}(n) - \tau \nabla F_e(\mathbf{s}(n))\Big] \qquad \mathbf{s}(0) \in \mathrm{S}$$

- $\nabla F_e(\mathbf{s}) = \mathrm{diag}(e^{s_1}, \ldots, e^{s_K})\nabla F(e^{\mathbf{s}})$:

$$\nabla F(\mathbf{p}) = (\mathbf{I} - \mathbf{V}^T \boldsymbol{\Gamma}(\mathbf{p}))\mathbf{g}(\mathbf{p})$$

  - $g_k(\mathbf{p}) = w_k \psi'(\mathrm{SIR}_k(\mathbf{p}))\mathrm{SIR}_k(\mathbf{p})/p_k$ (locally available)
  - $\boldsymbol{\Gamma}(\mathbf{p}) = \mathrm{diag}(\mathrm{SIR}_1(\mathbf{p}), \ldots, \mathrm{SIR}_K(\mathbf{p}))$

Computation of the Gradient Vector

$$\nabla F(\mathbf{p}) = \underbrace{(\mathbf{I} + \mathbf{\Gamma}(\mathbf{p}))\mathbf{g}(\mathbf{p})}_{\text{local variable}} - \underbrace{(\mathbf{I} + \mathbf{V}^T)\mathbf{\Gamma}(\mathbf{p})\mathbf{g}(\mathbf{p})}_{\text{global variable}}$$

- Problem is to obtain $\Sigma_k(\mathbf{p}) = \sum_l v_{l,k} m_l(\mathbf{p})$, $m_l(\mathbf{p}) = g_l(\mathbf{p})\mathrm{SIR}_l(\mathbf{p})$.
  - Distribute m_l using a flooding protocol (What about $v_{l,k}$?).
  - Estimate the sum Σ_k using an adjoint network.

The Notion of an Adjoint Network

Definition

Two networks with K links and gain matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ are referred to as being adjoint (to each other) if $\mathbf{V}_1 = \mathbf{V}_2^T$.

- Reverse the roles of transmitter and receivers is not sufficient

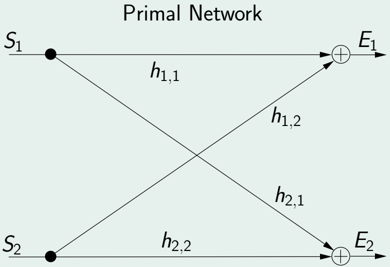
$$\underbrace{\mathbf{V}_1 = \mathbf{D}\mathbf{G}}_{\text{primal network}} \qquad \underbrace{\mathbf{V}_2 = \mathbf{D}\mathbf{G}^T}_{\text{reversed network}} \qquad \mathbf{V}_1 \neq \mathbf{V}_2^T$$

- $\mathbf{D} = \mathrm{diag}(\frac{1}{V_{1,1}}, \ldots, \frac{1}{V_{K,K}})$
- $\mathbf{G} = (V_{k,l})$ if $k \neq l$ and $(\mathbf{G})_{k,k} = 0$.

Adjoint Networks: A Simple Example

- Primal network

$$\mathbf{V}_1 = \begin{pmatrix} 0 & \frac{|h_{1,2}|^2}{|h_{1,1}|^2} \\ \frac{|h_{2,1}|^2}{|h_{2,2}|^2} & 0 \end{pmatrix}$$



Adjoint Networks: A Simple Example

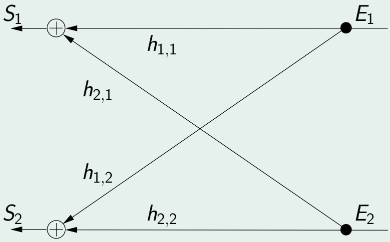
- Primal network

$$\mathbf{V}_1 = \begin{pmatrix} 0 & \frac{|h_{1,2}|^2}{|h_{1,1}|^2} \\ \frac{|h_{2,1}|^2}{|h_{2,2}|^2} & 0 \end{pmatrix}$$

- Reversed network + $X_k / h_{k,k}$

$$\mathbf{V}_2 = \begin{pmatrix} 0 & \frac{|h_{2,1}|^2}{|h_{2,2}|^2} \\ \frac{|h_{1,2}|^2}{|h_{1,1}|^2} & 0 \end{pmatrix}$$

Adjoint Network



The message m_k is contained in the transmit power

Min-max reformulation

$$\min_{\mathbf{s}} \max_{\mathbf{u}} \sum_k w_k \psi\left(\frac{e^{s_k}}{u_k}\right) \quad \text{subject to} \begin{cases} e^{\mathbf{s}} - \hat{\mathbf{p}} \le 0 \\ \mathbf{u} - \mathbf{t} \le 0 \\ \forall_k \; I_k(e^{\mathbf{s}}) - t_k = 0 \,. \end{cases}$$

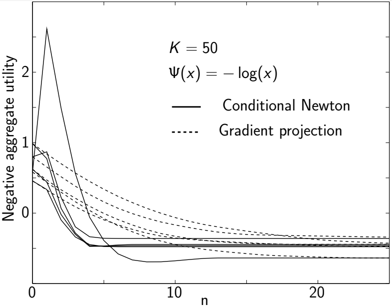
- Linear interference function: $I_k(e^{\mathbf{s}}) = (\mathbf{V}e^{\mathbf{s}} + \mathbf{z})_k$.
  - The Hessian is diagonal and its diagonals are given by the gradient.

Conditional Newton Algorithm

$$\begin{cases} \begin{pmatrix} \mathbf{s}(n+1) \\ \boldsymbol{\mu}(n+1) \end{pmatrix} = \begin{pmatrix} \mathbf{s}(n) \\ \boldsymbol{\mu}(n) \end{pmatrix} - \left(\nabla^2_{(\mathbf{s},\boldsymbol{\mu})} L(\mathbf{z}(n))\right)^{-1} \nabla_{(\mathbf{s},\boldsymbol{\mu})} L(\mathbf{z}(n)) \\ \nabla_{(\mathbf{u},\boldsymbol{\lambda}^u,\boldsymbol{\lambda},\mathbf{t})} L(\mathbf{z}(n+1)) = 0 \quad \text{can be solved explicitely} \end{cases}$$

$L(\mathbf{z}) = L(\mathbf{s}, \mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\lambda}^u, \boldsymbol{\lambda}, \mathbf{t})$: A modified Lagrangian function.

- Quadratic convergence.
- Global convergence if $\psi(x) = -\log(x)$ and $\psi(x) = 1/x$.
- No step size.
- Distributed implementation possible!



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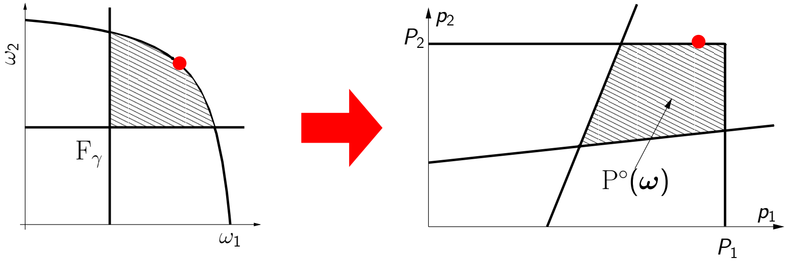
Problem Statement

$$\mathrm{s}^*(\boldsymbol{\omega}) := \arg\min_{\mathbf{s}\in \mathrm{S}} F_e(\mathrm{s}) \qquad \text{s.t.} \qquad \forall_k\ f_k(\mathrm{s}) := I_k(e^{\mathrm{s}})/e^{s_k} - 1/\gamma_k \leq 0$$



- The projection is not amenable to distributed implementation.

Primal-dual algorithm based on standard Lagrangian

Primal-dual iteration under individual power constraints

$$\begin{cases} s_k(n+1) = \min\left\{ s_k(n) - \delta e^{s_k(n)} \left[h_k(\mathrm{s}(n)) + \Sigma_k(\mathrm{s}(n), \boldsymbol{\mu}(n)) \right], \log(P_k) \right\} \\ \lambda_k(n+1) = \max\{0, \lambda_k(n) + \delta f_k(\mathrm{s}(n))\} \end{cases}$$

- $\Sigma_k(\mathrm{s}, \boldsymbol{\lambda}) = \sum_l v_{l,k} \left(\frac{\lambda_l}{e^{s_l}} + |\mathrm{SIR}_l(e^{\mathrm{s}}) g_l(\mathrm{s})| \right) = \sum_l v_{l,k} m_l(\mathrm{s}, \mu_l)$
  - Estimation of Σ_k using the adjoint network.

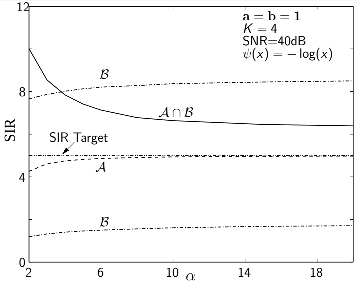
Soft QoS Support

Let $\mathcal{A}$ be pure QoS-links and $\mathcal{B}$ best-effort links. Consider

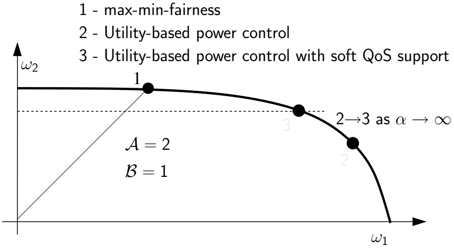
$$\tilde{F}_\alpha(\mathbf{s}) = \underbrace{\sum_{k\in\mathcal{A}} a_k \psi_\alpha\left(\frac{\mathrm{SIR}_k(e^{\mathbf{s}})}{\gamma_k}\right)}_{\text{penalty term}} + \underbrace{\sum_{k\in\mathcal{B}} b_k \psi\left(\mathrm{SIR}_k(e^{\mathbf{s}})\right)}_{\text{aggregate utility}} .$$

$$\tilde{\mathbf{s}}^*(\alpha) = \arg\min_{\mathbf{s}\in\mathbf{S}} \tilde{F}_\alpha(\mathbf{s})$$

- Convex problem.
- Slightly modified algorithms.



Soft QoS Support



Theorem

For any $\epsilon > 0$ and $\mathbf{V}$ (irreducible), there is $\alpha(\epsilon) \geq 1$ such that

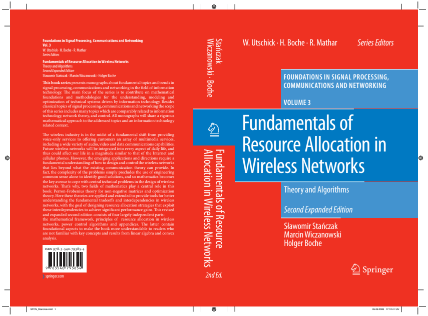
$$\forall_{\alpha \geq \alpha(\epsilon)} \qquad \max_{k \in \mathcal{A} \setminus \mathcal{B}} \mathrm{SIR}_k \leq \gamma_k + \epsilon \qquad \gamma_k - \epsilon \leq \min_{k \in \mathcal{A}} \mathrm{SIR}_k \, .$$

Conclusions and Outlook

- The power control problem is relatively well-understood.
- Throughput maximization in the low SINR regime is an open problem.

Outlook

- Joint optimization of
  - transmit powers,
  - schedulers (time+frequency),
  - physical-layer (single- and multi-mode transmission).
- Dynamic optimization over finite and infinite time horizon.
- Stochastic power control.
- ...



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