Distributed resource allocation in multi-cell OFDMA networks

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Outline

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Multi-cell OFDMA

- Multi-cell OFDMA:
  - Intra-cell allocation: subcarrier allocation, user's constraint (rate, buffer) fulfillment, cell power management
  - Inter-cell allocation: inter-cell interference management.
- Subcarriers cannot be seen as independent interference channels: joint optimization over several subcarriers.
  - Resource allocation in OFDMA: mainly studied in single-cell
  - Resource allocation on the interference channel: TDMA or CDMA
- Need to define distributed resource allocation methods: B3G and 4G cellular networks, ad hoc networks

Cross-layer resource allocation

- Two main service classes: Rate-constrained users and Best Effort users
    - Rate-constrained users require real-time services: provide a given target data rate at each time slot.
    - Best Effort users: avoid buffer overflows. Cross-layer resource allocation
- Rationale: providing distributed resource allocation methods for each service class.

Outline

Optimization problem

- Network $\mathcal{N}$: N_{BS} cells, K users, L orthogonal subcarriers.
- Sum power minimization, under user data rate and cell power constraints.

$$\begin{aligned}
&\min_{\vec{P},\vec{c}} \sum_{k=1}^{K}\sum_{l=1}^{L} c^{l}_{\text{BS}[k],k} P^{l}_{\text{BS}[k]} \\
&\text{subject to } \sum_{l=1}^{L} c^{l}_{\text{BS}[k],k} r^{l}_{k} - R_{k,\text{target}} \geq 0 \;\; \forall k \in [1,K] \\
&\text{subject to } \sum_{l=1}^{L} P^{l}_{j} - P_{\max} \leq 0 \;\; \forall j \in [1,N_{\text{BS}}]
\end{aligned} \tag{1}$$

Notations

- $c^l_{\mathrm{BS}[k],k}$: assignement index of user k served by BS $[k]$.
- $R_{k,\mathrm{target}}$: target data rate for user k
- $P^l_{\mathrm{BS}[k]}$: power transmitted by BS $[k]$ in subcarrier l
- r^l_k : data rate for user k in subcarrier l
- $G^l_{j,k}$: channel gain between BS j and user k in subcarrier l
- N_0: thermal noise
- $P_{\max}$: maximum transmit power per cell
- SINR of user k in subcarrier l:

$$\Gamma^l_k = \frac{P^l_{\mathrm{BS}[k]} G^l_{\mathrm{BS}[k],k}}{\sum_{j \neq \mathrm{BS}[k]}^{N_{\mathrm{BS}}} P^l_j G^l_{j,k} + N_0} \quad (2)$$

- γ^l_k: target SINR for user k in subcarrier l

Power control convergence, subcarrier *l*

$$\Gamma_k^l \geq \gamma_k^l,\ \forall k \iff (\mathbf{I} - \mathbf{D}^{\mathbf{l}}\mathbf{F}^{\mathbf{l}})\mathbf{P}^{\mathbf{l}} \geq \mathbf{v}^l \qquad (3)$$

- Where
  - $\mathbf{v}^l = [v_1^l, ..., v_{N_{\mathrm{BS}}}^l]'$ with $v_k^l = N_0\gamma_k^l / G_{\mathrm{BS}[k],k}^l$,
  - $\mathbf{D}^{\mathbf{l}} = \mathrm{diag}\{\gamma_1^l, ..., \gamma_{N_{\mathrm{BS}}}^l\}$
  - $\mathbf{F}^{\mathbf{l}}(k,j) = 0$ if $j = k$ and $\mathbf{F}^{\mathbf{l}}(k,j) = \frac{G_{j,k}^l}{G_{\mathrm{BS}[k],k}^l}$ if $j \neq k$.
- There exists a positive power allocation

$$\mathbf{P}^{\mathbf{l}} = (\mathbf{I} - \mathbf{D}^{\mathbf{l}}\mathbf{F}^{\mathbf{l}})^{-1}\mathbf{v}^l \qquad (4)$$

  if and only if $\left|\rho(\mathbf{D}^{\mathbf{l}}\mathbf{F}^{\mathbf{l}})\right| < 1$.
- Then the distributed iterative power control converges towards the optimal power solution [Foschini, 93] [Yates, 95].

Power control convergence requirement in OFDMA

- Determine the set of target SINR $\gamma_k = \left[\gamma_k^{n_{k,1}}, \ldots, \gamma_k^{n_{k,n_{\mathrm{SC},k}}}\right]'$ for user k
  - Feasible power control on each subcarrier
  - Achieve $R_{k,\mathrm{target}}$
- $\left|\rho(\mathbf{D^l F^l})\right| \leq \left\|\mathbf{D^l F^l}\right\|$ for any submultiplicative matrix norm $\|.\|$
- Distributed criterion fulfilled with the infinity norm:

$$\left\|\mathbf{D^l F^l}\right\|_\infty = \max_{1 \leq k \leq N_{\mathrm{BS}}} \left(\frac{\gamma_k^l \sum_{j=1, j \neq \mathrm{BS}[k]}^{N_{\mathrm{BS}}} G_{j,k}^l}{G_{\mathrm{BS}[k],k}^l} \right) \tag{5}$$

Power control convergence requirement in OFDMA

- Distributed convergence criterion:

$$\forall k \in [1, N_{\text{BS}}], \left(\frac{\gamma_k^l \sum_{j=1, j \neq \text{BS}[k]}^{N_{\text{BS}}} G_{j,k}^l}{G_{\text{BS}[k],k}^l} \right) < 1 \qquad (6)$$

- The target SINR must fulfill the condition $\gamma_k^l < E_k^l$, where $E_k^l = G_{\text{BS}[k],k}^l / \left(\sum_{j=1, j \neq \text{BS}[k]}^{N_{\text{BS}}} G_{j,k}^l \right)$

Subcarrier allocation

- Separate subcarrier allocation from power allocation to reduce complexity [Wong, 99] [Rhee, 00].
- Distributed subcarrier allocation per cell. All users have the same R_{target}
- Aim: power minimization under target data rate requirements (derived from [Rhee, 00]):
  - Initialization: for each user k, assign the subcarrier that maximimizes E_k^l.
  - Iterative process: for the user with lowest data rate, assign the subcarrier that maximizes E_k^l.
  - Stop when all users have reached R_{target} or there are no more subcarriers.

Power control

- Iterative process: Per cell power minimization under target data rate requirement, depending on the inter-cell interference from the previous step.
- The sum power constraint per cell is not directly considered in the iterative process.
- Final admission control step: while $\sum_{l=1}^{L} P_j^l > P_{\max}$, reject the users with highest power from cell j.

Power allocation: Power minimization

- Power required by Base Station BS $[k]$ to satisfy user k in subcarrier l:

$$P^l_{\mathrm{BS}[k]} = \frac{\gamma^l_k}{G^l_{\mathrm{BS}[k],k}} \left(N_0 + \sum_{j=1, j\neq \mathrm{BS}[k]}^{N_{\mathrm{BS}}} G^l_{j,k} P^l_j \right) = \gamma^l_k \left(\frac{I^l_k}{G^l_{\mathrm{BS}[k],k}} \right) \quad (7)$$

- Minimizing the sum power for user k is equivalent to the following target SINR allocation problem:

$$\begin{gathered} \min_{\gamma_k} \textstyle\sum_{l=1}^{n_{\mathrm{SC},k}} \gamma^l_k \left(\frac{I^l_k}{G^l_{\mathrm{BS}[k],k}} \right) \\ B_{\mathrm{SC}} \textstyle\sum_{l=1}^{n_{\mathrm{SC},k}} \log_2(1+\gamma^l_k) \geq R_{k,\mathrm{target}} \\ E^l_k > \gamma^l_k \geq 0 \end{gathered} \quad (8)$$

Power allocation: Power minimization

- (8) is a convex optimization problem that can be solved with the KKT conditions [Boyd, 04]:

$$\gamma_k^l = \min \left\{ \left[\lambda_2 \left(\frac{G_{\mathrm{BS}[k],k}^l}{I_k^l} \right) - 1 \right]^+ ; E_k^l - \epsilon \right\} \tag{9}$$

  - λ_2: constant value, set to fulfill the target data rate constraint.
  - ϵ: small strictly positive value, to avoid $E_k^l = \gamma_k^l$

Iterative power control

- Initialization: set all power values to 0.
- At each iteration:

1. SINR determination iteration: $\forall k$, compute γ_k given the power values obtained in the previous iteration.
2. For γ_k, perform distributed iterative power control on each subcarrier independently (convergence ensured by $E_k^l > \gamma_k^l$).
3. Update I_k^l and go to the next SINR determination iteration.

Numerical results

- Comparison with iterative water-filling, with two subcarrier allocation criteria: E, and the channel gain criterion G.
- A user is rejected if it cannot reach $R_{\text{target}} = 64$ kbits/s.

Table: Percentage of rejected users depending on the load (%)

N_{users}	< 128	128	160	192	224
Proposed method	0	1.88	7.58	14.82	20.02
Iterative water-filling, E	0	3.91	10.71	27.28	34.78
Iterative water-filling, G	0	4.78	18.42	30.76	38.43

- High rejected users rate for IWF at medium to high load due to power divergence. No power divergence with the proposed method.

Numerical results

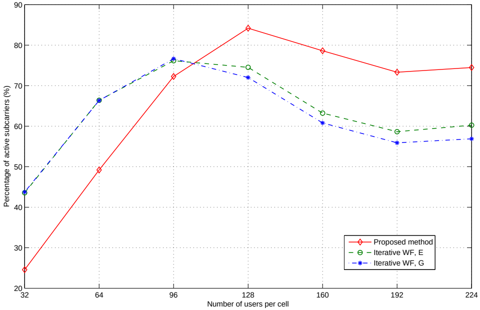

Figure: Percentage of active subcarriers, for varying load

- Lower subcarrier consumption than IWF at low to medium load: efficiency gain regarding subcarrier SINR setting.

Numerical results

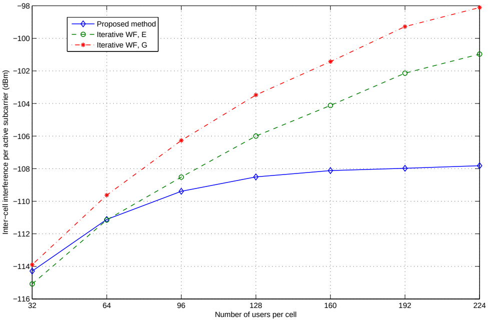

Figure: Interference per active subcarrier, for varying load

- Inter-cell interference is bounded with the proposed method.
- *E* criterion is more efficient than *G* criterion for inter-cell interference.

Outline

Information-theoretic results for MAC and broadcast channels

- Capacity region for the flat fading Gaussian MAC [Yeh, 04].
- Throughput-optimal RA: stabilizes the queues without knowledge of the queue's arrival rate.
  - Solution for MAC: max $\vec{\mu}\vec{R}$ with successive decoding [Tse, 98], where $\vec{\mu}$ is equal to the queue state: Longest Queue Receives the Highest Possible Rate (LQRHPR).
  - Similar solution for the broadcast channel with [Li, 01].
- Often referred to as Maximum Weight Matching Scheduling (MWMS).

MWMS for Best Effort users

- Best Effort users: avoid buffer overflows as much as possible.
- MAC and broadcast channel results do not extend to the interference channel...
- ... but MWMS intuitively seems well suited to balance the user's queues.

- The Maximum sum capacity issue has been studied on the interference channel [Qiu, 99] [Chiang, 05] and for single-user OFDM [Huang, 06].

MWMS on the interference channel

- K users on subcarrier l, α_k is the weight of user k.

$$\max_{\mathbf{P}} T_w(\mathbf{P}) = \max_{\mathbf{P}} \prod_{k=1}^{K} \left(1 + \frac{G^l_{BS[k],k} P^l_{BS[k]}}{I^l_k}\right)^{\alpha_k} \qquad (10)$$

$$P^l_k \in [P_{\min}, P_{\max}] \ \forall k$$

- At High SINR regime, MWMS is a Geometric Programming: unique global optimal solution, reachable via convex optimization.
- But in the general case, MWMS is not convex and may have several local optima.
- Simplified notations $G^l_{BS[k],k} = G^l_{k,k}$, $P^l_{BS[k]} = P^l_k$

MWMS on the interference channel

- Local optimum: $\frac{\partial T_w(\mathbf{P})}{\partial P_k^l} = 0$ or $P_k^l = P_{\min}$ or $P_k^l = P_{\max}$

$$\frac{\partial T_w(\mathbf{P})}{\partial P_k^l} = T_w(\mathbf{P}) \left[\frac{\alpha_k \gamma_k^l}{P_k^l(1+\gamma_k^l)} - \sum_{j \neq k} \frac{\alpha_j \gamma_j^l}{(1+\gamma_j^l)} \frac{G_{k,j}^l}{I_j^l} \right]$$

- Iterative process:

$$\begin{aligned} P_k^l(T+1) &= P_{\min} \text{ if } X_k^l(\mathbf{P^l}(T)) \leq P_{\min} \\ P_k^l(T+1) &= P_{\max} \text{ if } X_k^l(\mathbf{P^l}(T)) \geq P_{\max} \\ P_k^l(T+1) &= X_k^l(\mathbf{P^l}(T)) \text{ otherwise} \end{aligned} \quad (11)$$

$$X_k^l(\mathbf{P^l}(T)) = \frac{\alpha_k \gamma_k^l}{(1+\gamma_k^l)\left(\sum_{j \neq k} \frac{G_{k,j}^l \alpha_j}{I_j^l} \frac{\gamma_j^l}{(1+\gamma_j^l)}\right)} \quad (12)$$

MWMS on the interference channel

- $m_k^l = \frac{\alpha_k^l \gamma_l^l}{I_k^l\left(1+\gamma_k^l\right)}$ is the price charged by user k to the other users for generating interference in subcarrier l.
- $$X_k^l(\mathbf{P^l}(T)) = \frac{\alpha_k \gamma_k^l}{(1+\gamma_k^l)\left(\sum_{j\neq k} G_{k,j}^l m_j^l\right)} \tag{13}$$
- Distributed iterative process: after each computation of $P_k^l(T+1)$, each user sends m_k^l to all the other users for the next iteration.

Multi-channels MWMS

$$\max_{\mathbf{P}} \prod_{k=1}^{K} \prod_{l=1}^{L} \left(1 + \frac{G^{l}_{BS[k],k} P^{l}_{BS[k]}}{I^{l}_{k}} \right)^{\alpha_k . c^{l}_{BS[k],k}} \tag{14}$$

$$\sum_{l=1}^{L} P^{l}_{j} \leq P_{\text{max}} \; \forall j \in [1, N_{\text{BS}}] \text{ and } P^{l}_{k} \geq P_{\text{min}} \; \forall k$$

- With known subcarrier allocation, the problem can be solved in the dual space.
- Relax the sum power constraint by introducing a dual price per user, μ_k: L sub-problems, one per subcarrier.
- Convergence of the global algorithm in High SINR regime.

Multi-channels MWMS

- Initialization: at t=0, set the initial power and price, and dual price $\mu_k(0) \geq 0$.
- At each iteration:

1. Dual price update at t:

$$\mu_k(t) = \max \left\{ \mu_k(t-) + \kappa \left(\sum_{l=1}^{L} P_k^l(t-) - P_{\max} \right), 0 \right\}$$

2. Power update at t: user k updates its power on carrier l with (11) and $X_k^l(\mathbf{P^l}(T)) = \alpha_k \gamma_k^l / \left((1+\gamma_k^l) \left(\sum_{j \neq k} G_{k,j}^l m_j^l + \mu_k \right) \right)$
3. Price update at t, user k updates its price on channel l : m_k^l

Application to OFDMA

- Application to OFDMA: consider each cell as a unique user. Users are differentiated by their path loss.
- Separate subcarrier from power allocation.
- Queue-length based subcarrier allocation: number of requested subcarriers for user k with queue length S_k:

$$l_{\mathrm{SC},k} = \mathrm{int}\left[\frac{S_k}{\sum_{j=1}^{K[n_{\mathrm{BS}}]} S_j}\right] L$$

- Then allocate subcarriers to reach $l_{\mathrm{SC},k}$ with the aim to maximize E criterion.

Proposed 2 phases method

- First phase: in the general SINR regime, starting from $P_k^l = P_{\max} \forall (k, l)$, run algorithm (11) on each subcarrier.
- High SINR condition: keep only the users and subcarriers that are in high SINR with β precision.

$$\left\| \alpha_k \log_2(1+\gamma_k^l) - \alpha_k \log_2(\gamma_k^l) \right\| \leq \beta \Leftrightarrow \gamma_k^l \geq \frac{1}{\left(2^{\frac{\beta}{\alpha_k}} - 1\right)} \qquad (15)$$

- Second phase: for users in high SINR, use the dual algorithm with an additional constraint on $P_{k,\min}^l$ to ensure that the high SINR condition remains fulfilled:

$$P_{k,\min}^l = \frac{1}{\left(2^{\frac{\beta}{\alpha_k}} - 1\right)} \frac{\left(N_0 + \sum_{j \neq k} G_{j,k}^l P_k^l\right)}{G_{k,k}^l} \qquad (16)$$

Numerical results

Comparison with MWMS iterative water-filling: on each cell, MWMS with inter-cell interference considered as noise.

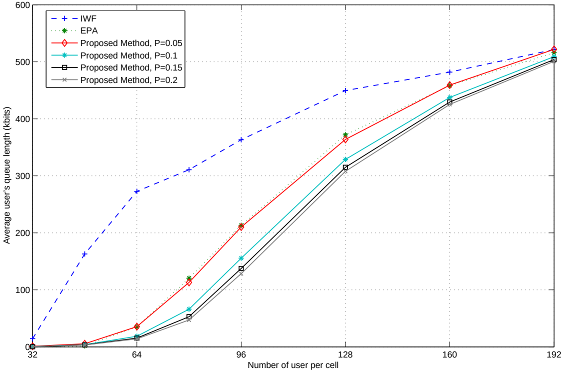

Figure: Average queue length per user, depending on the load

Outline

- Introduction
- Rate-constrained users
  - Optimization problem
  - Power minimization under convergence requirement
  - Numerical results
- Cross-layer optimization for Best Effort users
  - State of the art
  - MWMS in OFDMA
  - Numerical results
- Conclusions and future work

Conclusions and future work

- Conclusions
    - Sub-optimal resource allocation for rate-constrained users: quite effective when compared to iterative water-filling.
    - Cross-layer resource allocation for Best Effort users: extended the known results on TDMA to weighted sum capacity maximization in OFDMA.
- Future work
    - Characterization of the capacity region on the interference channel.
    - Deduce subcarrier allocation suited for MWMS to mitigate co-channel interference.
    - Extension to MIMO OFDMA.

Thank you for your attention !

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- Rate-constrained users:
  - $\mathcal{N}$: two rings of interfering Base Stations with omnidirectional antennas with same cell radius.
  - $N_{\text{users}} = [32, 64, 96, .., 224]$ users per Base Station.
  - $B = 10$ MHz, $N_{\text{FFT}} = 256$, $\text{d}_{\text{is}} = 1.212$ km
  - Path loss: Okumura-Hata, shadowing's standard deviation of 7 dB, thermal noise -105 dBm.
  - $R_{\text{target}} = 64$ kbits/s for all users.
- Best Effort users:
  - $\text{d}_{\text{is}} = 0.61$ km.
  - Poisson traffic arrival model,
    - Inter-arrival law: exponential law of average 200 time slots
    - Packet size: log-normal law with average 5kbits, s.d. 0.1 kbits.
  - Snap shots of 1000 time slots, 1 time slot = 2ms.