PARTIAL DECODING FOR SYNCHRONOUS AND ASYNCHRONOUS GAUSSIAN MULTIPLE RELAY CHANNELS

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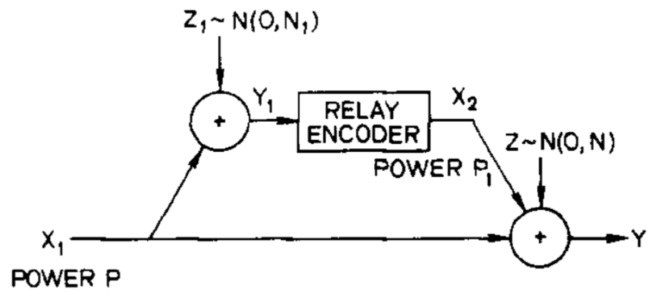
Motivation (I)

- Relaying increases capacity in wireless networks,
  - due to cooperative source-relay transmission [CovGam79].
  - Also improves reliability and delay.



T. Cover and A. El Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. on Information Theory*.

- How to operate? decode, compress or amplify?
  - It depends on topology, number of relays and SNR regime.
  - Decoding techniques performs good with low number of relays.

Motivation (II)

- We study the potentially better decoding technique, *i.e.*, *partial decoding*:
  - The relays only decode a part of the source message.
  - Transmission is then adapted to source-relay channels quality.
- Previous work on PD:
  - Capacity achieving for the *half-duplex* single-relay channel at the high SNR regime [HostZhang05]
  - *Idem* for the single-relay channel with orthogonal components [GamZah05].
- Our contribution
  - Application to the multiple parallel relay channel.

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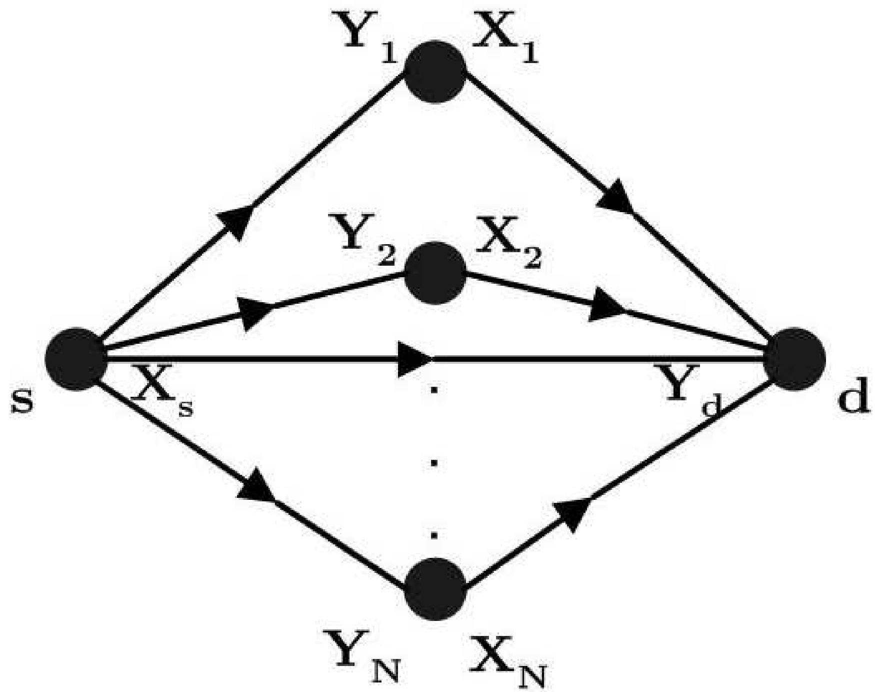
Channel Model (I)

We separately analyze two channels:

- The *full-duplex* multiple-relay channel



- *Remark*: parallel relays.

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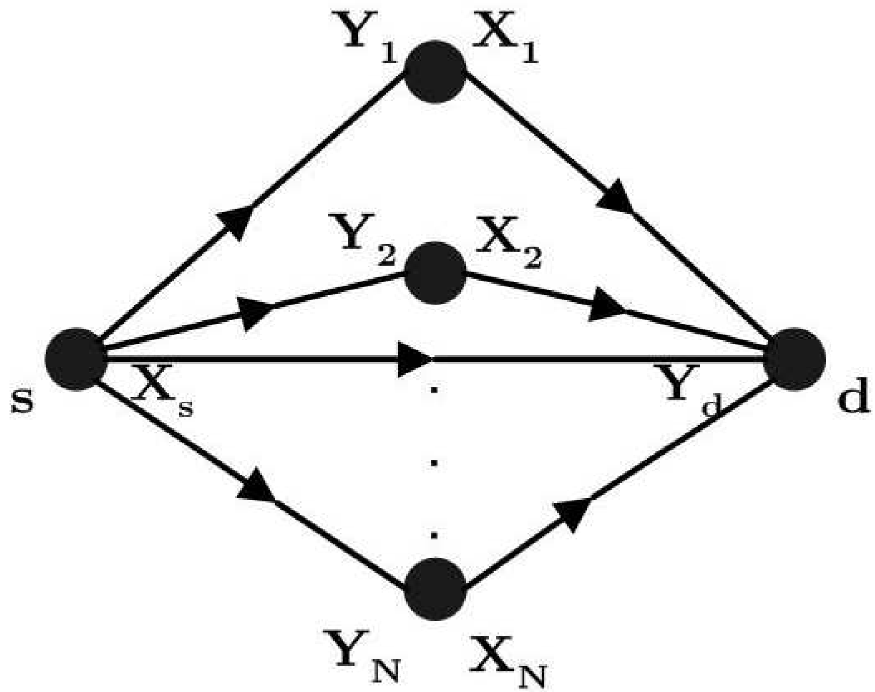

- The source transmits the complex signal X_s, while relays transmit X_i, $i = 1, \cdots, N$.

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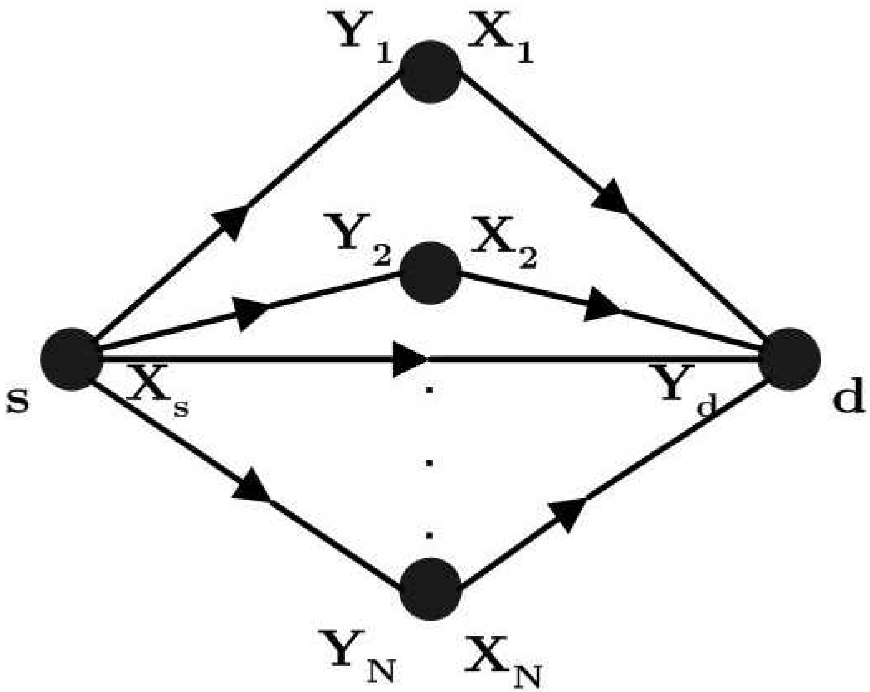

- Relays receive

$$Y_i = \sqrt{a_{\mathrm{s,i}}} \cdot X_s + Z_i$$

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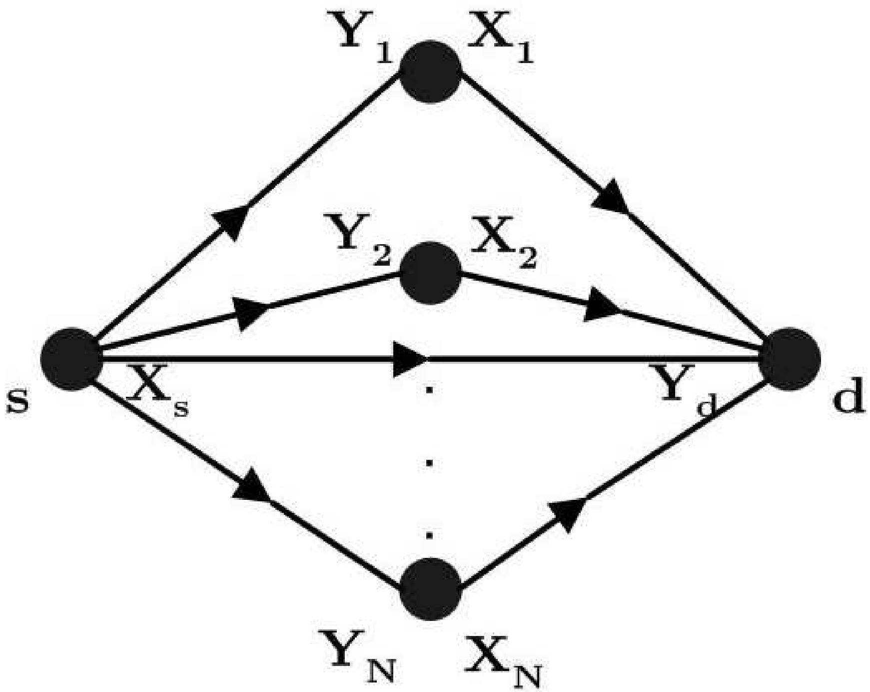
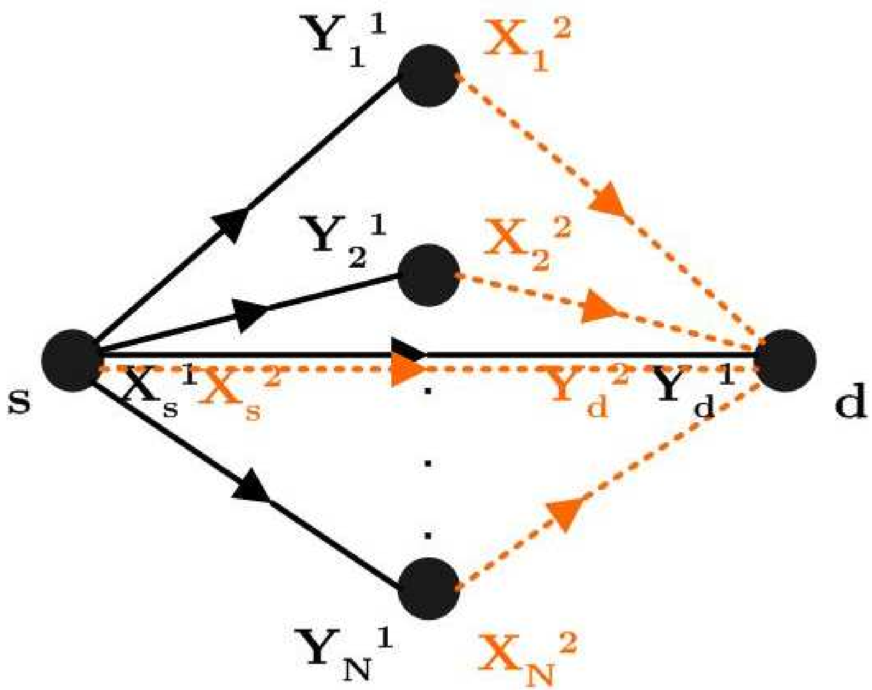


- The destination receives

$$Y_d = \sqrt{a_{\mathrm{s,d}}} \cdot X_s + \sum_{i=1}^{N} \sqrt{a_{\mathrm{i,d}}} \cdot X_i + Z_d \ .$$

Channel Model (II)

- The *half-duplex* multiple-relay channel



- During time slot 1, the source transmits the complex signal X_s^1 of duration α.
- During time slot 2, of duration $1-\alpha$, the source transmit X_s^2 while relays transmit X_i^2, $i = 1, \cdots, N$.

Channel Model (II)

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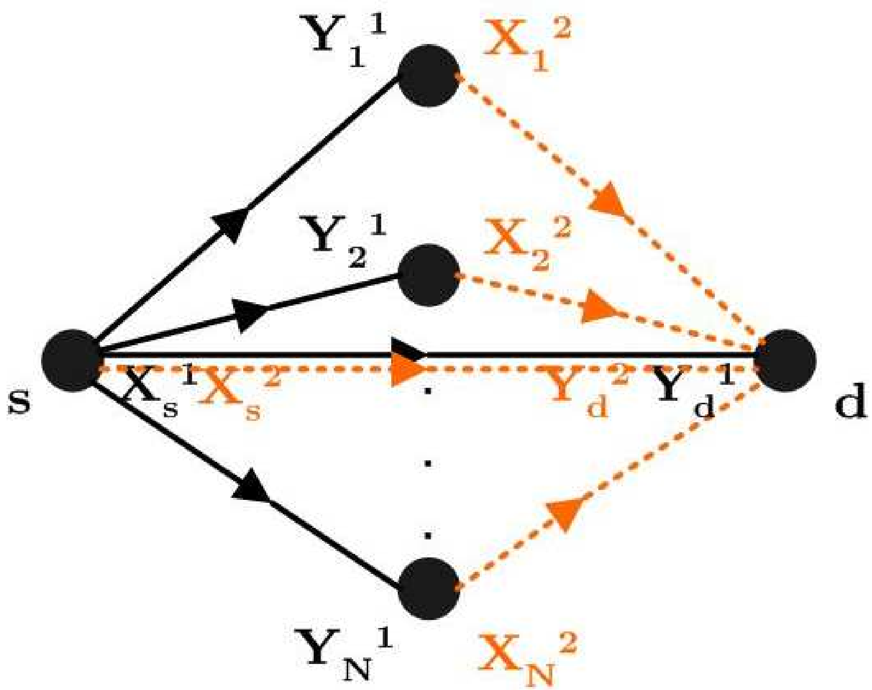

- Relays receive

$$Y_i^1 = \sqrt{a_{\mathrm{s,i}}} \cdot X_s^1 + Z_i^1$$

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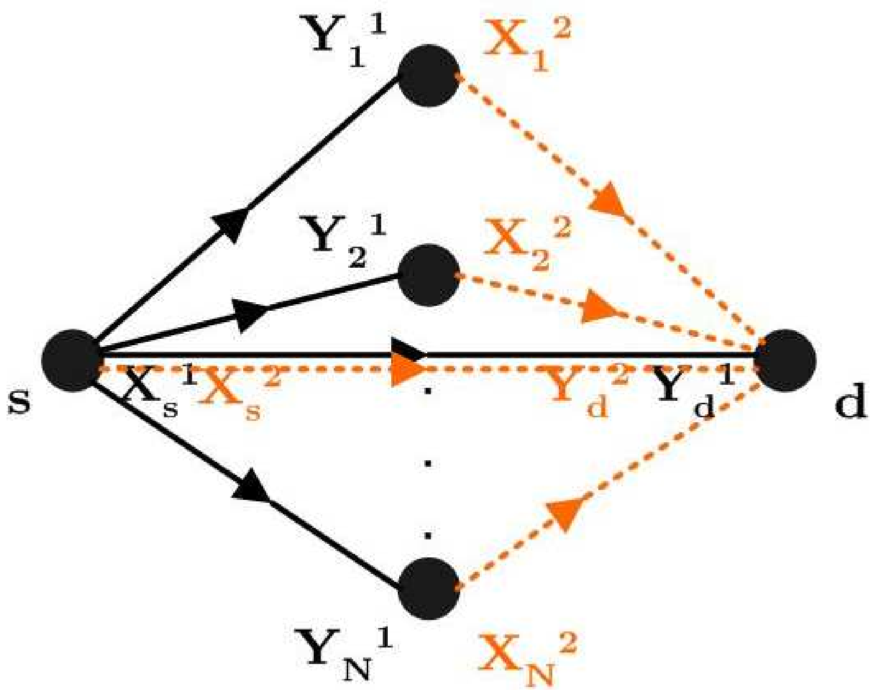

- The destination receives

$$Y_d^1 = \sqrt{a_{\rm s,d}} \cdot X_s^1 + Z_d^1$$

$$Y_d^2 = \sqrt{a_{\rm s,d}} \cdot X_s^2 + \sum_{i=1}^{N} \sqrt{a_{\rm i,d}} \cdot X_i^2 + Z_d^2$$

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Partial Decoding for the Full-duplex Channel (I)

Theorem 1 *The capacity of the channel is lower and upper bounded by*

$$C \geq \max_{1 \leq n \leq N} \min \left\{ I\left(X_s, \mathbf{X}_{1:n}; Y_d\right), \; I\left(U; Y_n | \mathbf{X}_{1:n}\right) + I\left(X_s; Y_d | U, \mathbf{X}_{1:n}\right) \right\}$$

$$C \leq \min \left\{ I\left(X_s, \mathbf{X}_{1:N}; Y_d\right), I\left(X_s; \mathbf{Y}_{1:N}, Y_d | \mathbf{X}_{1:N}\right) \right\}$$

where source-relays path gains have been ordered as:

$$a_{\mathrm{s,1}} \geq \cdots \geq a_{\mathrm{s,n}} \geq \cdots \geq a_{\mathrm{s,N}}.$$

Remark 1 *In the theorem, n denotes the cardinality of the decoding set.*

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Partial Decoding for the Full-duplex Channel (II)

Proof:

- The source splits its message into two parts: $\omega = \omega_r + \omega_d$.
  - Both parts are transmitted using superposition coding.
  - With codebooks $U(\cdot)$ and $V(\cdot)$, and rates R_r and R_d, respectively.

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- Assume that only the $n \leq N$ relays with the best source-relay channels are active.

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- The $n \leq N$ relays can decode ω_r if and only if:

$$R_r \leq I\left(U; Y_n | \mathbf{X}_{1:n}\right)$$

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$$R_r \leq I\left(U; Y_n | \mathbf{X}_{1:n}\right)$$

- The destination can decode ω_r if and only if

$$R_r \leq I\left(U, \mathbf{X}_{1:n}; Y_d\right)$$

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Partial Decoding for the Full-duplex Channel (II)

Proof:

- Finally, using SIC, the destination can decode ω_d if and only if

$$R_d \leq I\left(X_s; Y_d | U, \mathbf{X}_{1:n}\right)$$

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Partial Decoding for the Full-duplex Channel (II)

Proof:

- Finally, using SIC, the destination can decode ω_d if and only if

$$R_d \leq I\left(X_s; Y_d | U, \mathbf{X}_{1:n}\right)$$

- Considering that $R = R_d + R_r$ and that n can be optimally choosen, then it concludes the proof.

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Partial Decoding for the Full-duplex Channel (III)

- We evaluate the rate in Theorem 1 for a network with and without phase synchronism among relays.
- Overall power constraint: $\frac{\mathcal{E}\{X_s \cdot X_s^*\} + \sum_{i=1}^{N} \mathcal{E}\{X_i \cdot X_i^*\}}{\sigma_o^2} \leq \gamma$

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Partial Decoding for the Full-duplex Channel (III)

- PHASE SYNCHRONOUS NETWORK

Proposition 1 *The capacity of the channel is lower and upper bounded by*

$$C \geq \max_{1 \leq n \leq N} \max_{(\eta,\beta) \in [0,1]} \min \left\{ \mathcal{C}\left(\left(a_{\mathrm{s,d}} + \beta(1-\eta)\sum_{i=1}^{n} a_{\mathrm{i,d}}\right)\gamma\right), \right.$$
$$\left. \mathcal{C}\left(a_{\mathrm{s,n}}\beta\eta\frac{\gamma}{1 + a_{\mathrm{s,n}}(1-\beta)\gamma}\right) + \mathcal{C}\left(a_{\mathrm{s,d}}(1-\beta)\gamma\right)\right\}$$

$$C \leq \max_{\rho \in [0,1]} \min \left\{ \mathcal{C}\left(\left(a_{\mathrm{s,d}} + \rho\sum_{i=1}^{N} a_{\mathrm{i,d}}\right)\gamma\right), \right.$$
$$\left. \mathcal{C}\left(\left(a_{\mathrm{s,d}} + \sum_{i=1}^{N} a_{\mathrm{s,i}}\right)(1-\rho)\gamma\right)\right\}$$

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Partial Decoding for the Full-duplex Channel (IV)

- PHASE ASYNCHRONOUS NETWORK

Proposition 2 *The capacity of the channel is lower and upper bounded by*

$$C \geq \max_{1\leq n\leq N} \max_{(\eta,\beta)} \min \Big\{ \mathcal{C}\left(\left(a_{\mathrm{s,d}}\left(1-\beta+\beta\eta\right)+a^{+}_{[\mathrm{s},1:\mathrm{n}],d}\beta\left(1-\eta\right)\right)\gamma\right),$$

$$\mathcal{C}\left(a_{\mathrm{s,n}}\beta\eta\frac{\gamma}{1+a_{\mathrm{s,n}}\left(1-\beta\right)\gamma}\right)+\mathcal{C}\left(a_{\mathrm{s,d}}\left(1-\beta\right)\gamma\right)\Big\}$$

$$C \leq \max_{\rho\in[0,1]} \min \Big\{ \mathcal{C}\left(\left(a_{\mathrm{s,d}}\left(1-\rho\right)+a^{+}_{[\mathrm{s},1:\mathrm{n}],d}\cdot\rho\right)\gamma\right),$$

$$\mathcal{C}\left(\left(a_{\mathrm{s,d}}+\sum_{i=1}^{N}a_{\mathrm{s,i}}\right)\left(1-\rho\right)\gamma\right)\Big\}$$

Remark 1 *We set* $a^{+}_{[\mathrm{s},1:\mathrm{n}],d}=\max_{t=\{s,1,\cdots,n\}}a_{\mathrm{t,d}}$, *i.e., opportunistic relay transmission.*

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Partial Decoding for the Half-duplex Channel (I)

Theorem 2 *The capacity of the channel is lower and upper bounded by*

$$C \geq \max_{1 \leq n \leq N} \max_{\alpha} \min \left\{ I\left(\boldsymbol{X}_s, \boldsymbol{X}^2_{1:n}; \boldsymbol{Y}_d\right), \right.$$
$$\left. \alpha \cdot I\left(X^1_s; Y^1_n\right) + (1-\alpha) \cdot I\left(X^2_s; Y^2_d | U, \boldsymbol{X}^2_{1:n}\right) \right\}$$

$$C \leq \max_{\alpha} \min \left\{ I\left(\boldsymbol{X}_s, \boldsymbol{X}^2_{1:N}; \boldsymbol{Y}_d\right), \right.$$
$$\left. I\left(\boldsymbol{X}_s; \boldsymbol{Y}^1_{1:N}, \boldsymbol{Y}_d | \boldsymbol{X}^2_{1:N}\right) \right\}$$

where source-relays path gains have been ordered as:

$$a_{\mathrm{s},1} \geq \cdots \geq a_{\mathrm{s,n}} \geq \cdots \geq a_{\mathrm{s,N}}.$$

Proof: Equivalent to that of Theorem 1.

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Partial Decoding for the Half-duplex Channel (II)

- Evaluation for a Gaussian network with an overall power constraint, *i.e.*
  - Let $\gamma_1 = \frac{\mathcal{E}\{X_s^1(X_s^1)^*\}}{\sigma_o^2}$, $\gamma_2 = \frac{\mathcal{E}\{X_s^2(X_s^2)^*\}}{\sigma_o^2} + \frac{\sum_{i=1}^{N}\mathcal{E}\{X_i^2(X_i^2)^*\}}{\sigma_o^2}$
  - Then, $\alpha \cdot \gamma_1 + (1-\alpha) \cdot \gamma_2 \leq \gamma$

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Partial Decoding for the Half-duplex Channel (II)

- PHASE SYNCHRONOUS NETWORK

Proposition 3 *The capacity of the channel is lower and upper bounded by*

$$C \geq \max_{1 \leq n \leq N} \max_{\substack{(\alpha,\gamma_1,\gamma_2,\beta): \\ \alpha\cdot\gamma_1+(1-\alpha)\cdot\gamma_2=\gamma}} \min\left\{\alpha \cdot \mathcal{C}\left(a_{\mathrm{s,d}}\gamma_1\right) + (1-\alpha)\cdot\mathcal{C}\left(\left(a_{\mathrm{s,d}} + \beta \sum_{i=1}^{n} a_{\mathrm{i,d}}\right)\gamma_2\right), \alpha\cdot\mathcal{C}\left(a_{\mathrm{s,n}}\gamma_1\right) + (1-\alpha)\cdot\mathcal{C}\left(a_{\mathrm{s,d}}\left(1-\beta\right)\gamma_2\right)\right\}$$

$$C \leq \max_{\substack{(\alpha,\gamma_1,\gamma_2,\rho): \\ \alpha\cdot\gamma_1+(1-\alpha)\cdot\gamma_2=\gamma}} \min\left\{\alpha \cdot \mathcal{C}\left(a_{\mathrm{s,d}}\gamma_1\right) + (1-\alpha)\cdot\mathcal{C}\left(\left(a_{\mathrm{s,d}} + \rho \sum_{i=1}^{N} a_{\mathrm{i,d}}\right)\gamma_2\right), \alpha\cdot\mathcal{C}\left(\left(a_{\mathrm{s,d}} + \sum_{i=1}^{N} a_{\mathrm{s,i}}\right)\gamma_1\right) + (1-\alpha)\cdot\mathcal{C}\left(a_{\mathrm{s,d}}\left(1-\rho\right)\gamma_2\right)\right\}$$

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Partial Decoding for the Half-duplex Channel (III)

- PHASE ASYNCHRONOUS NETWORK

Proposition 4 *The capacity of the channel is lower and upper bounded by*

$$C \geq \max_{1\leq n\leq N} \max_{\substack{(\alpha,\gamma_1,\gamma_2,\beta): \\ \alpha\cdot\gamma_1+(1-\alpha)\cdot\gamma_2=\gamma}}$$

$$\min\left\{\alpha\cdot\mathcal{C}\left(a_{\mathrm{s,d}}\gamma_1\right)+(1-\alpha)\cdot\mathcal{C}\left(\left(a_{\mathrm{s,d}}\left(1-\beta\right)+a^{+}_{[\mathrm{s,1:n}],d}\beta\right)\gamma_2\right),\right.$$
$$\left.\alpha\cdot\mathcal{C}\left(a_{\mathrm{s,n}}\gamma_1\right)+(1-\alpha)\cdot\mathcal{C}\left(\left(1-\beta\right)a_{\mathrm{s,d}}\gamma_2\right)\right\}$$

$$C \leq \max_{\substack{(\alpha,\gamma_1,\gamma_2,\rho): \\ \alpha\cdot\gamma_1+(1-\alpha)\cdot\gamma_2=\gamma}}$$

$$\min\left\{\alpha\cdot\mathcal{C}\left(a_{\mathrm{s,d}}\gamma_1\right)+(1-\alpha)\cdot\mathcal{C}\left(\left(a_{\mathrm{s,d}}\left(1-\rho\right)+a^{+}_{[\mathrm{s,1:n}],d}\rho\right)\gamma_2\right),\right.$$
$$\left.\alpha\cdot\mathcal{C}\left(\left(a_{\mathrm{s,d}}+\sum_{i=1}^{N}a_{\mathrm{s,i}}\right)\gamma_1\right)+(1-\alpha)\cdot\mathcal{C}\left(a_{\mathrm{s,d}}\left(1-\rho\right)\gamma_2\right)\right\}$$

Numerical Results (I)

- We consider AWGN channels under unitary-mean i.i.d. Rayleigh flat-fading.
- Channel are assumed to be known and invariant during the entire frame duration
- Optimization problems have been solved numerically.
- The average achievable rate is plotted.

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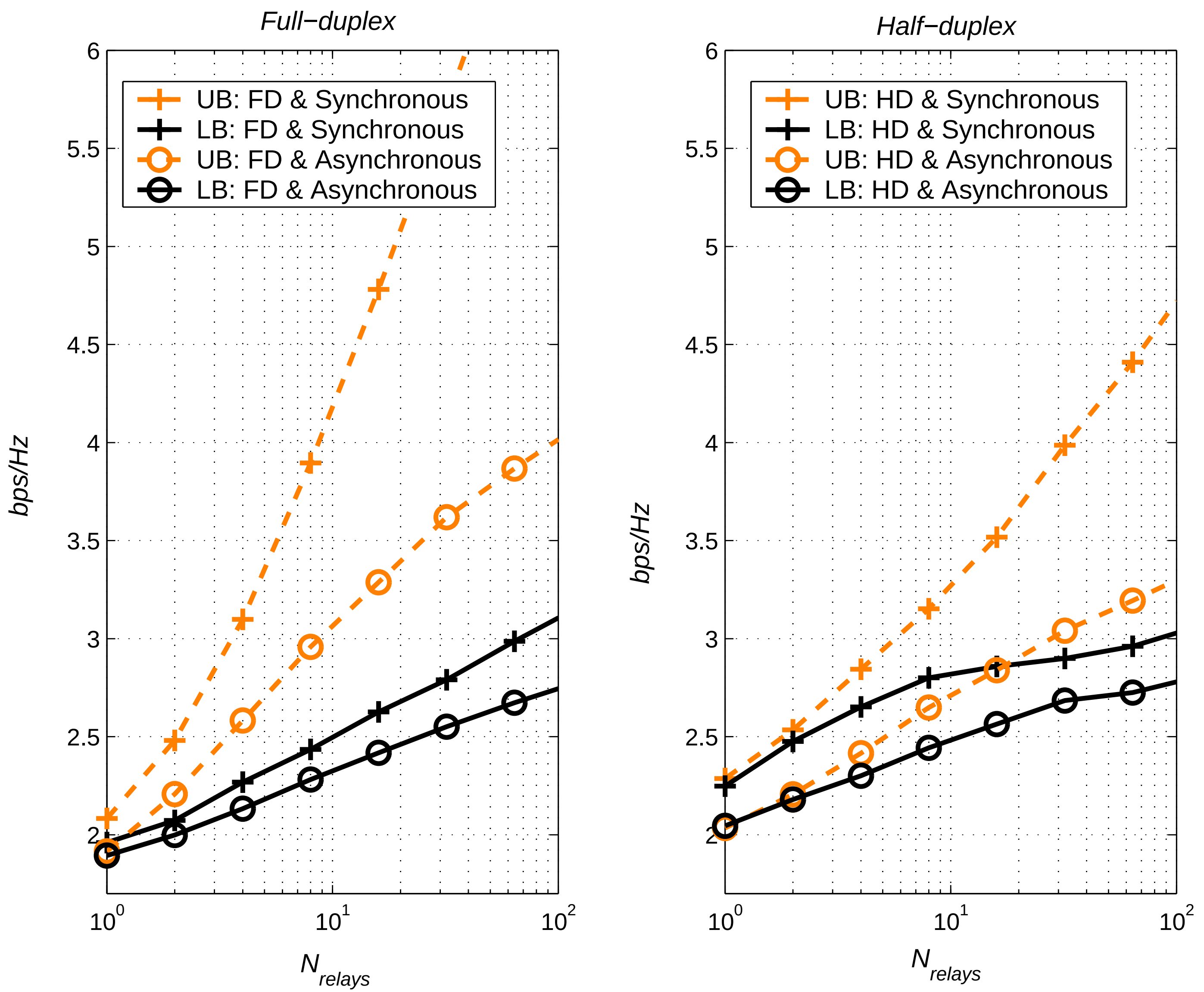
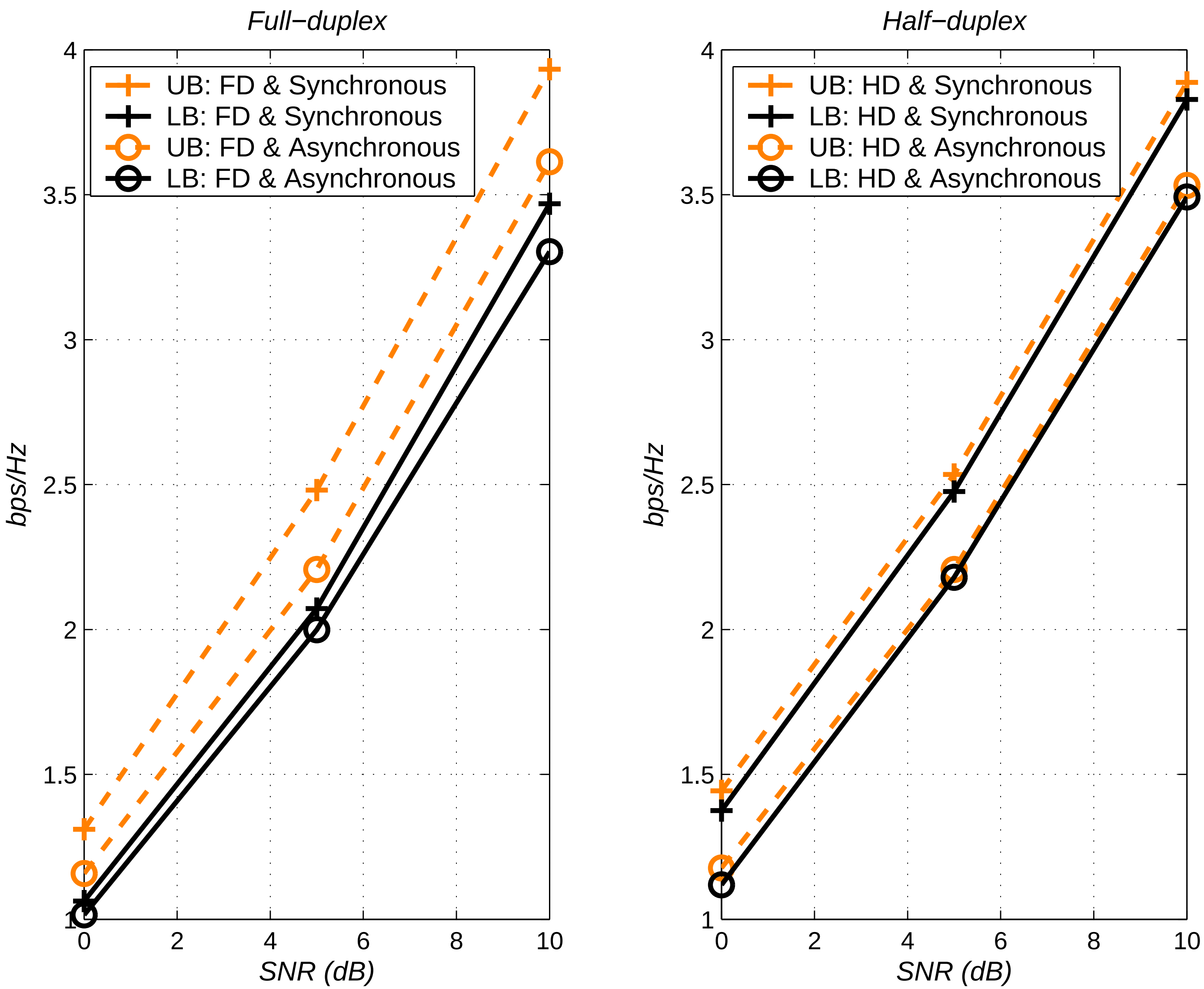
Numerical Results (I)



Numerical Results (II)



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Conclusions

- We have presented the achievable rate of partial decoding for both full-duplex and half-duplex multiple-relay channels.

- The evaluation of the *max-flow-min-cut* has also been obtained.

- Partial decoding is optimum for half-duplex channels with one and two relays.

- The half-duplex achievable rate outperforms the full-duplex one

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