ACHIEVABLE RATE FOR GAUSSIAN MULTIPLE RELAY CHANNELS WITH LINEAR RELAYING FUNCTIONS

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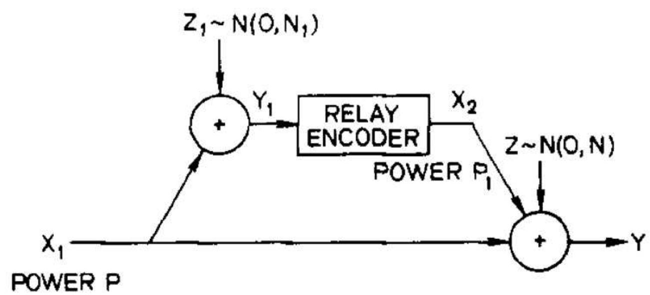
Motivation (I)

- Relaying increases capacity in wireless networks.
  - [CovGam79] cooperative source-relay transmission.
  - [WangGian06] opportunistic relaying.



T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. on Information Theory*.

- How to operate? decode, compress or amplify?
  - It depends on topology, number of relays and SNR regime.
  - [DanaHass03] amplification achieves capacity scaling law with $N \rightarrow \infty$.

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Motivation (II)

- We study the potentially better amplification scheme: *linear relaying* [Gam *et al.*06].

$$X_i^t = \sum_{j=0}^{t-1} d_i\left(t, j\right) Y_i^j, \quad i = 1, \cdots, N.$$

- Resemblance with AWGN channels with memory and colored noise [Gall68] [Hirt88].
  - Strategy: parallel channel decomposition and optimal power allocation.
  - It is not a channel with *time-invariant* ISI: no spectral *waterfilling* can be performed.

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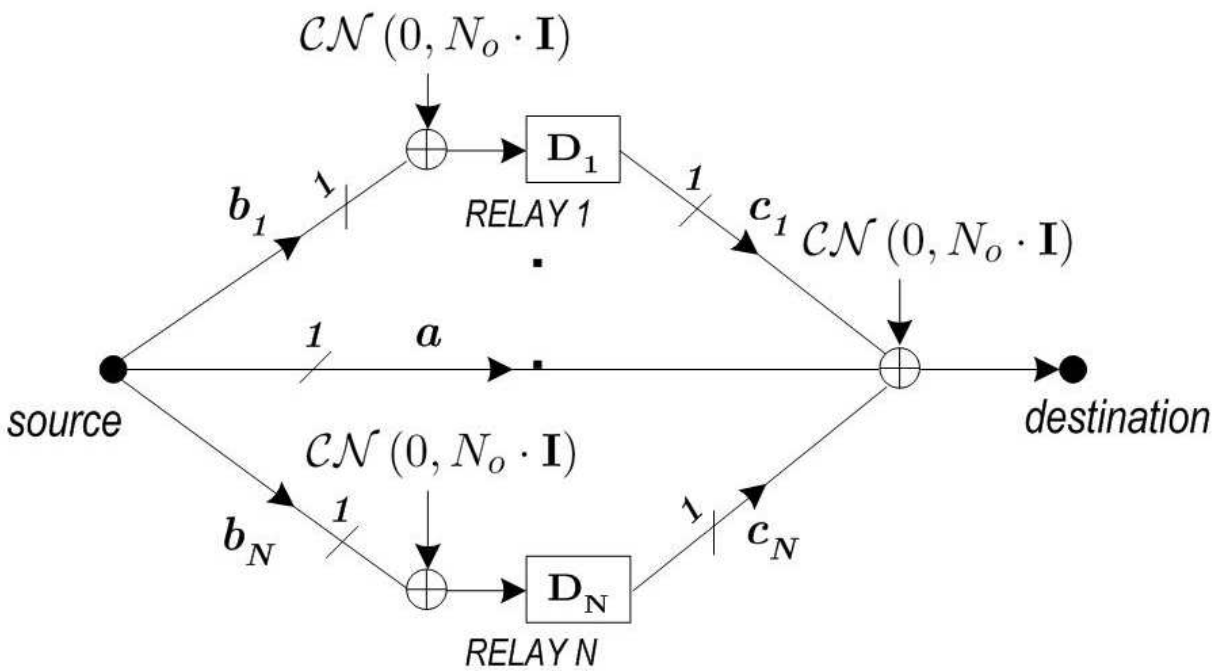
Signal Model and Capacity (I)

- AWGN multiple relay channel (MRC)



- *Remark*: parallel relays.

Signal Model and Capacity (I)

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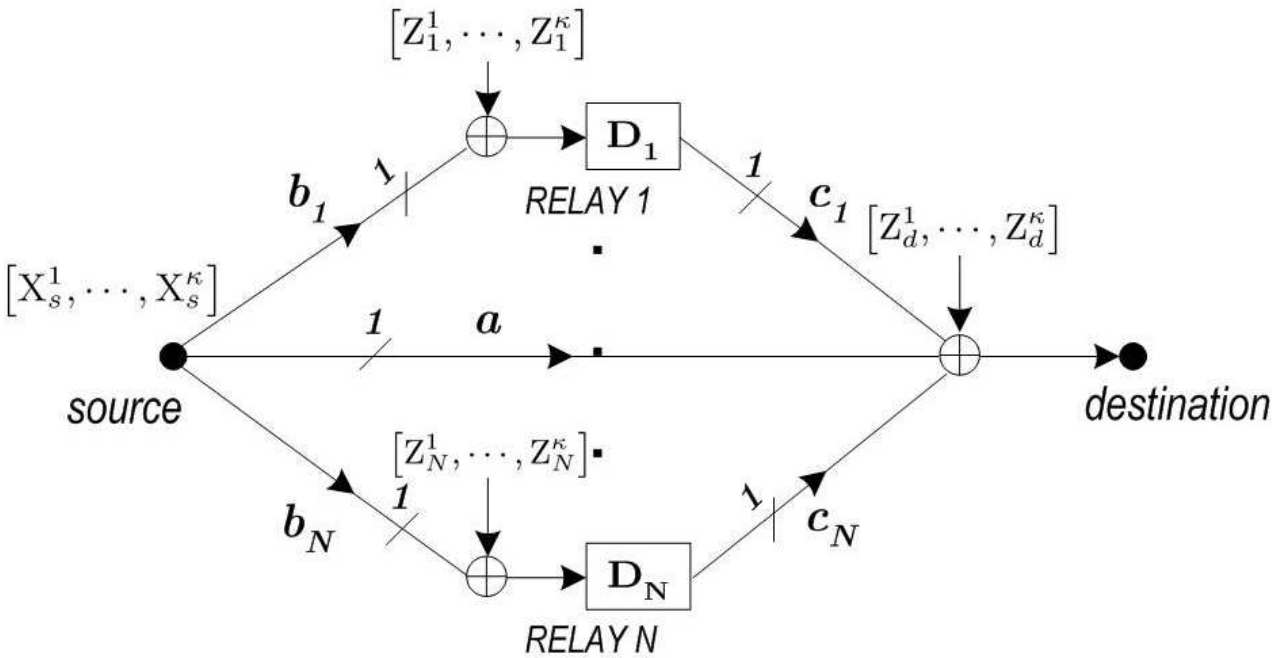

- The source transmits a sequence of κ symbols.

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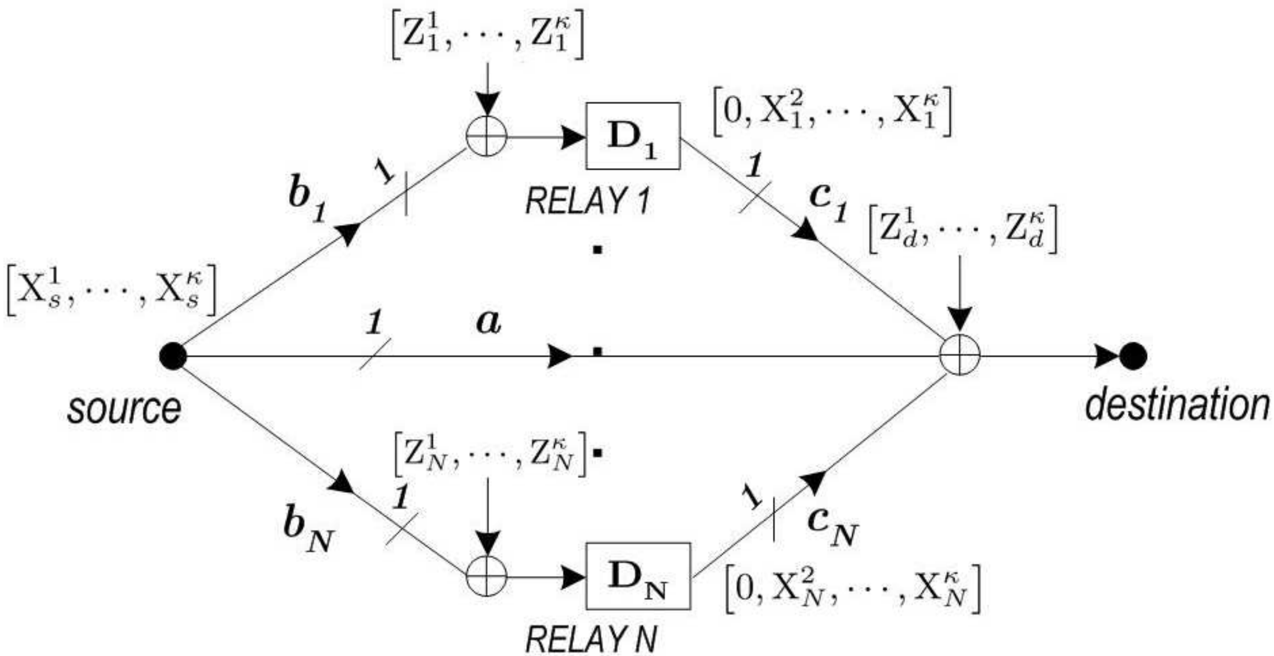

- Relays assist in the communication

$$\mathbf{X}_i^\kappa = \mathbf{D}_i \cdot \mathbf{Y}_i^\kappa = \mathbf{D}_i \cdot (b_i \cdot \mathbf{X}_s^\kappa + \mathbf{Z}_i^\kappa)$$

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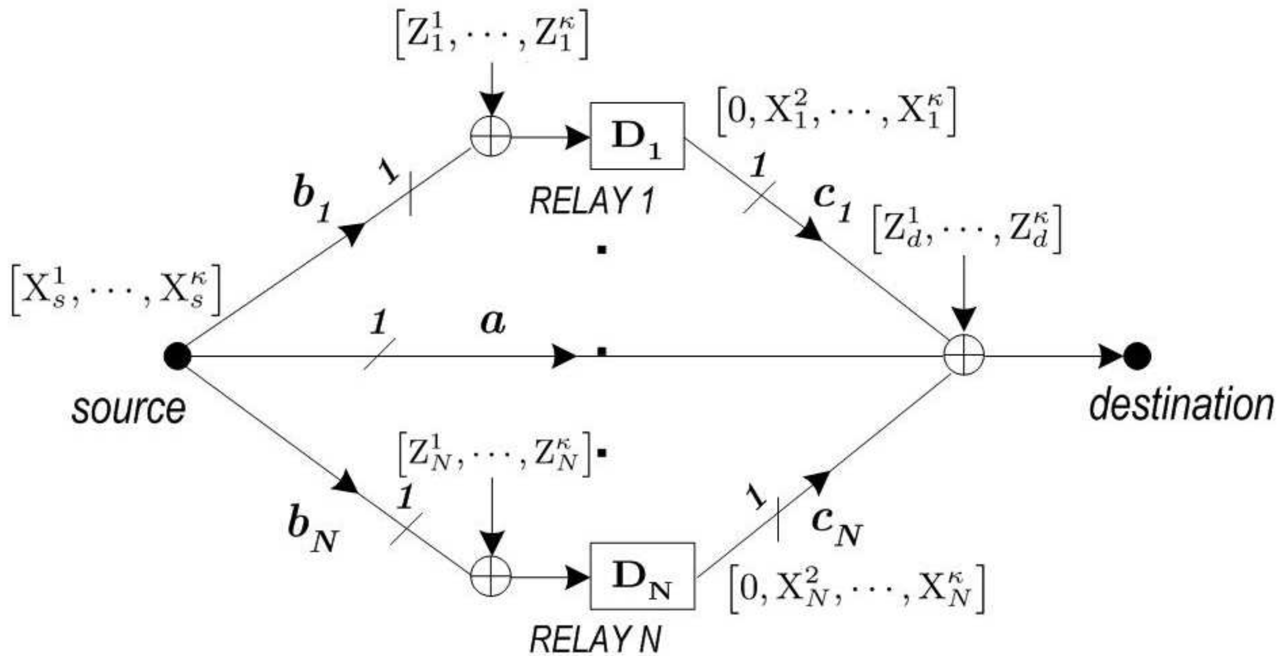

- The destination receives

$$\mathbf{Y}_d^\kappa = \left(a \cdot \mathbf{I} + \sum_{i=1}^N b_i c_i \mathbf{D}_i\right) \cdot \mathbf{X}_s^\kappa + \left(\mathbf{Z}_d^\kappa + \sum_{i=1}^N c_i \mathbf{D}_i \mathbf{Z}_i^\kappa\right)$$

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Signal Model and Capacity (II)

- Achievable rate: theory of discrete-time channels with memory [Hirt88]:

$$\mathcal{C} = \lim_{\kappa \to \infty} \max_{p_{\mathbf{X}_s^\kappa}, \mathcal{D}} \frac{1}{\kappa} \cdot I\left(\mathbf{X}_s^\kappa ; \mathbf{Y}_d^\kappa\right)$$

  with $\mathcal{D} = \{\mathbf{D}_1, \cdots, \mathbf{D}_N\}$.

- s.t. $\mathrm{tr}\left\{\mathrm{E}\left\{\mathbf{X}_s^\kappa \left(\mathbf{X}_s^\kappa\right)^H\right\}\right\} + \sum_{i=1}^N \mathrm{tr}\left\{\mathrm{E}\left\{\mathbf{X}_i^\kappa \left(\mathbf{X}_i^\kappa\right)^H\right\}\right\} \leq \kappa \mathrm{P}$

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with $\mathcal{D} = \{\mathbf{D}_1, \cdots, \mathbf{D}_N\}$.

- s.t. $\mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} \leq \kappa \mathrm{P}$ with

$$\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right) = \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \left(\mathbf{I} + \sum_{i=1}^{N} |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i\right) + \sigma_o^2 \sum_{i=1}^{N} \mathbf{D}_i^H \mathbf{D}_i .$$

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Signal Model and Capacity (II)

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$$\mathcal{C} = \lim_{\kappa\to\infty} \max_{\mathbf{\Sigma}_{\mathrm{X}_s^\kappa},\mathcal{D}} \frac{1}{\kappa}\cdot \log_2\left(\det\left(\mathbf{I}+\mathbf{H}_e\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}\mathbf{H}_e^H\right)\right)$$

$$\text{s.t.}\quad \mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa},\mathcal{D}\right)\right\}\leq \kappa \mathrm{P}\ ,\ \ \mathbf{\Sigma}_{\mathrm{X}_s^\kappa}\succeq 0$$

where

$$\mathbf{H}_e = \frac{1}{\sqrt{\sigma_o^2}}\left(\mathbf{I}+\sum_{i=1}^{N}|c_i|^2\mathbf{D}_i\mathbf{D}_i^H\right)^{-\frac{1}{2}}\left(a\cdot\mathbf{I}+\sum_{i=1}^{N}b_ic_i\mathbf{D}_i\right).$$

- Non-convex optimization for $\kappa > 2$

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Source Covariance Optimization (I)

Theorem 1 *Consider a given set* $\mathcal{D} = \{\mathbf{D}_1, \cdots, \mathbf{D}_N\} \in \mathrm{C}^{\kappa \times \kappa}$ *with SVD-Decomposition* $\mathbf{H}_e = \mathbf{U}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}^H$, *where* $\mathbf{\Lambda} = \mathrm{diag}([\lambda_1, \cdots, \lambda_\kappa])$. *The MRC achieves the rate*

$$\mathcal{C}(\kappa, \mathcal{D}) = \max_{\substack{\mathbf{\Sigma}_{\mathrm{X}_s^\kappa} : \mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} \leq \kappa \mathrm{P} \\ \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \succeq 0}} \frac{1}{\kappa} \cdot \log_2\left(\det\left(\mathbf{I} + \mathbf{H}_e \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)\right)$$

$$= \frac{1}{\kappa} \cdot \sum_{n=1}^{\kappa} \log_2\left(1 + \lambda_n \psi_n\right)$$

where

$$\psi_n = \left[\frac{1}{\mu \cdot (1+\phi_n)} - \frac{1}{\lambda_n}\right]^+, \quad \phi_n = \left[\sum_{i=1}^{N} \mathbf{V}^H |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i \mathbf{V}\right]_{n,n}$$

$$\sum_{n=1}^{\kappa} \left[\frac{1}{\mu} - \frac{1+\phi_n}{\lambda_n}\right]^+ = \kappa \mathrm{P} - \mathrm{tr}\left\{\sigma_o^2 \sum_{i=1}^{N} \mathbf{D}_i^H \mathbf{D}_i\right\}.$$

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Source Covariance Optimization (II)

Proof: The Lagrangian of the problem reads

$$\mathcal{L}\left(\boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa}, \boldsymbol{\Omega}, \mu\right) = \log\left(\det\left(\mathbf{I} + \mathbf{H}_e \boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)\right)$$
$$+\mathrm{tr}\left\{\boldsymbol{\Omega}\boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa}\right\} - \mu\left(\mathrm{tr}\left\{\mathbf{P}\left(\boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} - \kappa \mathrm{P}\right).$$

The Karush-Kuhn Tucker conditions are

i) $\mu\left(\mathbf{I} + \sum_{i=1}^{N} |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i\right) - \boldsymbol{\Omega} = \mathbf{H}_e^H \left(\mathbf{I} + \mathbf{H}_e \boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)^{-1} \mathbf{H}_e$

ii) $\mu\left(\mathrm{tr}\left\{\mathbf{P}\left(\boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} - \kappa \mathrm{P}\right) = 0$

iii) $\mathrm{tr}\left\{\boldsymbol{\Omega}\boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa}\right\} = 0$

For $\boldsymbol{\Sigma}_{\mathrm{X}_s^\kappa} = \mathbf{V}\boldsymbol{\Psi}\mathbf{V}^H$ the KKT conditions (necessary and sufficient for optimality) hold.

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Linear Functions Design

The optimum linear relaying matrices must satisfy

$$\mathcal{D}^{*} = \lim_{\kappa\longrightarrow\infty} \arg \max_{\mathcal{D}\in\mathbb{C}_{\mathrm{SLT}}^{\kappa\times\kappa}} \mathcal{C}\left(\kappa,\mathcal{D}\right).$$

- Non-convex optimization: untractable exhaustive search

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- Suboptimum design: $\mathcal{D}_{so} \triangleq \left\{\mathbf{D}_i = \eta_i \mathbf{D}_o\left(\beta\right),\ i = 1, \cdots, N\right\}$, with

$$\eta_i \triangleq \frac{b_i^* \cdot c_i^*}{\sqrt{\sum_{i=1}^N \left|b_i \cdot c_i\right|^2}}$$

$$\left[\mathbf{D}_o\left(\beta\right)\right]_{p,q} \triangleq \begin{cases} \sqrt{\frac{\beta\kappa}{\kappa-1}\frac{\mathrm{P}}{N_o}} & p = q+1;\quad 1 \leq q \leq \kappa - 1 \\ 0 & \text{elsewhere.} \end{cases}$$

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$$\eta_i \triangleq \frac{b_i^* \cdot c_i^*}{\sqrt{\sum_{i=1}^N |b_i \cdot c_i|^2}}$$

$$\mathbf{D}_o\left(\beta\right) \triangleq \begin{pmatrix} 0 & \cdots & 0 \\ \sqrt{\frac{\beta\kappa}{\kappa-1}\frac{\mathrm{P}}{N_o}} & \ddots & 0 \\ 0 & \sqrt{\frac{\beta\kappa}{\kappa-1}\frac{\mathrm{P}}{N_o}} & 0 \end{pmatrix}$$

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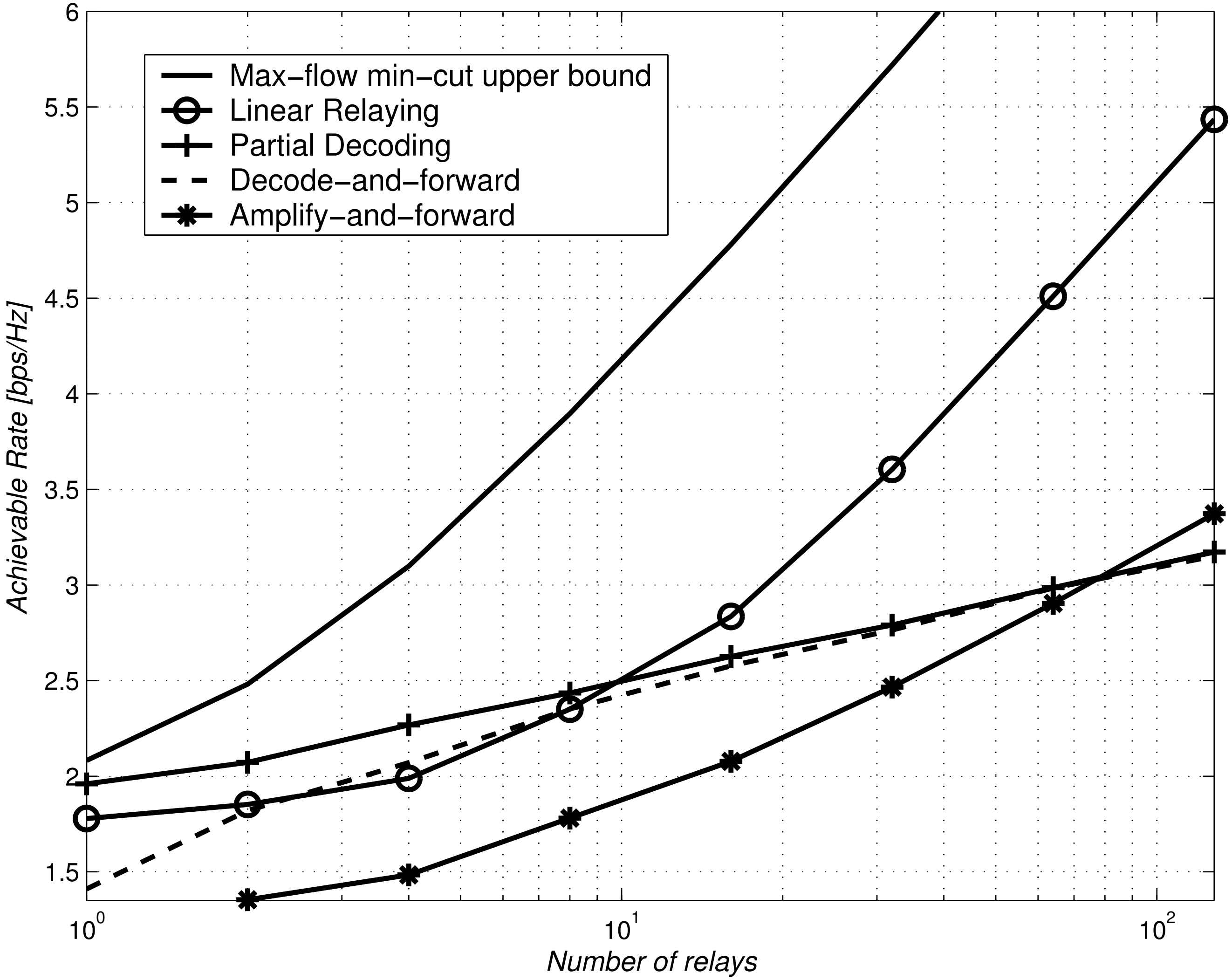
Numerical Results



Rayleigh-distributed, *time-invariant* channels, $\mathbb{E}\{\mathcal{C}\}$.

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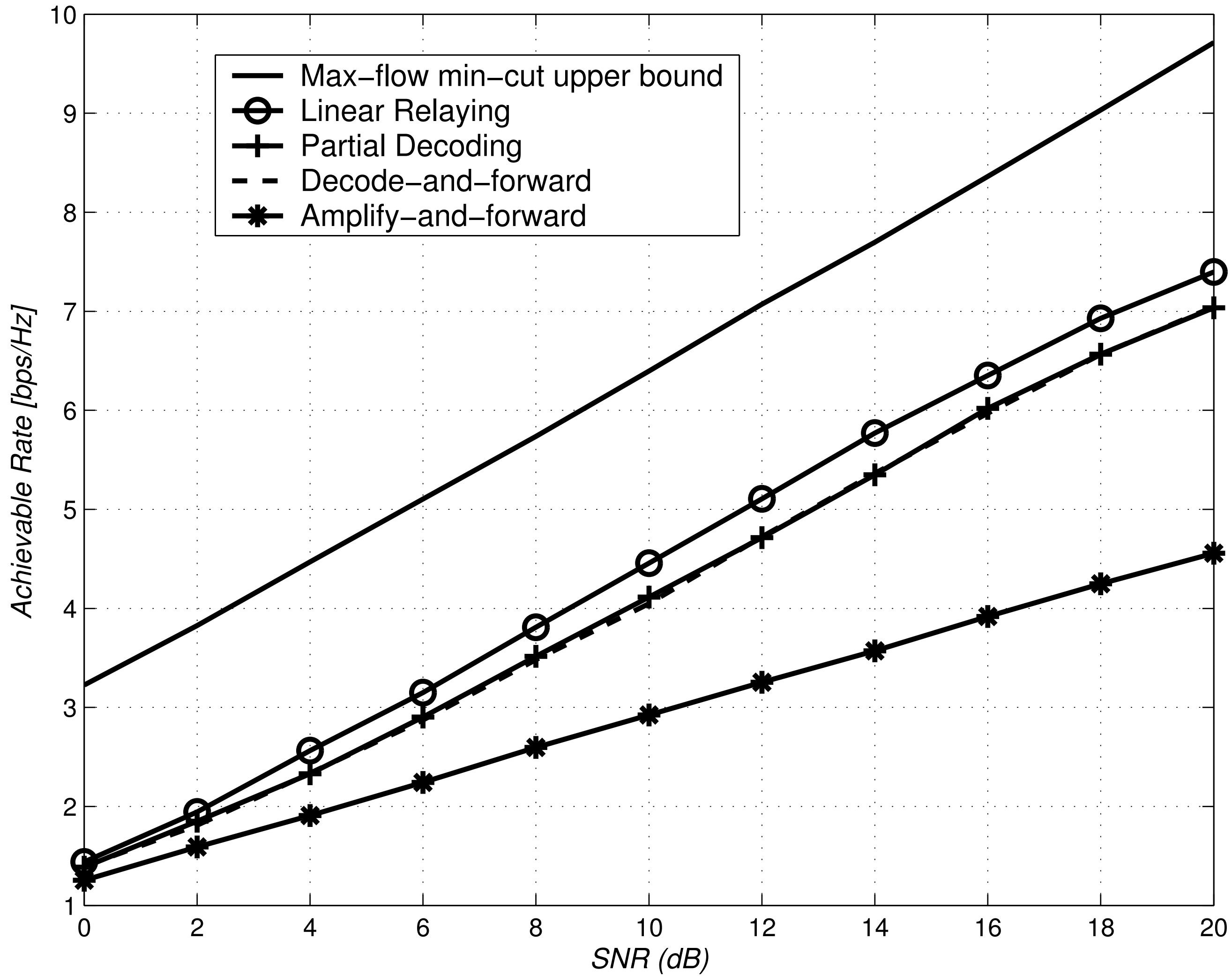

Rayleigh-distributed, *time-invariant* channels, $\mathbb{E}\left\{\mathcal{C}\right\}$.

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Conclusions

- Amplify-based relaying studied as AWGN channels with block memory and colored noise.

- *Linear relaying* is the most spectrally efficient technique for $N \to \infty$
  - Capacity doubles with respect to *amplify-and-forward*.

- Shortcoming: worse than *partial decoding* for low number of relays.

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