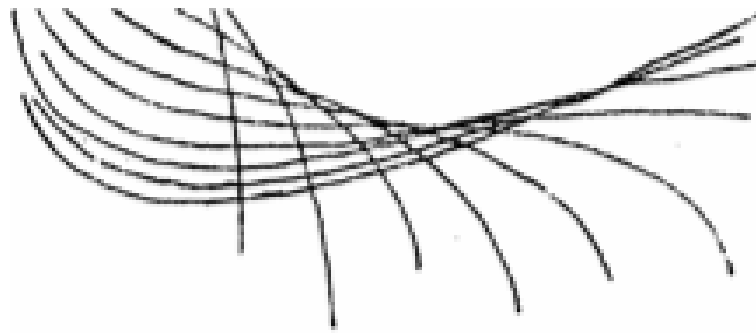
A photograph of St. Stephen's Cathedral in Vienna, Austria, featuring its prominent green dome and white facade. In the foreground, a MIMO antenna array is visible, consisting of several vertical poles with circular bases, arranged in a grid pattern on a paved surface. The scene is set against a blue sky with scattered clouds and green trees.

MIMO Systems: Methods



SPREAD SPECTRUM COMMUNICATIONS— MYTHS AND REALITIES

Andrew J. Viterbi

Coding is always beneficial and sometimes crucial for the suppression of interference in spread spectrum communications.

INTRODUCTION

Spread spectrum communication techniques date back to the early fifties. Since the earliest applications, system improvements have been more evolutionary than revolutionary. Like most improvements in electronic systems, these are due primarily to the availability of ever higher speed integrated circuit components, which translate in this case to wider spread spectra. In three decades the achievable spreading factor has grown by about three orders of magnitude¹ to the point that we are now limited more by bandwidth allocations than

PURPOSES

The purpose and applicability of spread spectrum techniques is threefold:

- Interference Suppression
- Energy Density Reduction
- Ranging or Time Delay Measurement

Foremost among these is the suppression of interference which may be characterized as any combination of the following:

- 1) Other Users: intentional (hostile or unintentional),
- 2) Multiple Access: spectrum sharing by "coordi-

Myths

QuickTime™ and a
decompressor
are needed to see this picture.

Max Weber 1943

Myth 1: In order to get any noticeable MIMO effect, the antennas may not be spaced much closer than half a wavelength.

This myth is a result of erroneous antenna matching.

No coupling

Matched coupling

Mis-matched coupling

QuickTime™ and a
decompressor
are needed to see this picture

Myth 2: In order to achieve a diversity gain you need to sacrifice on the multiplexing gain.

This myth is a result of excluding temporal diversity.

Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels

Lizhong Zheng, *Member, IEEE*, and David N. C. Tse, *Member, IEEE*

Abstract—Multiple antennas can be used for increasing the amount of diversity or the number of degrees of freedom in wireless communication systems. In this paper, we propose the point of view that both types of gains can be simultaneously obtained for a given multiple-antenna channel, but there is a fundamental tradeoff between how much of each any coding scheme can get. For the richly scattered Rayleigh-fading channel, we give a

work has concentrated on using multiple *transmit* antennas to get diversity (some examples are trellis-based space-time codes [6], [7] and orthogonal designs [8], [3]). However, the underlying idea is still averaging over multiple path gains (fading coefficients) to increase the reliability. In a system with m transmit and n receive antennas, assuming the path

We also assume that the channel matrix \mathbf{H} remains constant within a block of l symbols, i.e., the block length is much small than the channel coherence time. Under these assumptions, the

MULTIPLE antennas are an important means to improve the performance of wireless systems. It is widely understood that in a system with multiple transmit and receive antennas (multiple-input-multiple-output (MIMO) channel), the spectral efficiency is much higher than that of the conventional single-antenna channels. Recent research on multiple-antenna channels, including the study of channel capacity [1], [2] and the design of communication schemes [3]–[5], demonstrates a great improvement of performance.

Traditionally, multiple antennas have been used to increase *diversity* to combat channel fading. Each pair of transmit and receive antennas provides a signal path from the transmitter to

of freedom available for communication [2], [1]. Essentially, if the path gains between individual transmit–receive antenna pairs fade independently, the channel matrix is well conditioned with high probability, in which case multiple parallel *spatial channels* are created. By transmitting independent information streams in parallel through the spatial channels, the data rate can be increased. This effect is also called *spatial multiplexing* [5], and is particularly important in the high-SNR regime where the system is degree-of-freedom limited (as opposed to power limited). Foschini [2] has shown that in the high-SNR regime, the capacity of a channel with m transmit, n receive antennas, and i.i.d. Rayleigh-faded gains between each antenna pair is given by

*Myth 3: For high spectral efficiency,
one needs amplitude modulation.*

This myth
is a relict of the pre-MIMO era.

What is more expensive?

Alternative 1:

A **linear** increase of the number of antennas at the transmitter.

Alternative 2:

An **exponential** increase of the voltage the RF-chains have to cope with.

Myth 4: There is a (big) market for single user MIMO systems.

People (including researchers) like to believe what comforts them.

Myth 5: QPSK always tops BPSK.

This myth
is an invalid generalization from single-user
to multi-user communications.

*Myth 6:
Intersymbol interference is
something one should always
try to avoid.*

This myth
is valid on certain single-user SISO channels
with Gaussian input, but not in general.

On the Minimum Distance Problem for Faster-than-Nyquist Signaling

JAMES E. MAZO, MEMBER, IEEE, AND HENRY J. LANDAU, MEMBER, IEEE

Abstract—When pulses are sent through an ideal band-limiting channel at the Nyquist rate they do not interfere with one another, and so data can

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Gaussian noise. For signaling rates up to about 25 percent faster than the Nyquist rate, we show that the minimum distance does not drop below the value which it would have in the ideal case, when there is no intersymbol interference. Mathematically, the problem is to decide if the best L^2

sequences transmitted at a rate exceeding the Nyquist. This distance is the main parameter determining the error rate for recovery of the data in white Gaussian noise. For signaling rates up to about 25 percent faster than the Nyquist rate, we show that the minimum distance does not drop below the value which it would have in the ideal case, when there is no intersymbol interference. Mathematically, the problem is to decide if the best L^2 Fourier approximation to the constant 1 on the interval $(-\sigma\pi, \sigma\pi)$, $0 < \sigma \leq 1$, using the functions $\exp(inx)$, $n > 0$, with coefficients restricted to be ± 1 or 0, occurs when all coefficients are zero. We show this for $0.802 \dots \leq \sigma \leq 1$, and this is best possible.

I. DEDICATION

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the squared Euclidean distances,

$$\int_{-\infty}^{\infty} [W_i(t) - W_j(t)]^2 dt$$

between waveforms $W_i(t)$ and $W_j(t)$ corresponding to distinct data streams $\{a_k^{(i)}\}$ and $\{a_k^{(j)}\}$. In the ideal situation when the $\{s(t - \sigma k)\}$ are mutually orthogonal,

$$d_{\min}^2 = 4 \int_{-\infty}^{\infty} s^2(t) dt = 4E$$

achieved when exactly one data bit changes sign. Here and henceforth, the energy in the pulse $s(t)$ is denoted by $E \equiv \int_{-\infty}^{\infty} s^2(t) dt$.

*Myth 7:
Excess bandwidth like roll-off
cannot be utilized.*

This myth
is also an invalid generalization from single-
user to multi-user communications.

On the Capacity of Asynchronous CDMA Systems

Laura Cottatellucci
Institut Eurecom
Sophia Antipolis, France
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Ralf R. Müller
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7491 Trondheim Norway
Email: mueller@iet.ntnu.no

Mérouane Debbah
Supelec
Paris, France
Email: merouane.debbah@supelec.fr

Abstract— chip pulse access (CDMA) frequency-domain analysis in this paper is extended to asynchronous systems. The error (MMSE) is described by as the spectral per chip and in terms of relation between

CDMA systems with modulation based on sinc pulse waveforms and of synchronous CDMA systems is derived.

I. INTRODUCTION

The fundamental limits of synchronous CDMA systems have been thoroughly studied by modelling the spreading sequences by random sequences in [1], [2], [3]. However, this analysis is focused on synchronous CDMA systems while the assumption of synchronism is not realistic for the uplink of a CDMA system. Therefore, it is of theoretical and practical interest to extend the analysis of CDMA systems with random spreading to the asynchronous case.

The analysis of asynchronous CDMA systems limited to

Asynchronous multiuser systems have larger (or equal) capacity.

The larger the roll-off the larger the gap.

due to the use of asynchronous CDMA interference free asynchronous systems the analysis

system performance expressions of variance matrix

$\mathbf{H} \mathbf{H}^H$ (here \mathbf{H} is the transfer matrix of the system) while to derive the total capacity per chip of a large CDMA system or its SINR at the output of a linear MMSE detector the knowledge of the limit eigenvalue distribution is required (see [1], [2], [3]). Thus, the results in [7], [8] do not enable a large system analysis of the effects of asynchronism on these two relevant performance measures. In this work we investigate the fundamental limits of asynchronous CDMA systems in terms of both total capacity per chip constrained to a chip pulse waveform and the limit SINR of linear MMSE detectors.

Due to space restriction in this work we consider (i) CDMA systems with modulation based on chip pulse waveforms with bandwidth not greater than the Nyquist rate and constraint of

*Myth 8:
Power Control is a Good Idea to Deal
with Near-Far Effects.*

This myth
is the result of commercial interests.

On near-far gain in multiuser diversity systems

Kimmo Kansanen and Ralf R. Müller
Norwegian University of Science and Technology
NO-7491 Trondheim, Norway
email: kimmo.kansanen@iet.ntnu.no

Abstract—Multiuser diversity scheduling in a single-cell system is studied. The scheduler chooses a constant size subset of users with best short term fading gains and coordinates transmissions using superposition coding. Rate allocation is performed according to users' channel states assuming equal power allocation over users. Asymptotic analysis of system capacity is applied to study system behavior at low and high spectral efficiencies. It is found that if users have non-symmetric channel distributions, i.e. path loss is present, with large enough user population it becomes beneficial in the sum rate sense to schedule more than one user at a time.

I. INTRODUCTION

We consider the uplink of a wireless communications system with K users. Each user k experiences a propagation channel gain d_k that is the product of two random variables: the short term fading f_k and the path loss s_k . Short term fading is assumed fixed over the signaling interval but i.i.d. across users and signaling intervals. Path loss is assumed i.i.d. across users and fixed in time for all users. Throughput optimal scheduling [1] in a system with path loss results often in a severely unfair result: only the user with the best channel quality, usually very close to the access point, is scheduled. As a result, alternative scheduling methods providing fairness of scheduling decisions have been proposed. One of these is

ishingly small and asymptotically high spectral efficiencies, respectively. Numerical examples are presented in Section V.

The elements of a vector or a sequence \vec{x} are x_i . We denote by $x_{i:j}$ the i th smallest variable of j variables and the distribution of a random variable y by F_y . Expectation with respect to a random variable x is denoted by $E_x[\cdot]$ and with respect to a sequence of variables by $E_{\vec{x}}[\cdot]$. The base of logarithm is explicitly marked except for natural logarithms.

II. GROUP SCHEDULING

PFS allows the user with the relatively best channel to transmit without considering the resulting rate provided. In effect, the fairness is restricted to the scheduling decisions, and does not consider fairness in transmitted rate. Generalizing this approach to allowing K_A users with best short term fading channel realizations transmitting simultaneously, we come to the following formulation.

Let $\text{SNR} = 1/N_0$. At any given time t , the system capacity is expressed as a function of the channels of the set of scheduled users \mathcal{A} as

$$C(t) = \log \left(1 + \text{SNR} \frac{\sum_{k \in \mathcal{A}} d_k}{K_A} \right). \quad (1)$$

If we now define $u = \sum_{k \in \mathcal{A}} d_k / K_A$, we can express the

*Myth 9:
MIMO requires particular codes.*

This myth is a historical peculiarity.

LDPC Coded MIMO Multiple Access With Iterative Joint Decoding

Amichai Sanderovich, *Student Member, IEEE*, Michael Peleg, *Senior Member, IEEE*, and Shlomo Shamai (Shiz), *Fellow, IEEE*

Abstract—An efficient scheme for the multiple-access multiple-input multiple-output (MIMO) channel is proposed, which operates well also in the single user regime, as well as in a direct-sequence spread-spectrum (DS-CDMA) setting. The design features scalability and is of limited complexity. The system employs optimized low-density parity-check (LDPC) codes and an efficient iterative (belief propagation—BP) detection which combines linear minimum mean-square error (LMMSE) detection and iterative interference cancellation (IC). This combination is found to be necessary for efficient operation in high system loads $\alpha > 1$. An asymptotic density evolution (DE) is used to optimize the degree polynomials of the underlining LDPC code, and thresholds as close as 0.77 dB to the channel capacity are evident for a system load of 2. Replacing the LMMSE with the complex individually optimal multiuser detector (IO-MUD) further improves the performance up to 0.14 dB from the capacity. Comparing the thresholds of a good single-user LDPC code to the multiuser optimized LDPC code, both over the above multiuser channel, reveals a surprising 8-dB difference, emphasizing thus the necessity of optimizing the code. The asymptotic analysis of the proposed scheme is verified by simulations of finite systems, which reveal meaningful differences between the performances of MIMO systems with single and multiple users and demonstrate performance similar to previously reported techniques, but with higher system loads, and significantly lower receiver complexity.

(CDMA) is closely related to MIMO as a single mathematical model models both, with the MIMO propagation coefficients corresponding to the symbols of the CDMA spreading sequences [3]. This supports the use of the well-studied CDMA-based multiple-access techniques in MIMO channels (with either single or multiple access). Although [1] relates to ergodic MIMO channels, which are not block fading, if one assumes an asymptotically large system, then each block considered is an ergodic and stationary realization of the same process. Thus, the equality of the multiuser and single-user capacities is true only as long as an asymptotic number of antennas is considered, while for a finite number of antennas, similar conclusions require further study of the outage capacity, which is beyond the scope of this paper.

The asymptotic capacity of the CDMA channel is explicitly calculated in [4] for unconstrained signaling, where the number of users and chips is taken to infinity, while their ratio remains fixed. The *Replica method* is used by Tanaka in [5] to extend this result to a binary phase-shift keying (BPSK) signaling and to a quadrature phase-shift keying (QPSK) signaling in [6]. Since the asymptotic analysis assumes randomly generated

*Myth 10:
Constant envelop modulation can't
be equalized at high data rates.*

This myth is to be overthrown
if MIMO shall become the key
to the wireless revolution.

Realities

The background is an abstract composition. It features several horizontal bands of color: a dark red band at the top, a bright yellow band in the middle, and a dark blue band at the bottom. A grid of thin white lines is overlaid on the image. Three circles are also present: a white circle in the lower-left quadrant, a blue circle in the middle-right area, and a yellow circle to its right. The overall style is reminiscent of a technical drawing or a scientific diagram.

Antenna Spacing

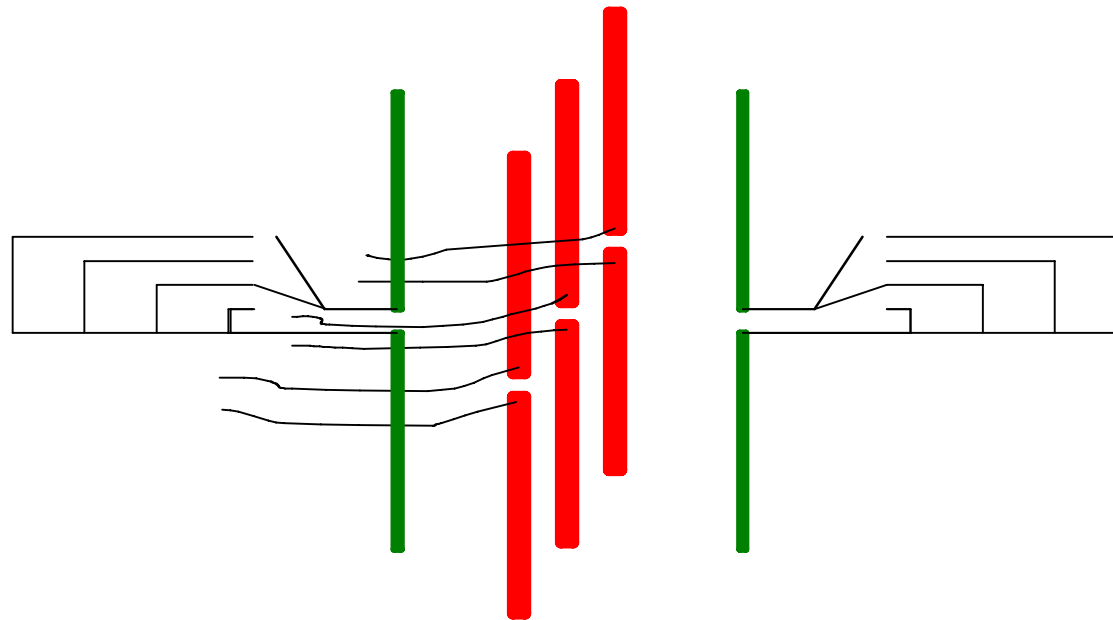
It is textbook knowledge in antenna theory that Bouwkamp and de Bruijn showed as early as 1946 that there is no theoretical limit to the directivity of any given aperture size.

Diversity vs. Multiplexing

- Multiplexing gains are expensive.
- Diversity almost comes for free.

Why should one sacrifice
multiplexing for diversity's sake?

Diversity on Discount



Parasitic antennas switch on kHz speeds.

Resource Pooling

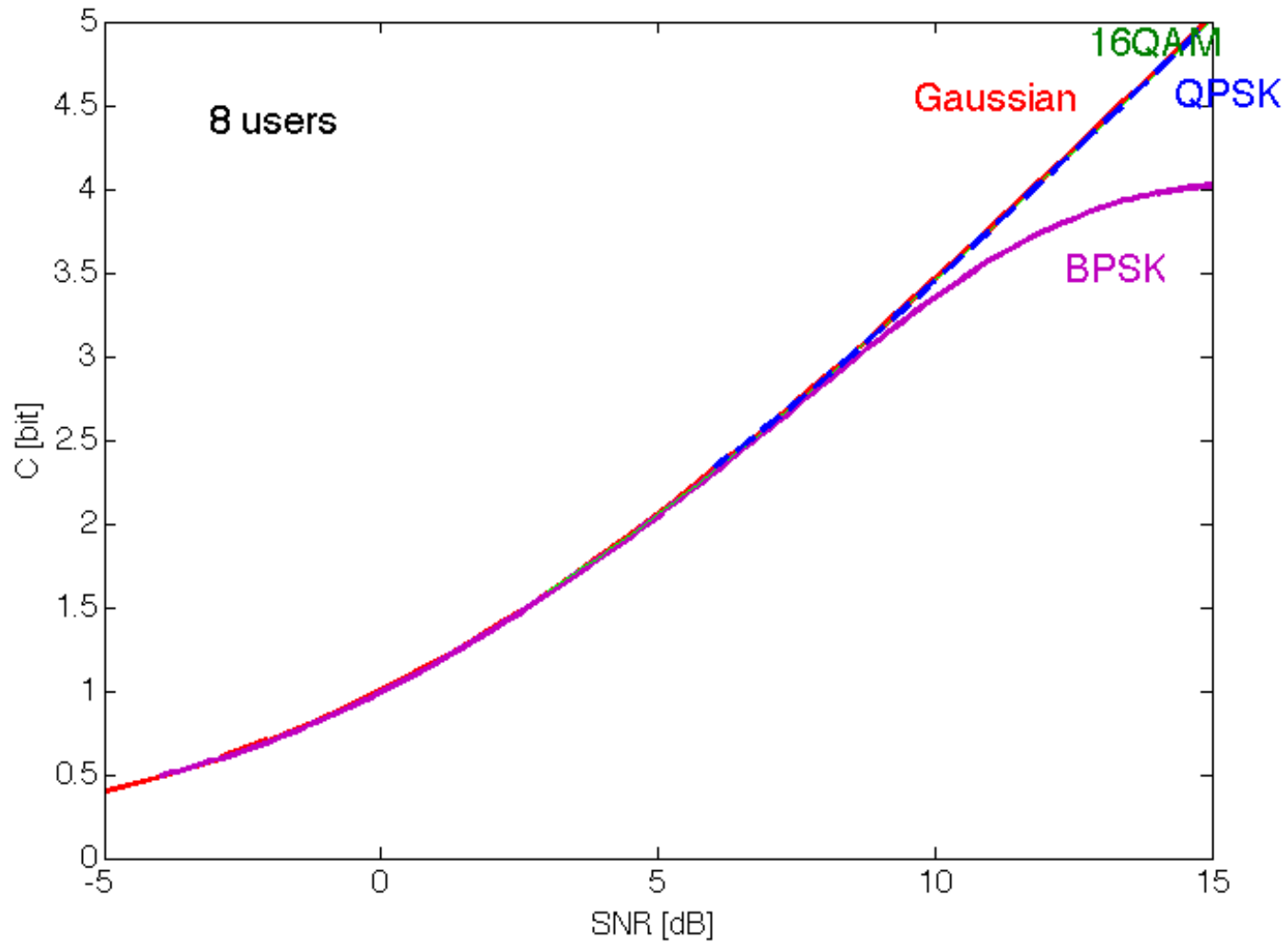
The antennas of all users are pooled together.

There is no need to distinguish between users and antennas.

The Resource Pooling View

MIMO is CDMA where the processing gain is provided by antennas.

Amplitude Modulation ?



Space-Time Coding ?

There are no multiuser space-time codes.

*For the downlink,
we need efficient dirty-paper codes.*

A Fatal Attraction

QuickTime™ and a
decompressor
are needed to see this picture.

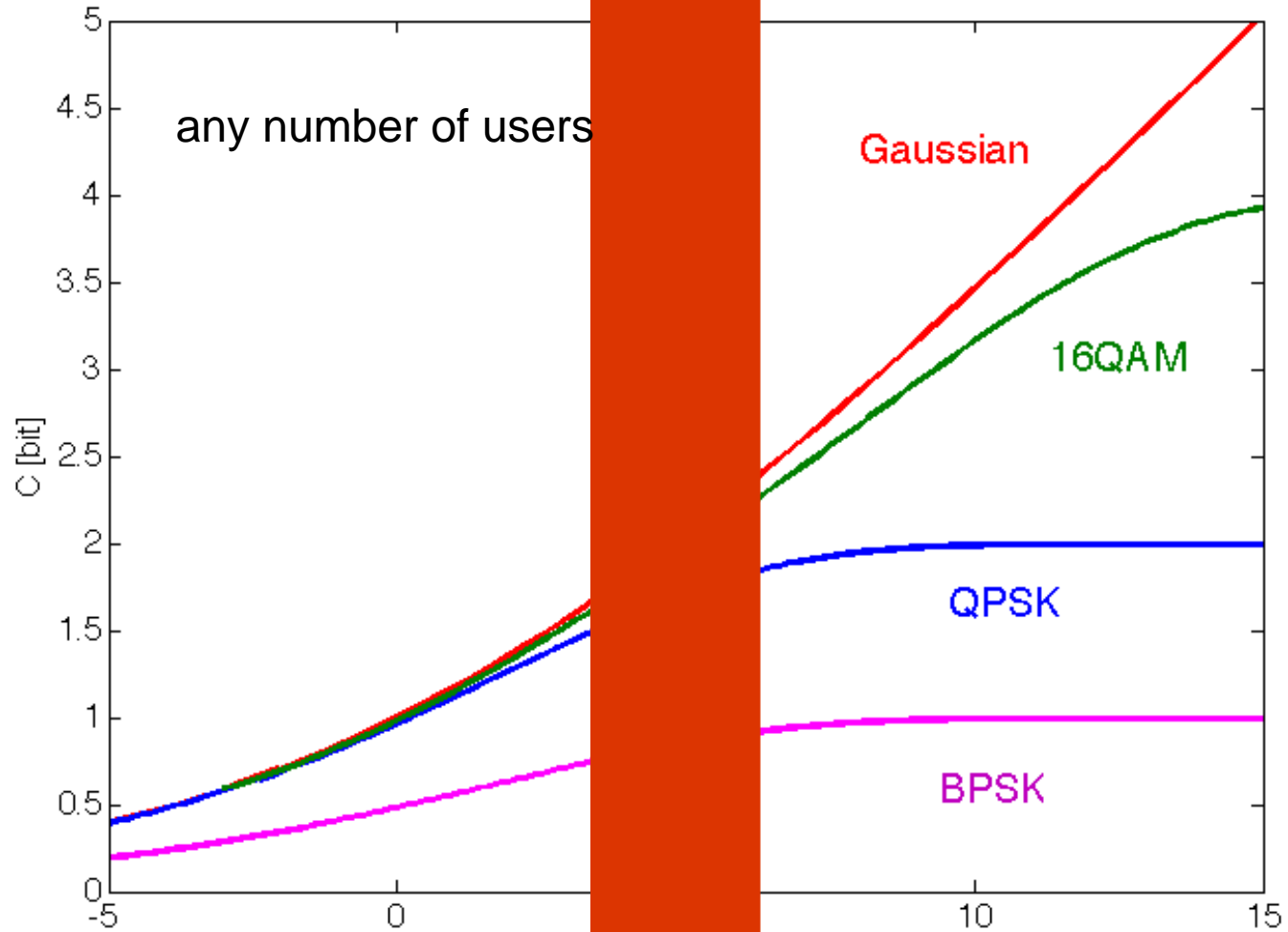


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Orthogonality lures by its beauty, but it
blinds your reason.

Capacity is achieved by **random** coding.

Amplitude Modulation



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What's wrong with **Orthogonality**?

capacity = output **entropy** – constant

entropy measures **disorder**

*Orthogonal*FDM ?

OFDM eases equalization.

OFDM prohibits constant envelope modulation like GMSK.

OFDM vs. CPM

- Simple equalization
- Few antennas
- Linear amplifiers
- Iterative equalization
- Many antennas
- Cheap amplifiers

MIMO

... is just an add on

... is the enabling tool.

My Personal Vision of MIMO

Dozens (hundreds) of closely spaced antennas

CPM modulation

High diversity order due to parasitic elements

Iterative processing of ISI, MUI, and FEC by belief propagation.

