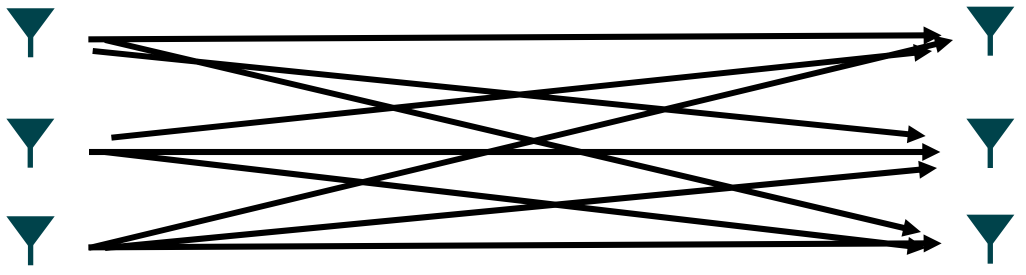
Rethinking MIMO for Wireless Networks: Linear Throughput Increases with Multiple Receive Antennas

Nihar Jindal
ECE Department, University of Minnesota

Joint work with Jeff Andrews & Steven Weber

MIMO in Point-to-Point Channels



$$\vec{y} = \boldsymbol{H}\vec{x} + \vec{z}$$

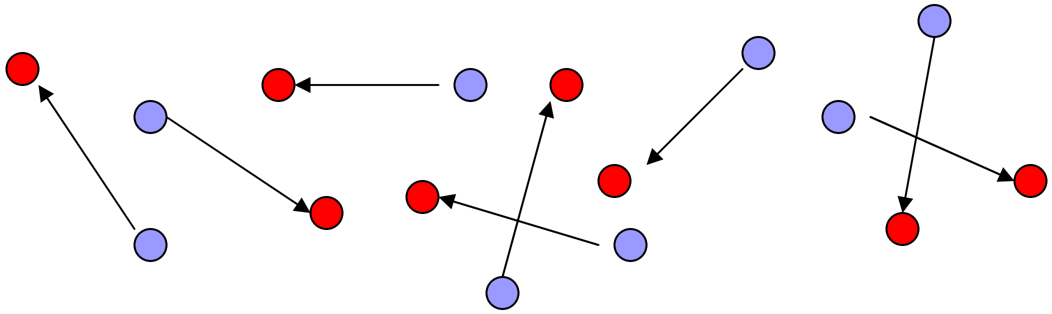
- Foschini-Gans '98: Capacity of a point-to-point MIMO channel scales linearly with min(N_t,N_r)

$$C \approx \min(N_t, N_r)\log_2 SNR$$

  - Transmit min(N_t,N_r) data streams, each at rate ~ $\log_2$(SNR)
  - No increase in power or bandwidth

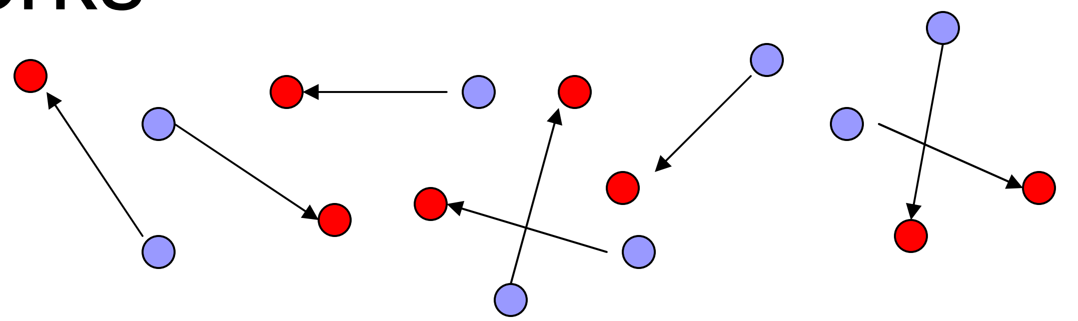
- MIMO an integral part of every contemporary high-rate wireless systems (LTE, 802.11n, WiMax)

MIMO in Ad-Hoc Networks



- Many TX-RX pairs simultaneously communicating, mutually interfering
- If each node has N antennas, use point-to-point MIMO techniques to increase per-link rate linearly with N
  - Interference power does not increase with N
  - Spatial color of interference increases rates
- Linear increase in system throughput: transport capacity & area spectral efficiency (ASE)
- Is this the only way?

Multiple RX Antennas in Ad-Hoc Networks



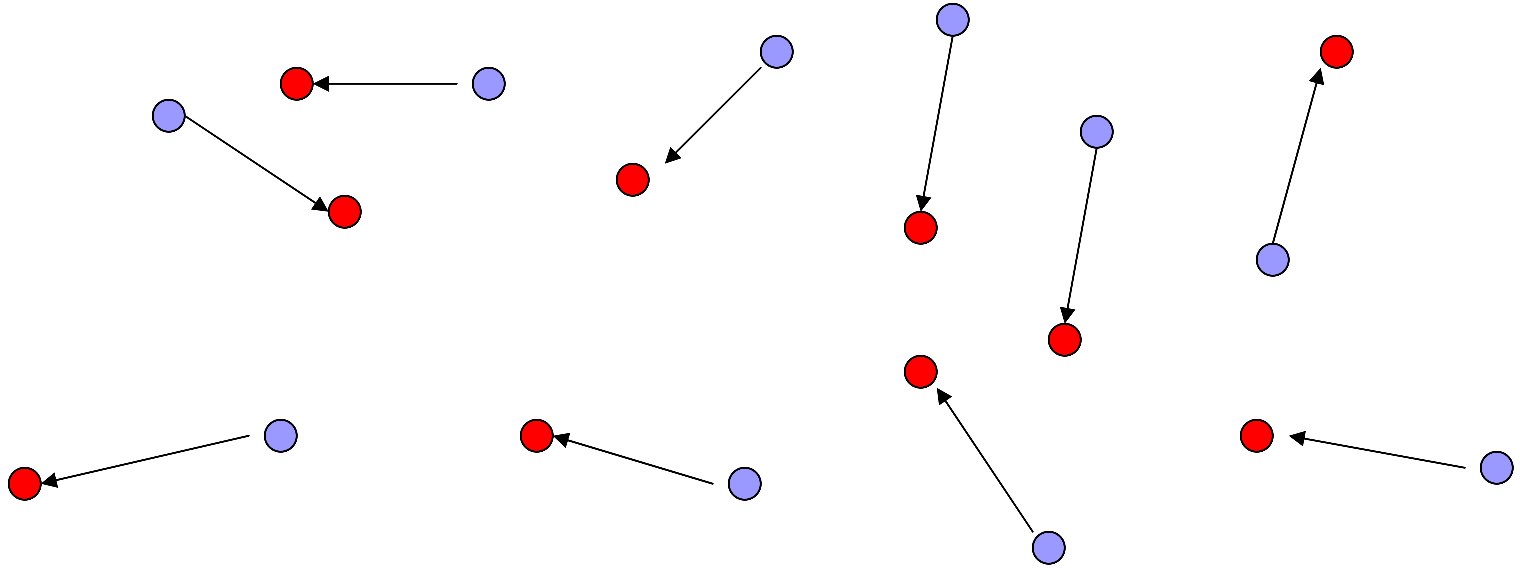
- Each TX has 1 antenna, each RX has N antennas
- Main Result: System throughput scales linearly with N

- Use RX antennas to simultaneously:
  - Cancel nearby interferers
  - Increase received signal power (array gain)
- Maintain a constant per-link rate (SINR) & increase spatial density of simultaneous transmissions linearly with N

- Linear increase in density, constant per-link rate -> linear increase in system throughput

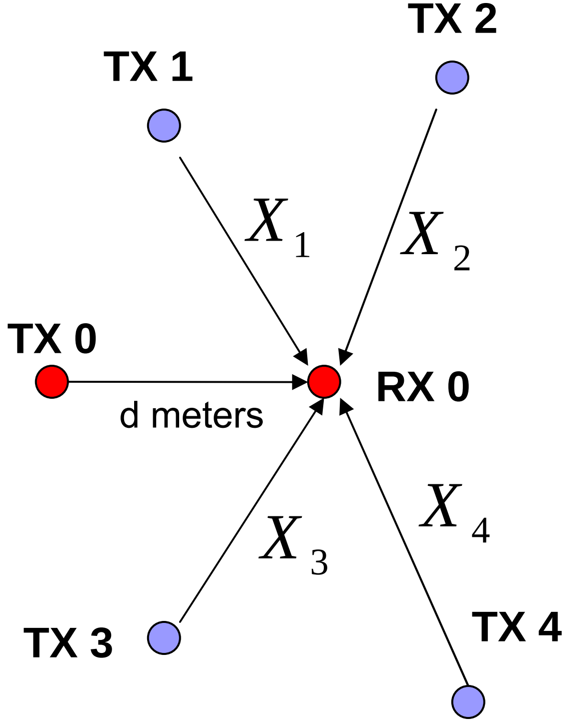
Network Model

- Decentralized network with randomly located TX-RX pairs (single-hop)
- TX's randomly located according to 2-D Poisson process (density λ)
- Each TX associated with a RX a distance d meters away
- Models network using ALOHA



Network Model (2)

- Consider reference pair TX0-RX0
  - Interferers still form a 2-D Poisson process (density λ)
- TX-RX distance: d meters
- α : path-loss exponent:
- X_i:distance to i-th nearest interferer
- h_i :vector channel to i-th interferer
  - iid complex Gaussian components



- Received signal (N-dim vector):

$$\vec{y} = \vec{h}_0 d^{-\alpha} u_0 + \sum_{i=1} \vec{h}_i X_i^{-\alpha} u_i$$

- SIR after RX filter v

$$SIR = \frac{\left|\vec{v}^H \vec{h}_0\right|^2 d^{-\alpha}}{\sum_{i=1} \left|\vec{v}^H \vec{h}_i\right|^2 X_i^{-\alpha}}$$

Performance Metrics

- Outage probability relative to SINR threshold β:

$$P_{out}(\lambda, N, \beta) = P[SIR \leq \beta]$$

  - Increasing in λ, decreasing in N (good RX filter)
  - Corresponding spectral efficiency: $\log_2(1+\beta)$ bps/Hz

- Transmission Capacity [Weber, Andrews, et. al '05]: maximum density λ such that outage no larger than ε

$$\lambda_\varepsilon(N, \beta) = P_{out}^{-1}(\varepsilon)$$

- Objective: Show λ_ε (N,β) increases linearly with N

RX Filter Design

$$SIR = \frac{\left|\vec{v}^H \vec{h}_0\right|^2 d^{-\alpha}}{\sum_{i=1} \left|\vec{v}^H \vec{h}_i\right|^2 X_i^{-\alpha}}$$

- Maximum-ratio combining (MRC): [Hunter-Andrews-Weber] max signal power by choosing filter in direction of desired channel

$$\vec{v} = \vec{h}_0 \left(1 / \left\|\vec{h}_0\right\|\right)$$

- Full Zero-Forcing (ZF): [Huang-Andrews-Heath-Guo-Berry] minimize int. power by choosing filter orthogonal to channels of nearest N-1 interferers

$$\vec{v} \perp \vec{h}_1, \ldots, \vec{h}_{N-1}$$

- MMSE: maximizes SIR

$$\vec{v} = \left(\sum_{i=1} X_i^{-\alpha} \vec{h}_i \vec{h}_i^H \right)^{-1} \vec{h}_0$$

Prior Work

- [Hunter-Andrews-Weber '07]: Maximum ratio combining allows density to increase with N as:

$$\lambda = O\left(N^{2/\alpha}\right)$$

- [Huang-Andrews-Heath-Guo-Berry '08]: Full zero-forcing allows density to increase with N as

$$\lambda = O\left(N^{1-2/\alpha}\right)$$

- [Govindasamy-Bliss-Staelin '07]: For fixed density, with MMSE expected SINR scales with N as

$$E[SINR] = O\left(N^{\alpha/2}\right)$$

Partial Zero Forcing

- Choose filter orthogonal to k (≤ N-1) nearest interferers

$$\vec{v} \perp \vec{h}_1, \ldots, \vec{h}_k$$

$$\Rightarrow \vec{v} = \text{Projection of } \vec{h}_0 \text{ on Nullspace } \left(\vec{h}_1, \ldots, \vec{h}_k\right)$$

- Signal term

$$\left|\vec{v}^H \vec{h}_0\right|^2 \sim \chi^2_{2(N-k)}$$

Interference terms

$$\left|\vec{v}^H \vec{h}_i\right|^2 = 0 \quad i = 1, \ldots, k$$

$$\left|\vec{v}^H \vec{h}_i\right|^2 \sim \chi^2_2 \quad i = k+1, \ldots$$

$$SIR \simeq \frac{\chi^2_{2(N-k)} d^{-\alpha}}{\sum_{i=k+1}^{\infty} \chi^2_2 X_i^{-\alpha}}$$

Signal & Interference Power

$$SIR \sim \frac{\chi^2_{2(N-k)} d^{-\alpha}}{\sum_{i=k+1} \chi^2_2 X_i^{-\alpha}}$$

- Expected signal power:

$$E[\chi^2_{2(N-k)}] = N - k$$

- Expected interference power:
  - Ordered squared-distances ~ 1-D PPP(πλ)
  - Distance-squared to i-th nearest interferer ~ $(\pi\lambda)^{-1}\chi^2_{2i}$

$$E\left[\sum_{i=k+1} X_i^{-\alpha}\right] < (\pi\lambda)^{\alpha/2}\left(\frac{\alpha}{2}-1\right)^{-1}\left(k-\left\lceil\frac{\alpha}{2}\right\rceil\right)^{1-\alpha/2}$$

Outage Probability Upper Bound

- For fixed density and N:

$$P[SIR \le \beta] \le \frac{\beta\kappa(\alpha)\lambda^{\alpha/2}\left(k - \lceil \alpha/2 \rceil\right)^{1-\alpha/2}}{(N-k+1)}$$

- Proof: Use Markov inequality to upper bound probability interference power is too large

$$\begin{aligned}
P\left[I \ge S/\beta\right] &= \int_0^{\infty} P\left[I \ge x/\beta\right] f_S(x)\,dx \\
&\le \int_0^{\infty} \frac{E[I]}{x/\beta} f_S(x)\,dx \\
&\le \int_0^{\infty} \frac{\beta c(\alpha)\lambda^{\alpha/2}}{x}\left(k - \lceil \alpha/2 \rceil\right)^{1-\alpha/2} f_S(x)\,dx \\
&= \beta c(\alpha)\lambda^{\alpha/2}\left(k - \lceil \alpha/2 \rceil\right)^{1-\alpha/2} E\left[1/S\right] \\
&= \beta c(\alpha)\lambda^{\alpha/2}\left(k - \lceil \alpha/2 \rceil\right)^{1-\alpha/2}\left(N-k+1\right)^{-1}
\end{aligned}$$

Density Lower Bound

- Invert outage probability upper bound to get max-density lower bound:

$$\lambda_{\varepsilon} \geq \frac{\varepsilon^{2/\alpha}(\alpha/2-1)^{2/\alpha}}{\pi\beta^{2/\alpha}} \underbrace{(N-k+1)^{2/\alpha}}_{\text{Array Gain}} \underbrace{(k-\lceil \alpha/2 \rceil)^{1-2/\alpha}}_{\text{Int. Cancellation}}$$

- Need to choose k such that bound is linear in N

Achieving Linear Density Scaling

- Use a <u>strict fraction</u> of RX degrees of freedom to cancel interference:

$$k = \theta N \qquad \text{for any } 0 < \theta < 1$$

- Max-density lower bound:

$$\lambda_\varepsilon \geq \frac{\varepsilon^{2/\alpha}(\alpha/2-1)^{2/\alpha}}{\pi\beta^{2/\alpha}}(1-\theta)^{2/\alpha}\theta^{1-2/\alpha}$$
$$\times\ (N + (1-\theta)^{-1})^{2/\alpha}\left(N - \theta^{-1}\lceil \alpha/2 \rceil\right)^{1-2/\alpha}$$

- Lower bound scales linearly with N

Intuition

- Expected signal power $$d^{-\alpha}(N-k)$$
- Expected int power (cancel k) $$\sim\left(\frac{\pi^{\alpha/2}}{\alpha/2-1}\right)\lambda^{\alpha/2}k^{1-\alpha/2}$$
- Fixed outage requires constant ratio between signal power & int power
  - MRC (k=0) [H-A-W

    $$\text{Signal} \sim N,\ \text{Int} \sim \lambda^{\alpha/2} \quad \Rightarrow \quad \lambda = O\left(N^{2/\alpha}\right)$$

  - Full ZF (k=N-1) [H-A-H-G-B]

    $$\text{Signal} \sim 1,\ \text{Int} \sim \lambda^{\alpha/2} N^{1-\alpha/2} \quad \Rightarrow \quad \lambda = O\left(N^{1-2/\alpha}\right)$$

  - Partial ZF (k=θN for θ<1)

    $$\text{Signal} \sim N(1-\theta),\ \text{Int} \sim \lambda^{\alpha/2}\left(\theta N\right)^{1-\alpha/2} \quad \Rightarrow \quad \lambda = O\left(N\right)$$

Optimization of Partial ZF

- Density lower bound converges to:

$$N \frac{\varepsilon^{2/\alpha} \left(\alpha / 2 - 1\right)^{2/\alpha}}{\pi\beta^{2/\alpha}} \left(1 - \theta\right)^{2/\alpha} \theta^{1-2/\alpha}$$

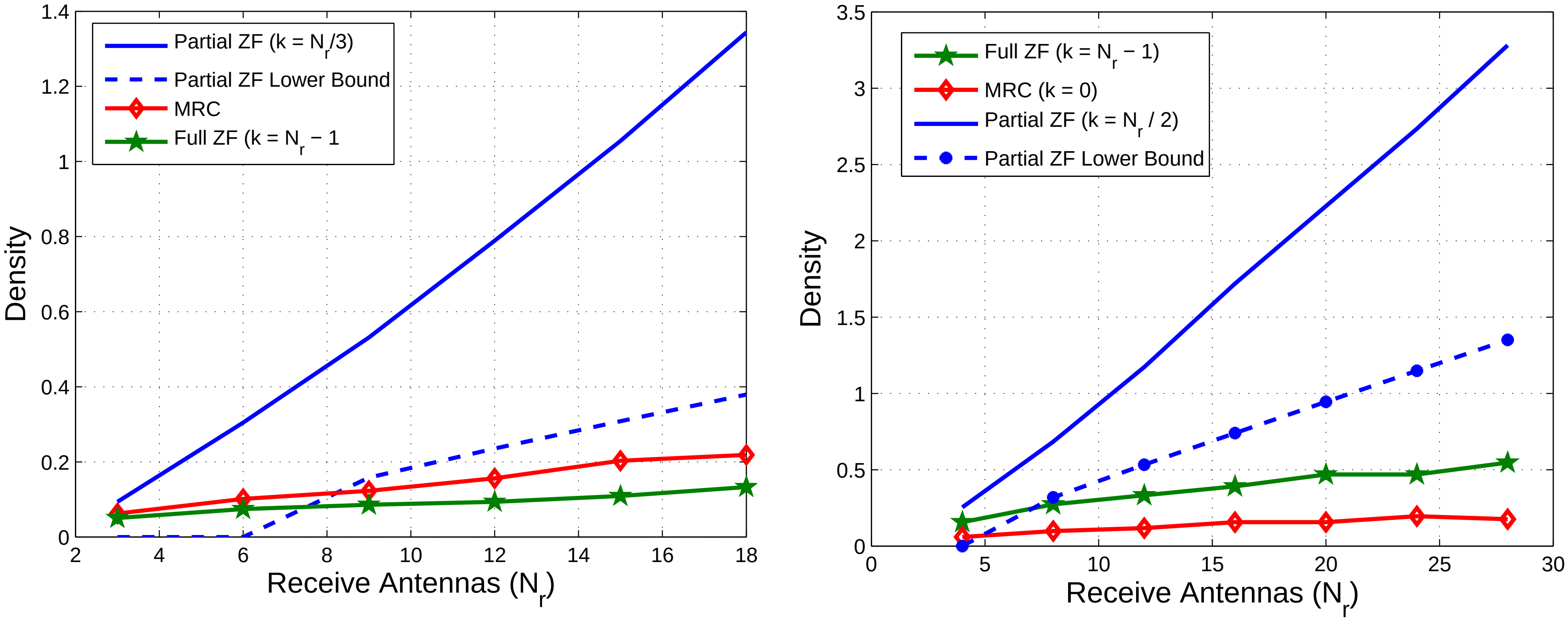
- Maximize with respect to θ:

$$\boxed{\theta^* = 1 - \frac{2}{\alpha}}$$

  - Small PL exponent: far interferers powerful, so no point in canceling nearby ones
  - Large PL exponent: nearby interferers much more powerful, so get benefit by canceling

Numerical Results

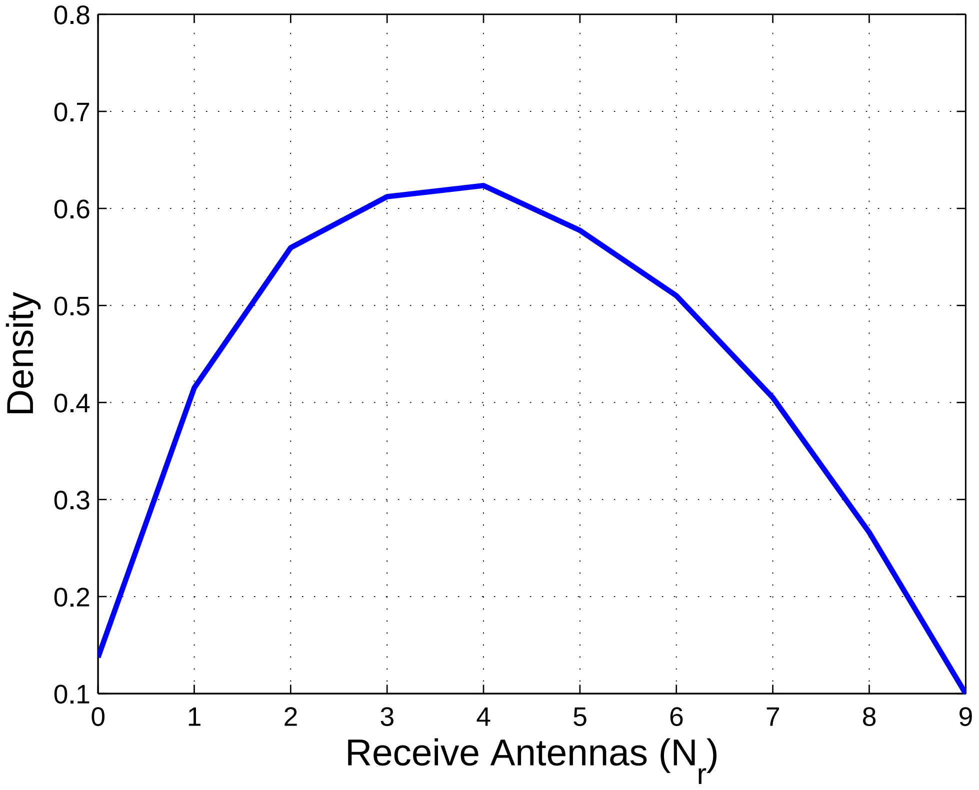
- Maximum density vs. N (ε=0.1)



- β=1,d=1 m, α=3 (left), α=4 (right)
- Very large density gains even for small N

Numerical Results (2)

- Maximum density vs. # cancelled, N=10, α=3, ε=0.1



- Consistent with $\theta^*=1/3$

Rate vs. Density Increase

- [Govindasamy-Bliss-Staelin] considered almost identical model for fixed density, optimal MMSE receiver
  - Expected SINR scales superlinearly: $N^{\alpha/2}$
  - Expected spectral efficiency scales logarithmically: $(\alpha/2) \log(N)$

- If density kept constant and RX antennas used to increase per-link rate: throughput ~ log(N)
- Increasing density sub-linearly (with N) leads to sub-linear throughput scaling

- Implication of increasing density:
  - In ALOHA network, can increase contention probability with N
  - In general, can be more aggressive with simultaneous transmissions

Conclusion

- RX cancellation allows transmitter density to grow linearly with # of RX antennas, without requiring multiple TX antennas
  - Constant per-link rate, linearly increasing density
- Results suggest MIMO can be very powerful in ad-hoc networks
- Spatial interference cancellation changes perspective on multi-user interference: is careful transmission scheduling needed?
- Extensions:
  - Evaluate benefit in multi-hop setting
  - MMSE receiver
  - Imperfect CSI