Generalized Secrecy Capacity

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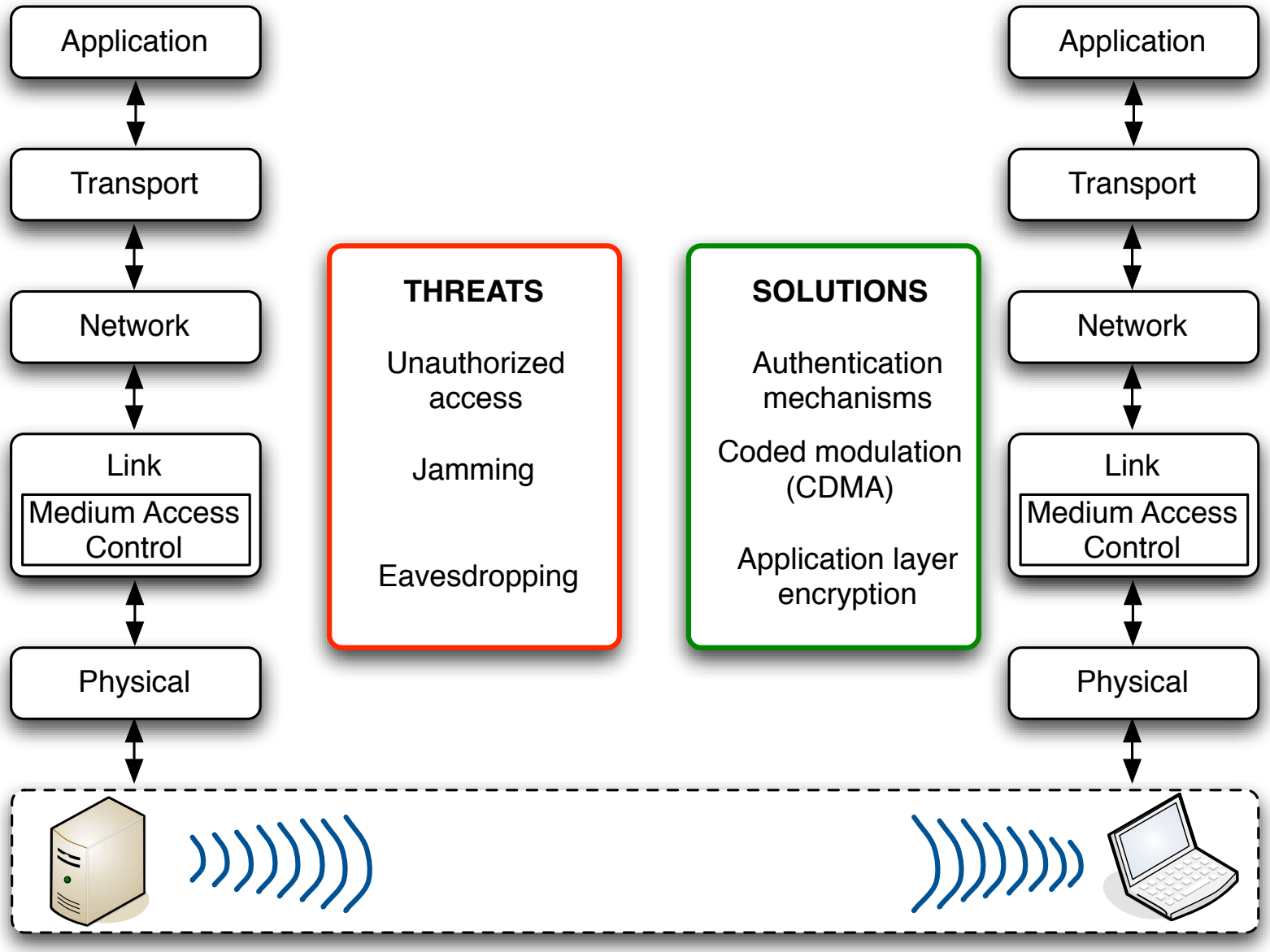
joint work with J. Nicholas Laneman

Agenda

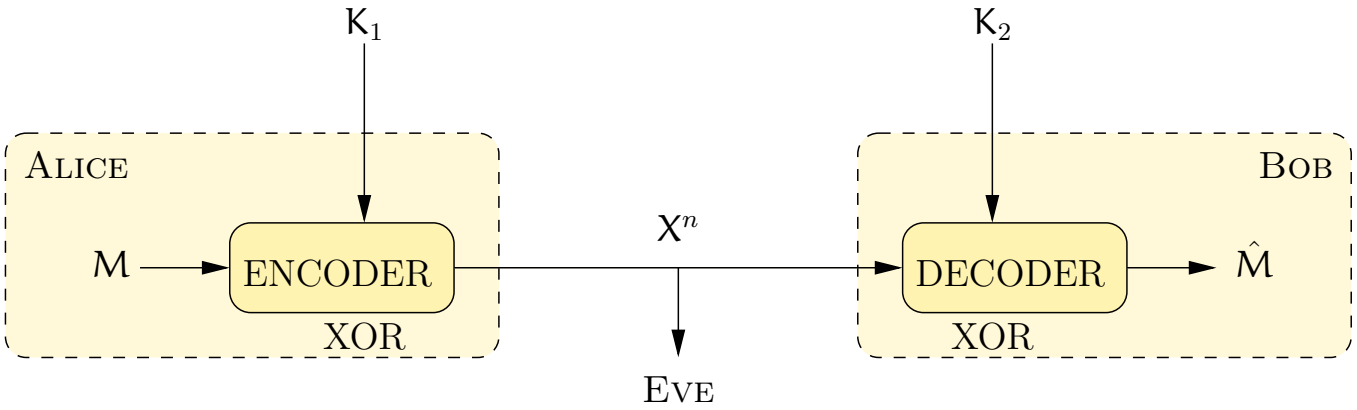
1. Physical-Layer Security
   - Computational security vs. information-Theoretic Security
   - Wiretap channel model
2. Secrecy Capacity of Arbitrary Wiretap Channels
   - Verdú-Han Information-Spectrum Approach
   - Security Criteria
   - Security and Resolvability
3. Applications
   - Discrete Memoryless Channels
   - Mixed Channels
   - Wiretap Channels with Channel State Information
4. Conclusion and Perspectives

Physical-Layer Security

Motivating example



Notions of security



Computational security

- security based on mathematical conjectures
- assumptions on attacker's computational power
- effective in most situations

Perfect security

- strictest notion of security: $p(\mathrm{m}|\mathrm{x}^n) = p(\mathrm{m})$ $(\mathbb{I}(\mathrm{M};\mathrm{X}^n) = 0)$
- requires one-time pad and secret key such that $\mathbb{H}(\mathrm{K}) \geq \mathbb{H}(\mathrm{M})$

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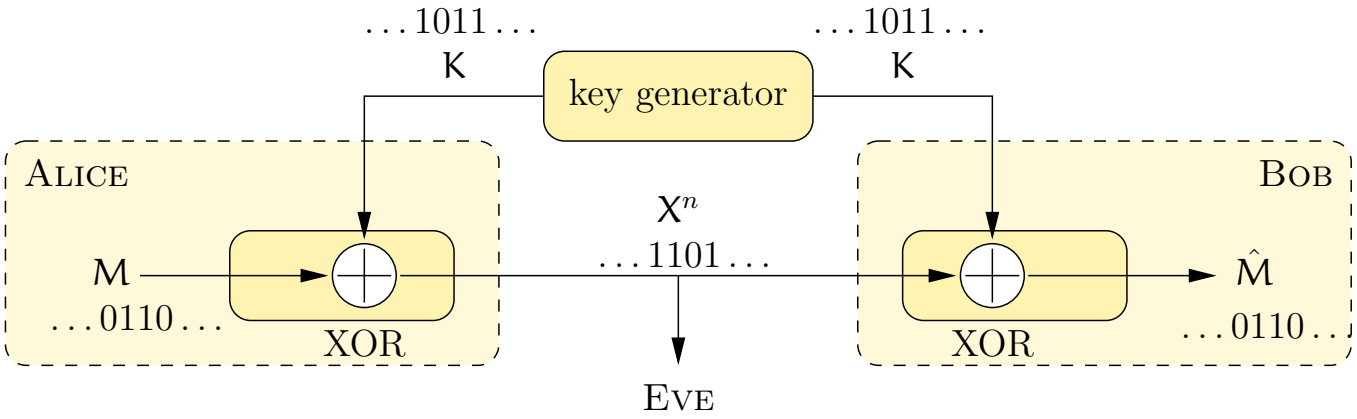

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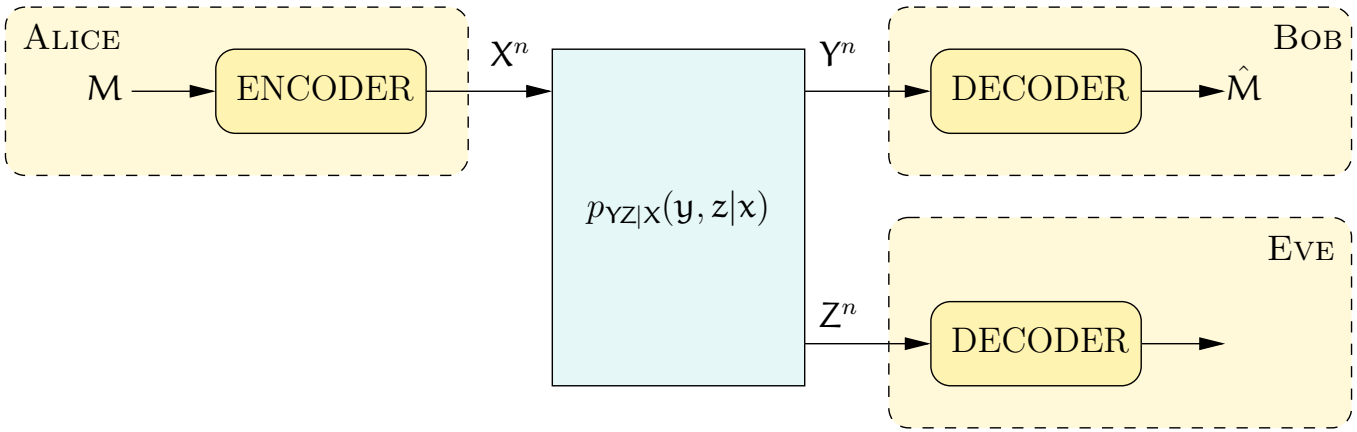
Communication over Wiretap Channel



- reliability constraint: $\mathbb{P}\left[M \neq \hat{M}\right] < \epsilon$
- secrecy constraint: $\frac{1}{n}\mathbb{H}(M|Z^n) \geq R - \epsilon$

Secrecy capacity of wiretap channel

- highest rate R_s such that constraints satisfied for all ϵ

$$C_s = \max_{V \to X \to YZ} \left[\mathbb{I}(V;Y) - \mathbb{I}(V;Z)\right]$$

- for noiseless channels $C_s = 0$

Caveats of Wiretap Channel Model

1. Knowledge of channel
   - wiretap channel model assumes perfect knowledge of channel
   - mainly results for memoryless case
2. Authentication
   - no protection against man-in-the-middle attacks
3. Passive attacker model
   - attacker cannot tamper with channel
4. Perfect random number generation
   - perfect randomness necessary for full secrecy
5. Definition of secrecy
   - common security criterion is weak
6. Scalability
   - model is hardly tractable in multi-user settings

Overcoming Caveats...

Understanding model is critical

Applicability of physical-layer security depends on assumptions.
$\Rightarrow$ Assumptions need to be credible

In this talk

1. Provide general framework for wiretap channel models
   - multi-user situations should be tractable
2. Strengthen existing results
   - use strongest secrecy criterion
3. Evaluate role of channel state information
   - how precise need channel state information be ?

Secrecy Capacity of Arbitrary Channels

Verdú-Han Information-Spectrum Approach

Information-Spectrum approach

- mutual information as a random variable

$$\mathrm{I}(\mathsf{X};\mathsf{Y}) := \log \frac{p_{\mathsf{XY}}(\mathsf{X},\mathsf{Y})}{p_{\mathsf{X}}(\mathsf{X})\, p_{\mathsf{Y}}(\mathsf{Y})} \qquad \mathbb{I}(\mathsf{X};\mathsf{Y}) = \mathbb{E}[\mathrm{I}(\mathsf{X};\mathsf{Y})]$$

- (rate)-information spectrum := density of $\frac{1}{n}\mathrm{I}(\mathsf{X};\mathsf{Y})$
- results expressed in terms of limsup and liminf in probability of random processes

A general formula for channel capacity

Arbitrary channel characterized by $\left\{p_{\mathsf{Y}^n|\mathsf{X}^n}(\mathsf{y}^n|\mathsf{x}^n)\right\}_{n=0}^{\infty}$

$$C = \max_{\{\mathsf{X}^n\}_{n=0}^{\infty}} \text{p-liminf}_{n\to\infty} \frac{1}{n}\mathrm{I}(\mathsf{X}^n;\mathsf{Y}^n)$$

Secrecy criteria

Message M is secure if M and Z^n are statistically independent.

- ideally: $\exists n_0 \geq 0$ such that $p_{\mathrm{M}\mathrm{Z}^{n_0}}(\mathrm{m}, \mathrm{z}^{n_0}) = p_{\mathrm{M}}(\mathrm{m})\, p_{\mathrm{Z}^{n_0}}(\mathrm{z}^{n_0})$
- realistically: $p_{\mathrm{M}\mathrm{Z}^{n}}(\mathrm{m}, \mathrm{z}^{n}) \approx p_{\mathrm{M}}(\mathrm{m})\, p_{\mathrm{Z}^{n}}(\mathrm{z}^{n})$ as $n \to \infty$

Secrecy criteria

$$\text{p-lim}_{n\to\infty}\, \mathbb{I}(\mathrm{M};\mathrm{Z}^n) = 0 \quad (1) \quad \text{strong secrecy}$$

$$\text{p-lim}_{n\to\infty}\, d(\mathrm{M},\mathrm{Z}^n) = 0 \quad (2) \quad \text{strong convergence in variational distance}$$

$$\text{p-lim}_{n\to\infty}\, |\mathrm{I}(\mathrm{M};\mathrm{Z}^n)| = 0 \quad (3) \quad \text{strong convergence of information spectrum}$$

$$\text{p-lim}_{n\to\infty}\, \frac{\mathbb{I}(\mathrm{M};\mathrm{Z}^n)}{n} = 0 \quad (4) \quad \text{weak secrecy}$$

$$\text{p-lim}_{n\to\infty}\, \frac{d(\mathrm{M},\mathrm{Z}^n)}{n} = 0 \quad (5) \quad \text{weak convergence in variational distance}$$

$$\text{p-lim}_{n\to\infty}\, \frac{|\mathrm{I}(\mathrm{M};\mathrm{Z}^n)|}{n} = 0 \quad (6) \quad \text{weak convergence of information spectrum}$$

Ordering of Secrecy Criteria

Ordering law

Criterion (i) is stronger than criterion (j) ($(i) \succeq (j)$) if and only if

$$(i)\,\text{satisfied} \;\Rightarrow\; (j)\,\text{satisfied}$$

Previous criteria can be ordered as

$$(1) \succeq (2) \succeq (3) \succeq (4) \succeq (5) \succeq (6)$$

Key issue

Which criteria are acceptable from a cryptographic perspective ?

Ordering of Secrecy Criteria

Example (Oviously flawed)

Let $\mathrm{u}^n = (\mathrm{u}_1, \ldots, \mathrm{u}_n)$ be a message. Let $\mathrm{k}^{n-k} = (\mathrm{k}_1, \ldots, \mathrm{k}_{n-k})$ be a uniformly distributed secret key. Encode as

$$\mathrm{x}^n = (\mathrm{u}_1 \oplus \mathrm{k}_1, \ldots, \mathrm{u}_{n-k} \oplus \mathrm{k}_{n-k}, \mathrm{u}_{n-k+1}, \ldots, \mathrm{u}_n)\,.$$

For this (dummy) scheme

$$\begin{aligned} \mathbb{I}(\mathrm{U}^n; \mathrm{X}^n) &= k \\ d\,(\mathrm{U}^n, \mathrm{X}^n) &> 0 \\ \frac{\mathbb{I}(\mathrm{U}^n; \mathrm{X}^n)}{n} \leq \frac{k}{n} &\leq \epsilon \quad \text{for } n \text{ large enough} \end{aligned}$$

Usual information-theoretic secrecy criterion is questionable !

Ordering of Secrecy Criteria

Example (Non-uniformity)

Let $\mathrm{u}^n = (\mathrm{u}_1, \ldots, \mathrm{u}_n)$ be a message. Let $\mathrm{k}^n = (\mathrm{k}_1, \ldots, \mathrm{k}_n)$ be a non-uniformly distributed secret key.

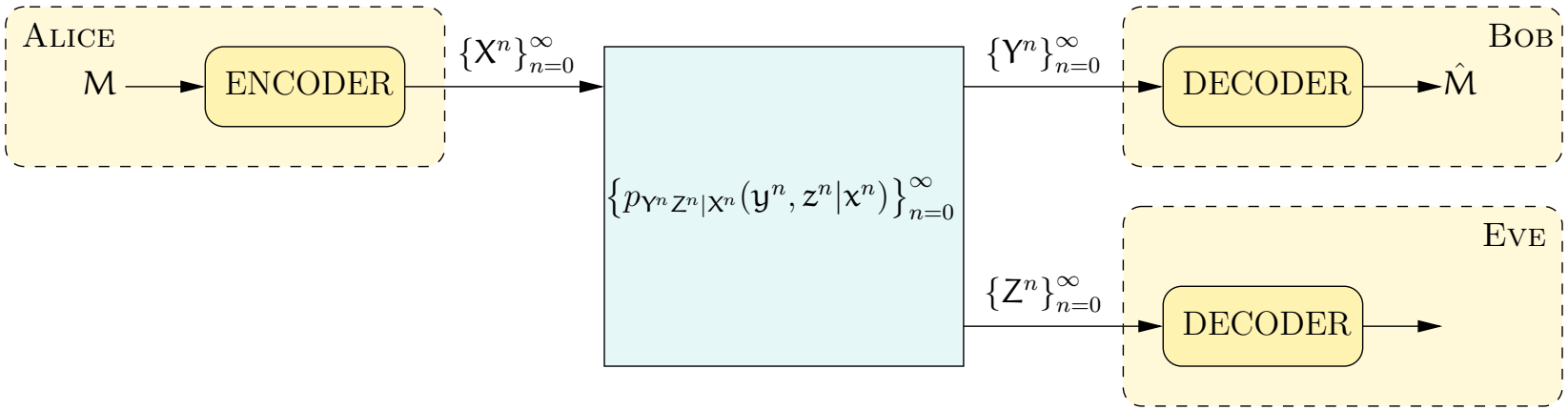
$$\mathbb{P}[\mathrm{K}^n = 0^n] = \frac{1}{n} \qquad \mathbb{P}[\mathrm{K}^n = \mathrm{k}^n] = \frac{1 - \frac{1}{n}}{2^n - 1}$$

Encode as $\mathrm{x}^n = (\mathrm{u}_1 \oplus \mathrm{k}_1, \ldots, \mathrm{u}_n \oplus \mathrm{k}_n)$
One can show

$$\mathbb{I}(\mathrm{U}^n; \mathrm{X}^n) \geq 1 - \epsilon$$
$$d\left(\mathrm{U}^n, \mathrm{X}^n\right) \to 0$$
$$\frac{\mathbb{I}(\mathrm{U}^n; \mathrm{X}^n)}{n} \to 0$$

Strong variational distance criterion is also questionable !

Arbitrary Wiretap Channels



Secrecy capacity

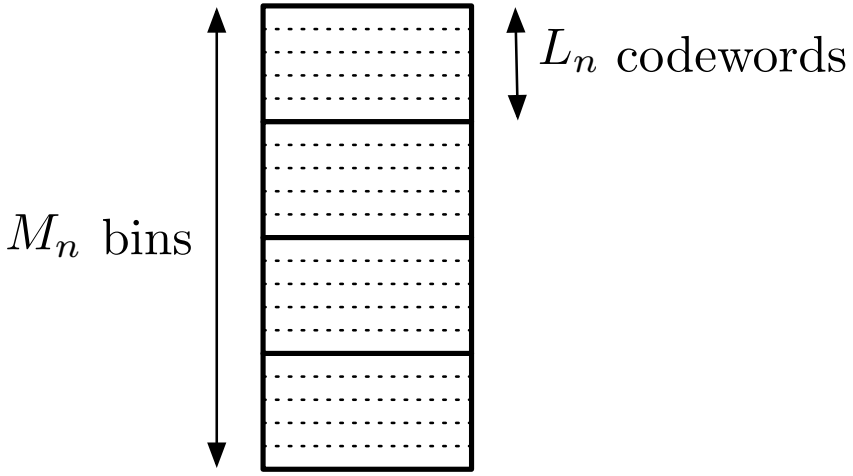
The secrecy capacity of an arbitraty wiretap channel under secrecy criteria (2)-(6) is

$$C_s = \max_{\{V^n, X^n\}_{n=1}^{\infty}} \left(\text{p-liminf}_{n\to\infty} \frac{1}{n} \mathrm{I}(V^n; Y^n) - \text{p-limsup}_{n\to\infty} \frac{1}{n} \mathrm{I}(V^n; Z^n) \right),$$

where $\{V^n, X^n\}_{n=1}^{\infty}$ is subject to $V^n \to X^n \to Z^n Y^n \quad \forall n \in \mathbb{N}^*$.

Secrecy and Resolvability

Key ingredients of wiretap coding: binning and stochastic encoding



$$d\left(p_{\mathsf{M}\mathsf{Z}^n}, p_{\mathsf{M}} p_{\mathsf{Z}^n}\right) \leq 2 \sum_{i=1}^{M_n} \frac{1}{M_n} d\left(p_{\mathsf{Z}^n|i}, \sum_{i=1}^{M_n} \frac{1}{M_n} p_{\mathsf{Z}^n|i}\right)$$

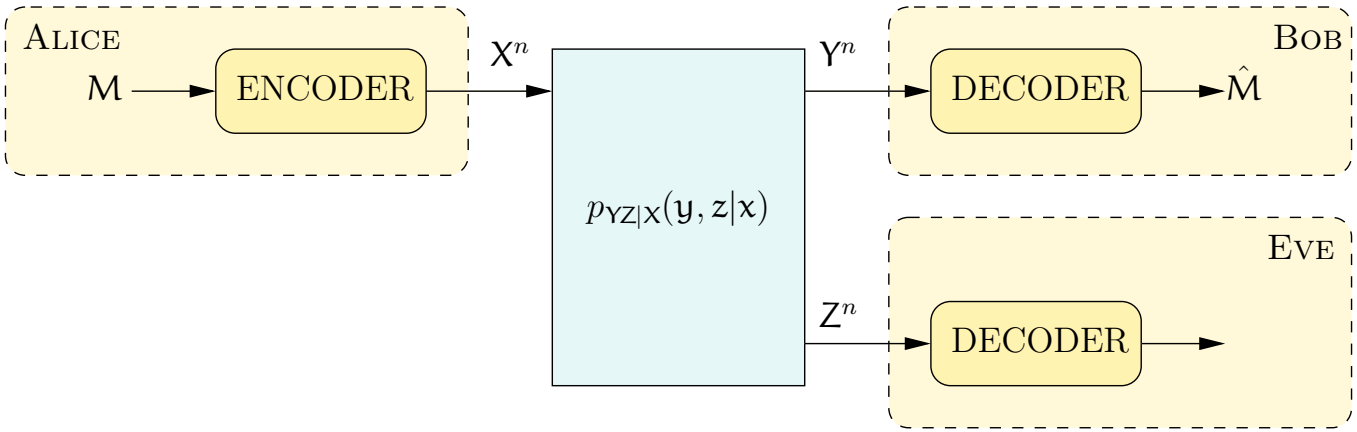
Each bin should induce same distribution at output of the channel.

- channel resolvability problem

$$\frac{1}{n} \log L_n \geq \text{p-limsup}_{n\to\infty} \frac{1}{n} \mathrm{I}(\mathsf{X}^n; \mathsf{Z}^n)$$

Applications

Discrete Memoryless Channels



Secrecy capacity follows from general formula

$$C_s = \max_{V \to X \to YZ} [\mathbb{I}(V;Y) - \mathbb{I}(V;Z)]$$

- achievability part is straightforward by law of large numbers
- converse does not follow easily

Mixed Wiretap Channel

Channel transition probabilities defined as

$$p_{\mathsf{Y}^n\mathsf{Z}^n|\mathsf{X}^n}(\mathsf{y}^n,\mathsf{z}^n|\mathsf{x}^n) = \alpha_1 p_{\mathsf{Y}_1^n\mathsf{Z}_1^n|\mathsf{X}^n}(\mathsf{y}^n,\mathsf{z}^n|\mathsf{x}^n) + \alpha_2 p_{\mathsf{Y}_2^n\mathsf{Z}_2^n|\mathsf{X}^n}(\mathsf{y}^n,\mathsf{z}^n|\mathsf{x}^n)\,.$$

Secrecy capacity of mixed channel

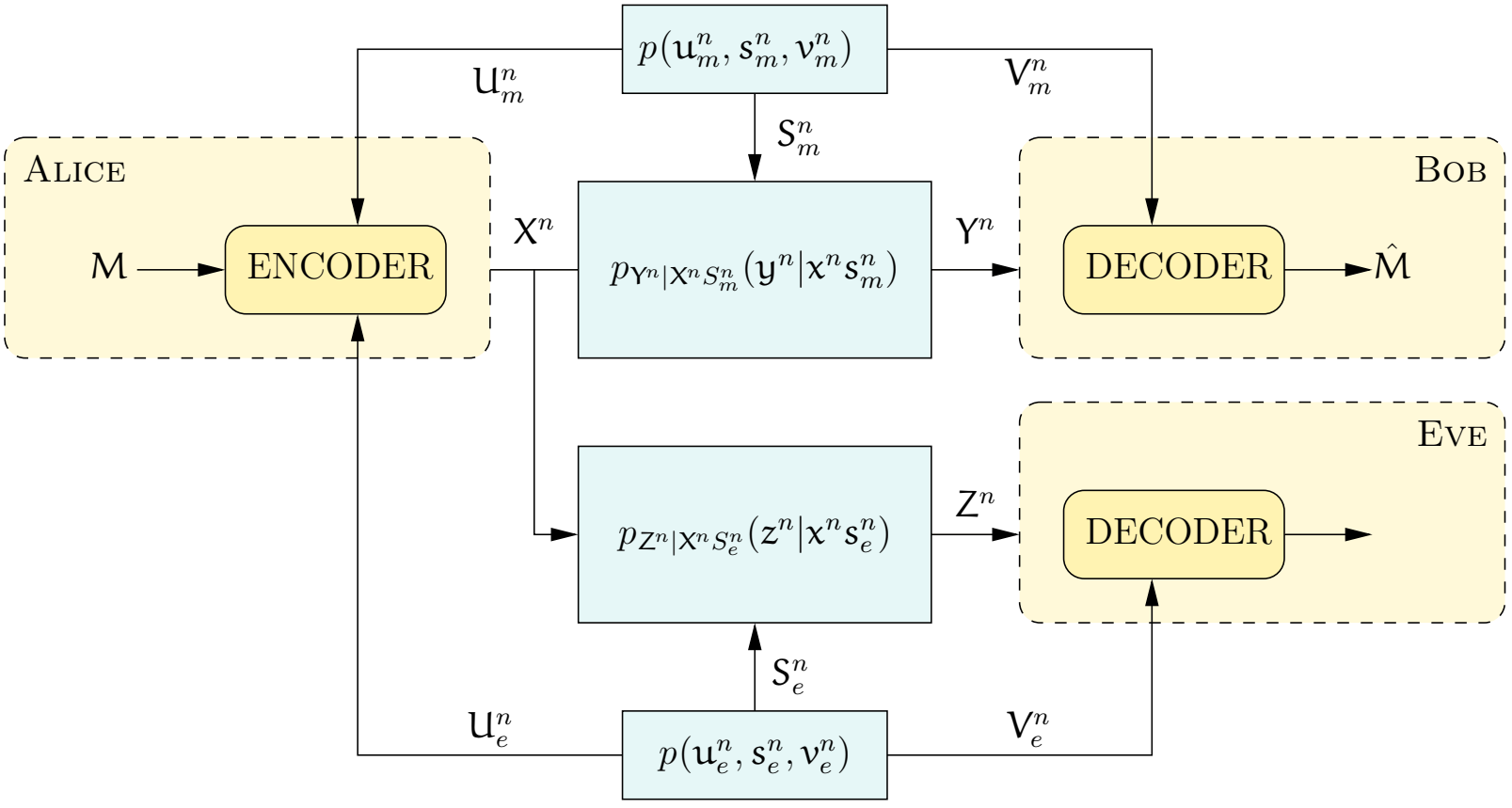
$$\max_{\{\mathsf{V}^n,\mathsf{X}^n\}_{n=1}^{\infty}} \left(\min_{i\in\{1,2\}} \text{p-liminf}_{n\to\infty} \frac{1}{n}\mathrm{I}(\mathsf{V}^n;\mathsf{Y}_i^n) - \max_{j\in\{1,2\}} \text{p-limsup}_{n\to\infty} \frac{1}{n}\mathrm{I}(\mathsf{V}^n;\mathsf{Z}_j^n) \right)$$

Secrecy capacity of memoryless mixed channels

$$C_s \geq \sup_{\mathsf{V}\to\mathsf{X}\to\mathsf{Y}_i\mathsf{Z}_j} \left(\min_{i\in\{1,2\}} \mathbb{I}(\mathsf{V};\mathsf{Y}_i) - \max_{j\in\{1,2\}} \mathbb{I}(\mathsf{V};\mathsf{Z}_j) \right)$$

Applications: parallel wiretap channels, broadcasting to many users, etc.

Wiretap Channels with Channel State Information



$$C_s = \max_{\{\mathsf{R}^n, \mathsf{T}^n\}_{n=1}^{\infty}} \left(\text{p-liminf}_{n\to\infty} \frac{1}{n} \mathrm{I}(\mathsf{R}^n; \mathsf{Y}^n | \mathsf{V}_m^n) \; - \text{p-limsup}_{n\to\infty} \frac{1}{n} \mathrm{I}(\mathsf{R}^n; \mathsf{Z}^n | \mathsf{V}_w^n) \right)$$

Gaussian Fading Wiretap Channel

Channel model

$$\begin{cases} Y_n = \sqrt{S_n} X_n + N_n^m & \text{with } N_n^m \sim \mathcal{N}(0,1) \\ Z_n = \sqrt{S'_n} X_n + N_n^e & \text{with } N_n^e \sim \mathcal{N}(0,1) \end{cases}$$

- $S_n \in \mathbb{R}^+$ and $S'_n \in \mathbb{R}^+$ are stationary ergodic processes
- Noisy CSIT: U_n and U'_n are noisy estimates of S_n and S'_n
- Perfect CSIR at both receivers: knowledge of received average SNR

$$V_n = S_n \mathbb{E}\left[|X_n|^2 | U_n\right] \qquad V'_n = S'_n \mathbb{E}\left[|X_n|^2 | U'_n\right]$$

- Average input power constraint $\mathbb{E}\left[X_n^2\right] \leq P$

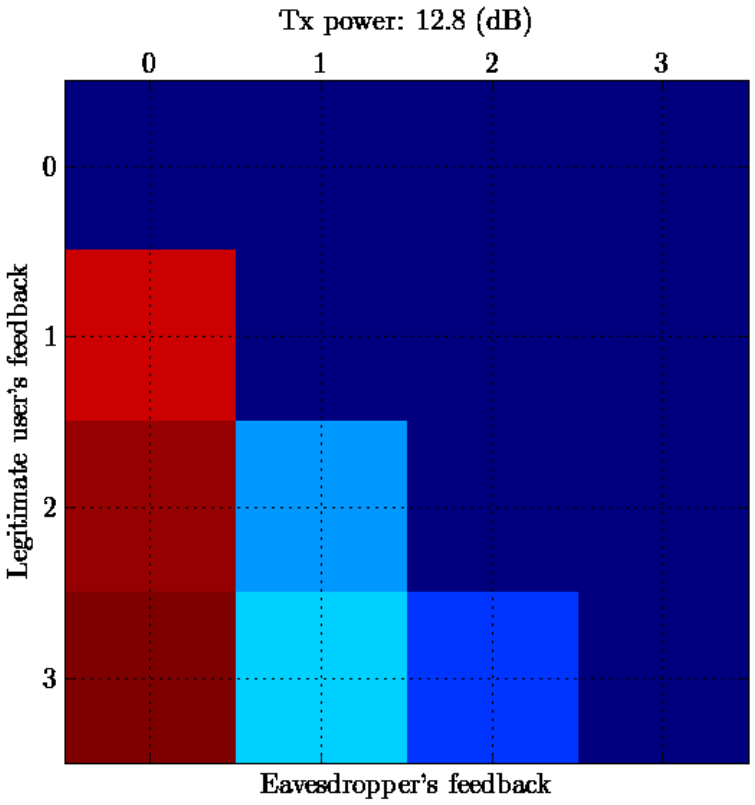
Lower bound on secrecy capacity

$$C_s \geq \max_{\gamma} \mathbb{E}_{S,S',U,U'} \left[\frac{1}{2} \log \left(\frac{1 + S\gamma(U,U')}{1 + S'\gamma(U,U')} \right) \right]$$

Gaussian Fading Wiretap Channel

Numerical example

- Uniform fading on $[0, 2]$ for both channels
- U_n is b_m-bits uniform quantization feedback
- U'_n is b_w-bits quantization feedback



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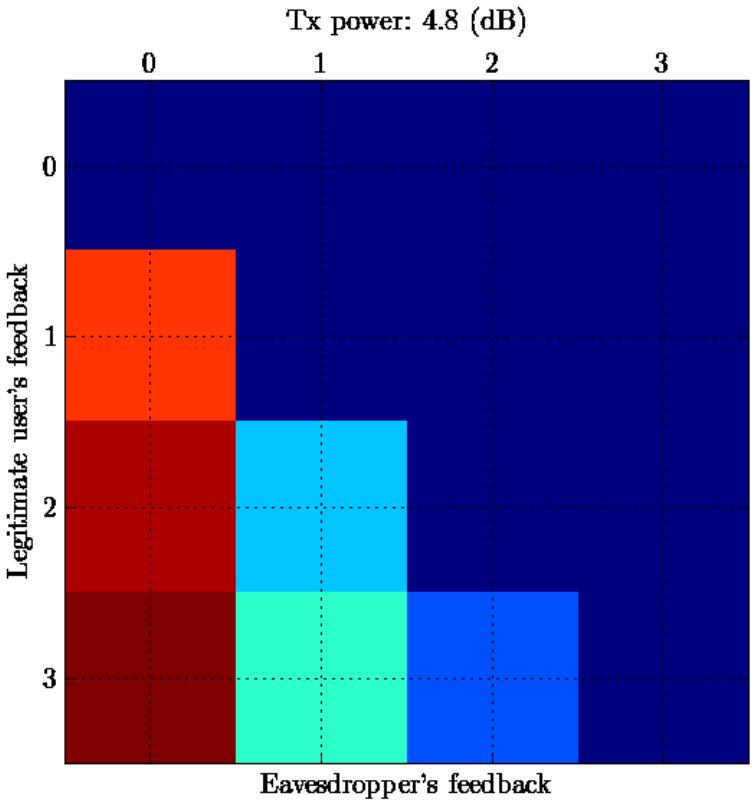

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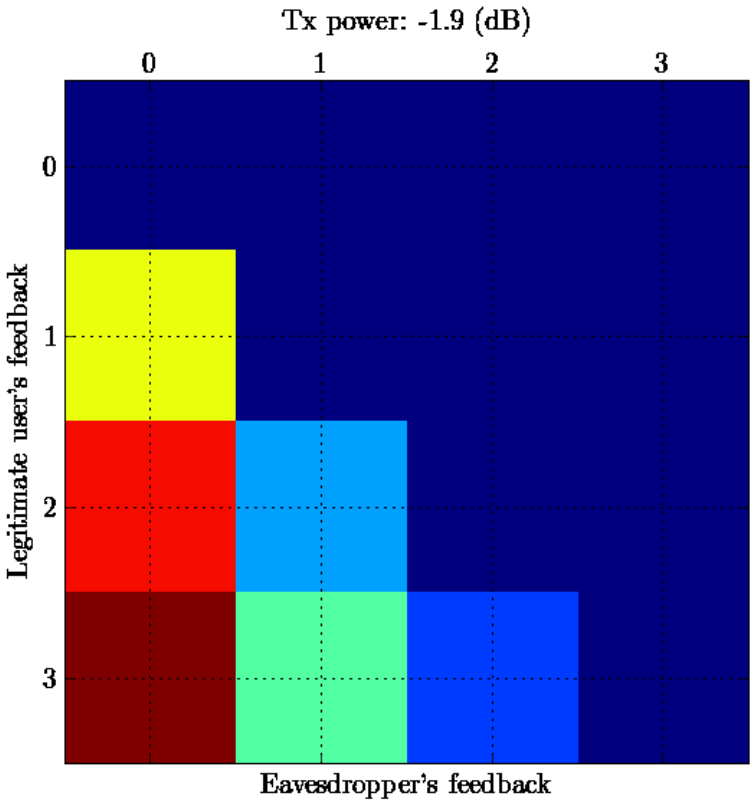

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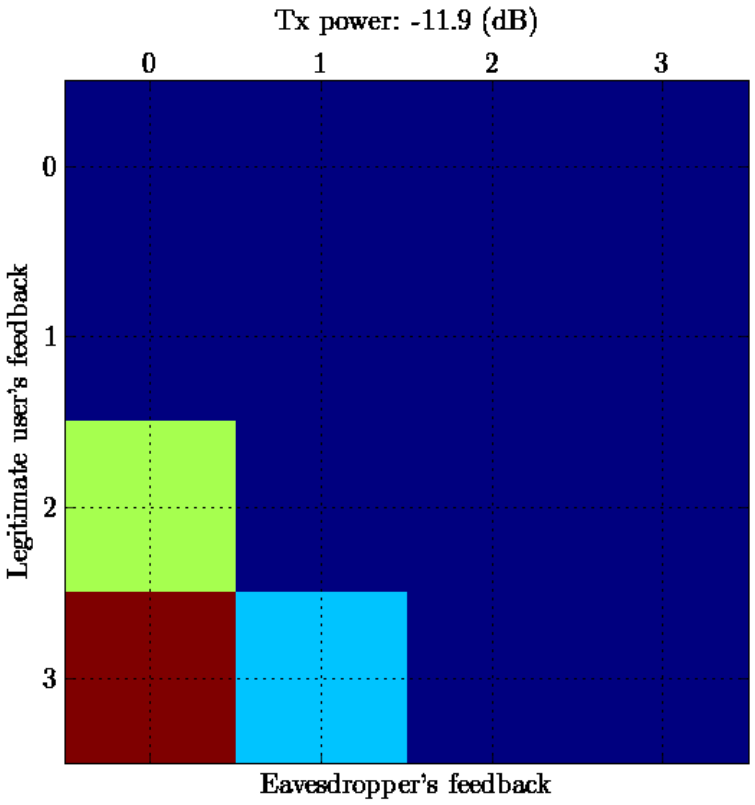

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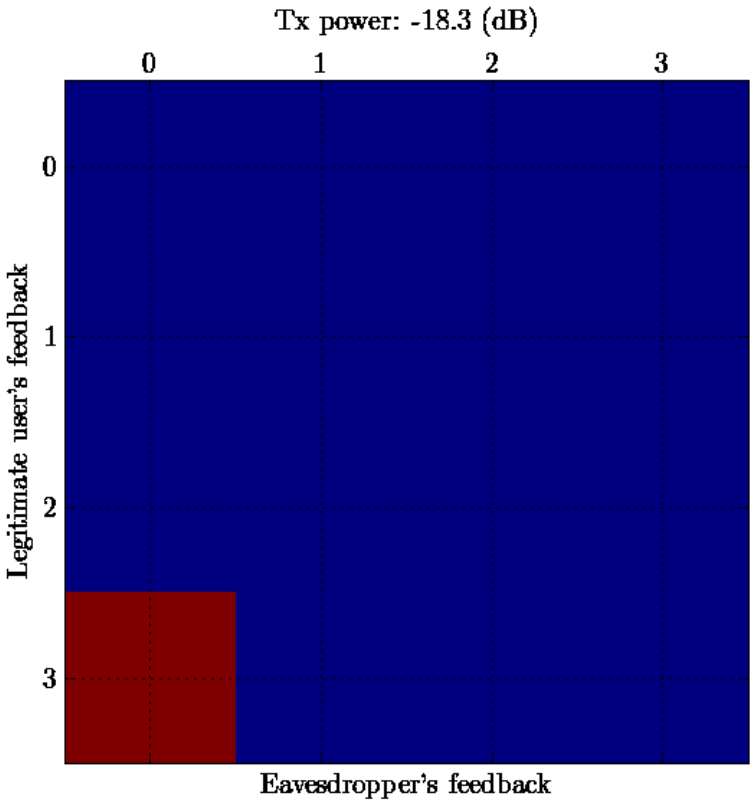
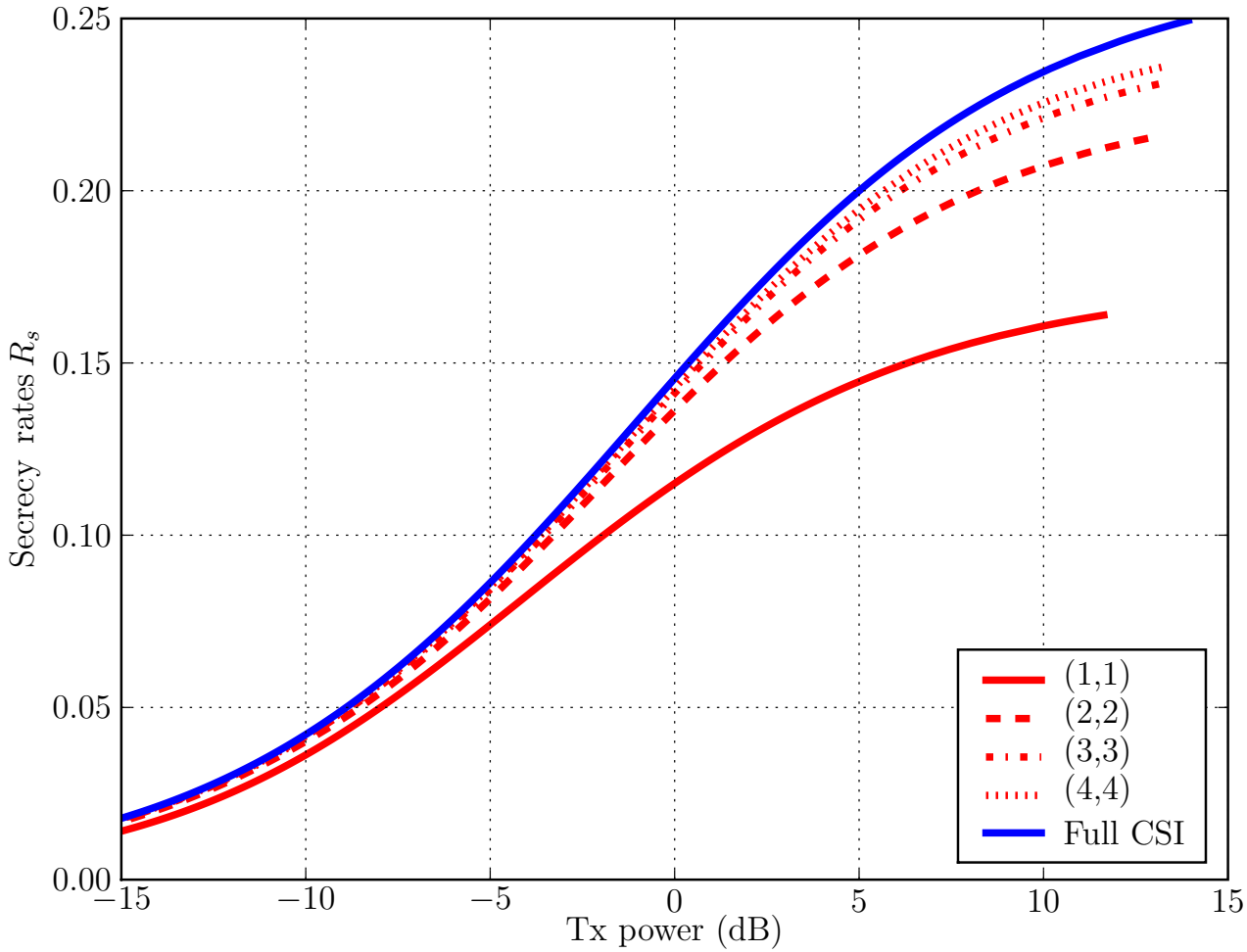


Gaussian Fading Wiretap Channel

Impact of number of feedback bits.



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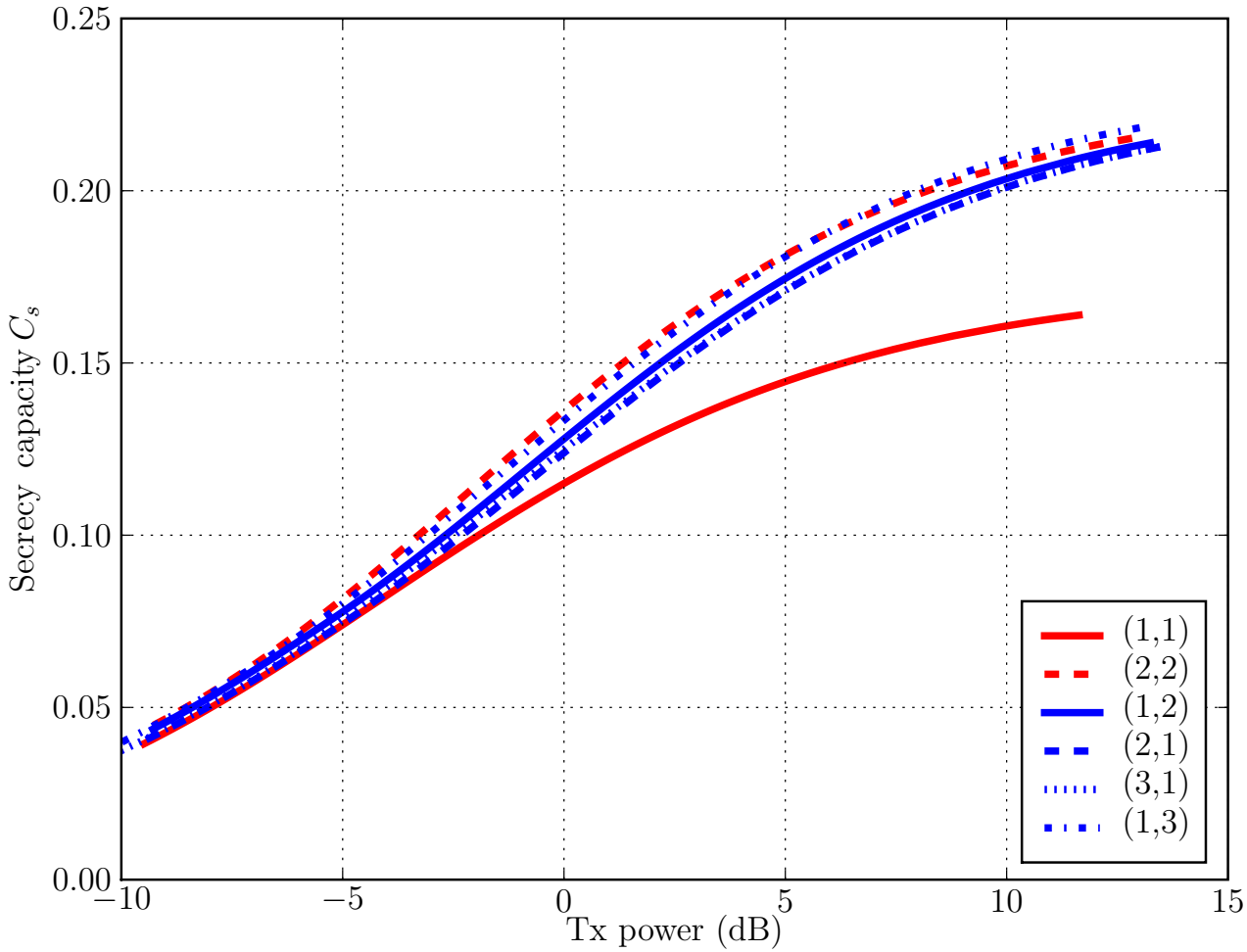

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Conclusion

Information-spectrum approach is useful

- generalizes many wiretap channel models
- fundamentally different approach from Csiszár and Körner (channel resolvability)
- convenient way of computing achievable secure rates
- BUT usefulness somewhat limited for converse proofs

Channel state information impacts secrecy

- may not necessarily need much information
- may not require same precision for feedback

Perspectives

Feedback and security

- notion of feedback not captured in previous models
- feedback known to increase secrecy rates
- feedback may compensate absence of channel state information

Practical code constructions

- current results mostly of theoretical interest
- resolvability might provide an alternative approach

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