Capacity of Wireless Ad Hoc Networks with Multiple Antennas

Theory and Practical Approaches

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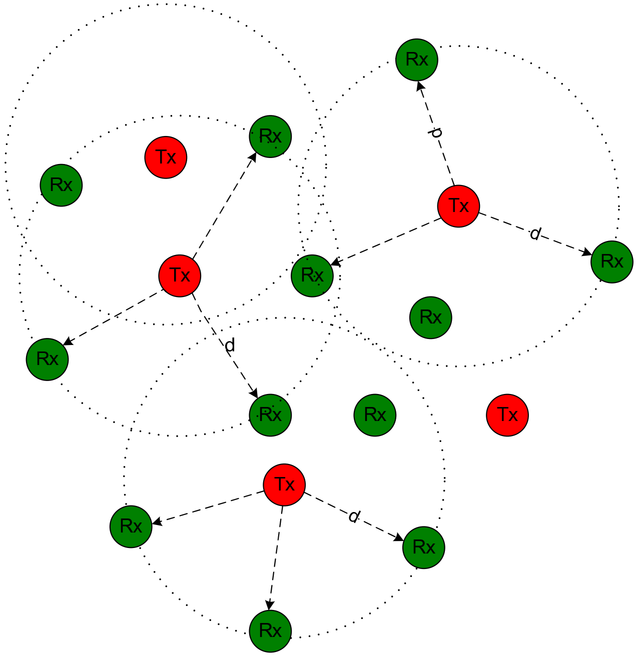
We consider a wireless ad hoc network with multi-antenna nodes



Question: How to efficiently use multiple antennas (degrees of freedom) to enhance network performance?

Answer:

- receive antenna processing is of cardinal importance.
- choice depends on the availability of CSI.

Talk Outline

- Wireless Ad Hoc Networks: Introduction & Motivation
- PART I: Transmission Capacity: what, why and how?
- PART II: SDMA Multi-Antenna Ad Hoc Networks
  - Perfect Channel knowledge
    - Dirty Paper Coding
    - Linear Precoding
  - Limited feedback
- Concluding Remarks & Perspectives

Wireless Ad Hoc Networks

- Ad hoc networks are peer-to-peer with no pre-existing infrastructure
- The most general wireless networks: single-hop, relay, interference, mesh, and star networks are special cases
- Variety of applications:
  - Home networking
  - Emergency/rescue and medical operational networks
  - Military communications
  - Sensor networks
  - Metropolitan mesh networks
- Attractive characteristics:
  - Decentralized and mobile
  - Dynamic, highly flexible, and self-organizing
  - Large scale

Network Information Theory

What is the capacity of ad hoc networks?

- The Holy Grail: general end-to-end capacity results for wireless ad hoc networks are difficult to derive!
- Shannon IT was a Success Story:
  - fundamental limits & design driver
  - relevant benchmark for single-user, BC, MAC channels
- BUT ... links & networks have different capacity characteristics
- What changes:
  - unbounded delay and reliability not allowed (T-D-R region)
  - spatially dependent nature of the interference
  - spatio-temporal network dynamics (need for decompositions)
  - bursty traffic sources, e2e delay constraints, multihop routing, mobility → constantly changing network topology

Recent Approaches

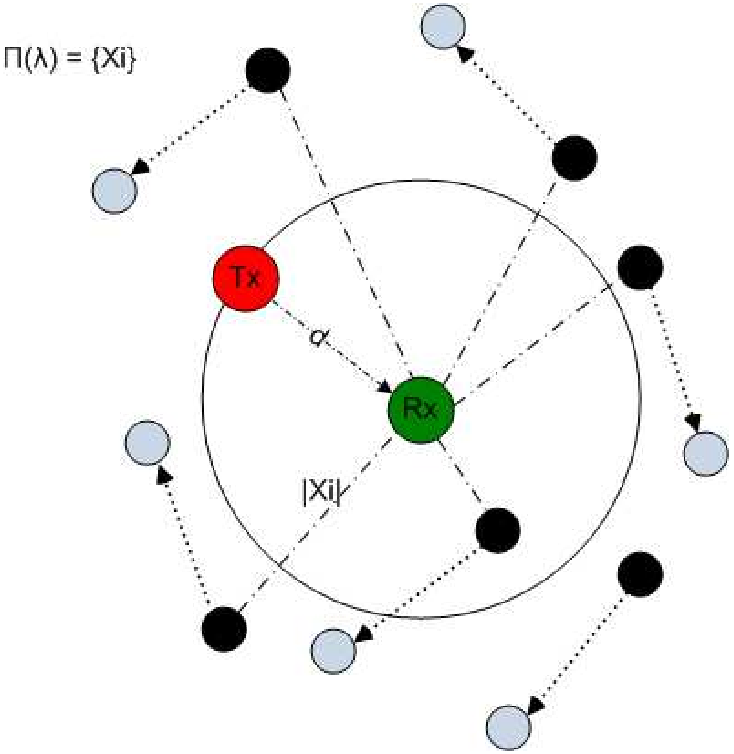
- Seminal result [GupKum'00]: for K nodes
  - Transport capacity scales at best as $\Theta(\sqrt{K})$ bit-m/s^2
  - sublinear in K & per-node capacity $\rightarrow$ 0 as $K \rightarrow \infty$
- Several results so far:
  - Mobility increases capacity (O(K) if delay is not an issue) [GroTse'02]
  - Hierarchical Cooperation, virtual MIMO [OzgLevTse'07]
  - Electromagnetic limits [FraMigMin'07]
  - 'Wireline' Tree Network idea [NieGupSha'08]
- Deterministic Model & Approximate Capacity for Interference Channels [AveDigTse'07]

Issue: Scaling laws and numerical simulations do not provide sufficient insights

$\longrightarrow$ Transmission Capacity comes to supplement the puzzle!

Network Model

Consider one snapshot in time (single hop)



- Transmission & end-to-end distances are roughly constant
- Uncoordinated network (e.g. ALOHA)
- Tx locations: stationary Poisson point process (PPP) on the plane (*stochastic geometry framework*)

Outage Probability

- Given a choice of system parameters, outage events capture the main network behavior
- Outage Probability for a *typical Tx-Rx pair*:

$$P_{\text{out}} = \mathbb{P}\left\{ \frac{\rho H_{00} d^{-\alpha}}{\sum_{i \in \Pi(\lambda)} \rho H_{0i} \left|X_i\right|^{-\alpha} + \eta} < \beta \right\} \leq \epsilon$$

ρ	transmission power
H_{ij}	fading coefficient from Tx i to Rx j
d	distance between Tx-Rx pair
α	pathloss exponent ($\alpha > 2$)
η	noise power
$\left\|X_i\right\|$	distance from interference i
$\Pi(\lambda) = \{X_i\}$	Poisson point process of interferers with intensity λ (per m^2)
β	SINR/SIR requirement for successful reception
ϵ	outage probability constraint

Transmission Capacity Framework

- <u>Definition</u>: *the maximum intensity* λ *of successful concurrent transmissions per unit area for which each transmission is successful with probability* $1-\epsilon$

Key metrics:

- (Network-wide) Outage Probability

$$\pi(\lambda) \equiv \mathbb{P}\left\{\mathrm{SINR} < \beta\right\}$$

$\pi(\lambda) \in [0,1]$ - continuous monotone increasing in λ

- Transmission capacity

$$\mathcal{C}(\epsilon) = \pi^{-1}(\epsilon)(1-\epsilon) \quad 0 < \epsilon < 1$$

$\pi^{-1}(\epsilon) \equiv \lambda_\epsilon$: max. density of transmissions (users/m^2)

- Area Spectral Efficiency

$$\mathrm{ASE} = \lambda_\epsilon(1-\epsilon)\log_2(1+\beta)\ \mathrm{bps/Hz/m}^2$$

TC depends on interferers' positions & random channel effects

Implications of TC Framework

- TC framework allows for quantification of achievable single-hop rates by focusing on a simplified PHY/MAC model
- TC allows for intuitive closed-form expressions
- Stochastic geometry models to quantify the multiuser interference
  $\longrightarrow$ relationship between the spatial density and P_{succ} can be determined
- Homogeneous Poisson distribution of nodes assumption (tractability - more sophisticated results exist)
- Gives crisp insights into cross-layer design problems (FH vs. DS-CDMA, SIC, power control, bandwidth partitioning, threshold scheduling, network coexistence, cognitive policies)

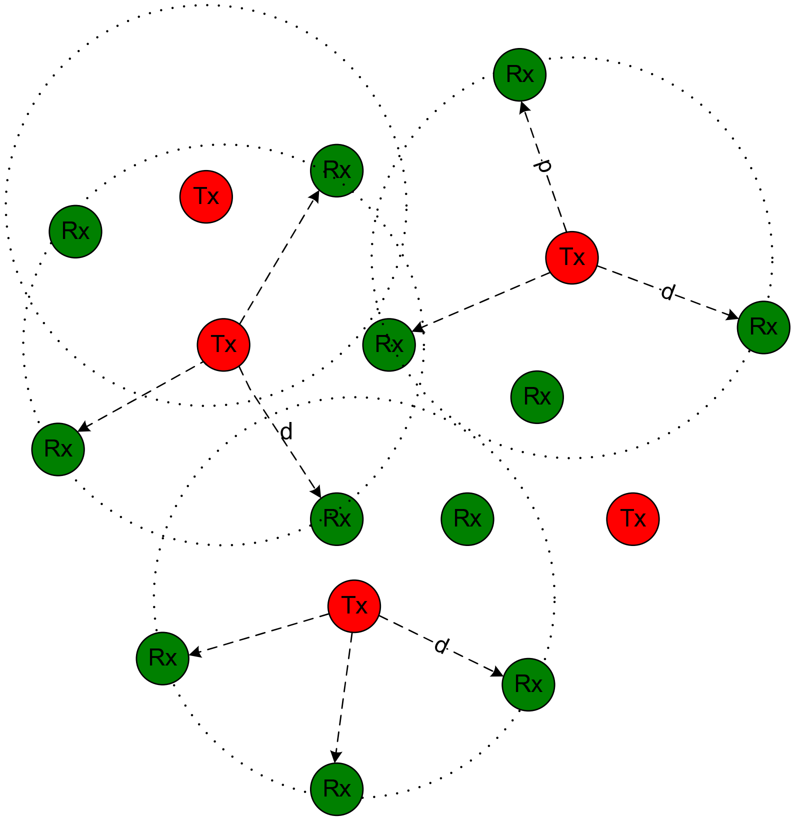
For details: [WebAndJin'07], [WebAndJin'08]

Multi-Antenna Wireless Ad Hoc Networks: point-to-multipoint MIMO links

Motivation and Goals

- Extend transmission capacity framework to take into account multi-receiver (SDMA) transmissions
- Derive the fundamental limits of SDMA communications in wireless ad hoc networks
- Quantify the throughput gains (*if any*) by sending multiple streams to different receivers (multiuser MIMO)
- Provide insights on how to use multiple antennas in SDMA ad hoc networks
- Effect of limited feedback in multi-antenna transmission
- Any gains in terms of scheduling, delay, and packet forward progress?

SDMA Ad Hoc Networks



- Each transmitter with M antennas communicates simultaneously with $K \leq M$ receivers, each with N receive antennas.
- Perfect CSIR and error-free, zero-delay feedback channels

SDMA Network Model

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SDMA Ad Hoc Networks

SDMA Network Model

Perfect Channel Knowledge

Linear Precoding with CSIT

Imperfect Channel Knowledge

The Way Forward

The received signal $\mathbf{y}_k$ at reveiver $k \in \mathcal{K}$ is given by

$$\mathbf{y}_k = \sqrt{\rho} d^{-\alpha/2} \mathbf{H}_{0k} \mathbf{x}_k + \sqrt{\rho} \sum_{i \in \Pi(\lambda)} |X_i|^{-\alpha/2} \mathbf{H}_{ik} \mathbf{x}_i + \mathbf{n} \qquad (1)$$

where $\mathbf{H}_{0k} \in \mathbb{C}^{N \times M}$ is the channel between T_0 and receiver k

$\mathbf{H}_{ik} \in \mathbb{C}^{N \times M}$ is the channel between receiver k and interfering transmitters T_i

$\mathbf{x}_k$ is the normalized transmit signal vector

$\mathbf{n}$ is complex additive Gaussian noise.

Key metric: Assuming transmission at the Shannon target rate $R = \log_2(1+\beta)$ bps/Hz, the area spectral efficiency is defined as

$$\mathcal{C}_\epsilon = K \lambda_\epsilon (1-\epsilon) R \quad \text{bps/Hz/m}^2 \qquad (2)$$

and depends on the number $1 \leq K \leq M$ spatial streams sent by each source node.

Dirty Paper Coding (1/2)

Multi-antenna Receivers

Lemma 1: The maximum contention density under DPC is upper bounded by

$$\lambda_{\text{DPC}} \leq \frac{(4MN)^{2/\alpha}}{\mathcal{I}_M \beta^{2/\alpha} d^2} \left[-\log(1-\epsilon) + \frac{\eta \beta d^\alpha}{4MN\rho} \right] \qquad (3)$$

where

$$\mathcal{I}_M = \frac{2\pi}{\alpha} \sum_{m=0}^{M-1} \binom{M}{m} B(m + 2/\alpha, M - (m + 2/\alpha)) \qquad (4)$$

Since $\mathcal{I}_M \sim \pi\Gamma(1-2/\alpha)M^{2/\alpha}$ for large M:

Lemma 2: The DPC capacity scales superlinearly as

$$\mathcal{C}_{\text{DPC}} = O(MN^{2/a}) \qquad (5)$$

For $M = N$: $\mathcal{C}_{\text{DPC}} = O(N^{1+2/a})$

→ DPC allows for $N^{2/\alpha}$ more concurrent transmissions per unit area as compared to single-stream MIMO communications.

Dirty Paper Coding (2/2)

Single-antenna Receivers

Proposition 1: Transmission to M single-antenna Rx using DPC results in a maximum contention density $\lambda_{\mathrm{DPC}}^{\mathrm{miso}}$ of

$$\lambda_{\mathrm{DPC}}^{\mathrm{miso}} = \frac{\mathcal{F}_M \epsilon}{\mathcal{I}_M \beta^{2/\alpha} d^2} e^{-\frac{\beta d^2}{\rho}} \qquad (6)$$

where

$$\mathcal{F}_M = \left[\sum_{k=0}^{M-1} \sum_{j=0}^{k} \binom{k}{j} \left(\frac{\eta}{\rho}\right)^{k-j} \frac{1}{j!} \prod_{m=0}^{j-1} (m - 2/\alpha) \right]^{-1} \qquad (7)$$

- For large M, $\frac{\mathcal{F}_M}{\mathcal{I}_M} = O(1)$ → capacity exhibits linear scaling (i.e. $\mathcal{C}_{\mathrm{DPC}}^{\mathrm{miso}} = O(M)$)
- Lack of Rx processing → no diversity or interference cancellation gain → per-user outage probability is $O(1)$

Linear Precoding with CSIT (1/3)

Zero-forcing Beamforming with Antenna Combining

For $M \geq KN$ with $N > 1$

Inter-user interference constraint at each Rx antenna n:
$\mathbf{h}_{k,n}\mathbf{w}_{j,l} = 0, \forall j \neq k, \forall n, l \in [1, N]$ and $\mathbf{h}_{k,n}\mathbf{w}_{k,l} = 0, \forall l \neq n$.

Proposition 2: The capacity in the small outage constraint regime scales as

$$\mathcal{C}_{\mathrm{ZF}} = O\left(\frac{(M - KN + 1)^{2/\alpha}}{(KN)^{2/\alpha - 1}}\right) \qquad (8)$$

For $M = KN$, we have a scaling of $O((KN)^{2/\alpha})$ (full diversity [HunAndWeb'08]).

For $N > M$

Extra DoF at the Rx side can be exploited to eliminate the inter-node interference.
$\rightarrow \mathcal{C}_{\mathrm{ZF}} = O(M(\frac{N-M+1}{M})^{\frac{2}{\alpha}}) = O(M^{1-2/\alpha}) < O(N^{1-2/\alpha})$.

Linear Precoding with CSIT (2/3)

Zero-forcing Beamforming with Antenna Selection

The maximum contention density (for $\epsilon \to 0$) is given by

$$\lambda_{\mathrm{ZF}}^{as} = \frac{\epsilon}{\mathcal{S}'_N \mathcal{I}_M \beta^{2/\alpha} d^2} e^{-\frac{\beta d^\alpha}{\rho}} \tag{9}$$

where $\mathcal{S}'_N = \sum_{n=1}^{N} \sum_{j=1}^{n} \binom{n}{j} \binom{d}{j} \left(\frac{\eta}{\rho}\right)^{(n-j)} (-1)^{j+1} j^{2/\alpha}$ *and* $\mathcal{I}_M$ *is given by (4).*

- The capacity scales as $\mathcal{C}_{\mathrm{ZF}}^{as} = O(\mathcal{S}_N^{-1} M^{1-\frac{2}{\alpha}})$
- For $M = N \to \mathcal{C}_{\mathrm{ZF}}^{as} = O(M)$ (selection improves the typical channel without amplifying interference)
- Since order statistics (due to selection) provides an $M^{2/\alpha}$-fold increase of the received signal power → linear capacity growth.

Linear Precoding with CSIT (3/3)

Zero-forcing Beamforming with Single-antenna Receivers

BF vector of Rx k: $\mathbf{h}_j \mathbf{w}_k = 0, \forall j \in \mathcal{K}, j \neq k$.

Proposition 3: For a random access wireless network in which the Tx spatially multiplexs M single-antenna receivers using ZFBF, the maximum contention density is given by

$$\lambda_{\mathrm{ZF}} = \frac{-\log(1-\epsilon)}{\mathcal{I}_M \beta^{2/\alpha} d^2} + \frac{\eta \beta^{1-\frac{2}{\alpha}} d^{\alpha-2}}{\rho \mathcal{I}_M} \qquad (10)$$

where $\mathcal{I}_M$ is given in (4).

- For large M, the ASE scales as $\mathcal{C}_{\mathrm{ZF}} = O(M^{1-\frac{2}{\alpha}})$ (same scaling as interference-aware beamforming [HuaAndHeaGuoBer'08]).
- Regularized channel inversion (MMSE precoding) provides the same $O(M^{1-\frac{2}{\alpha}})$ scaling but achieving higher SINR target β per user stream.

DPC Performance Evaluation

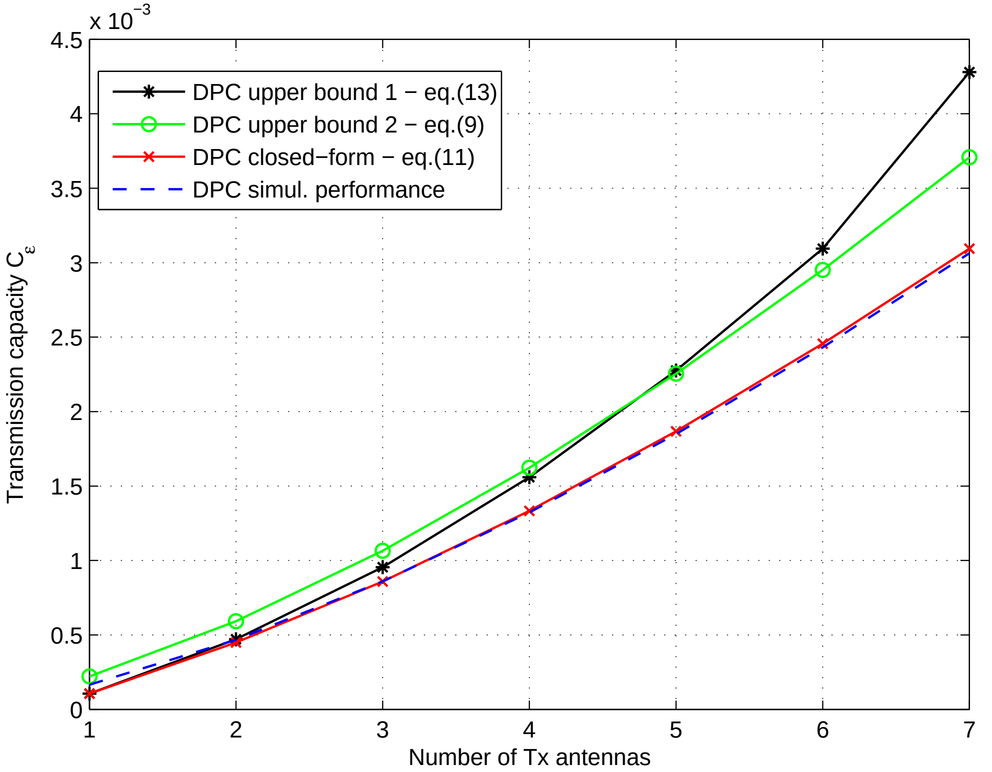

Figure 1: DPC capacity scaling $\mathcal{C}_{\mathrm{DPC}}$ vs. nb. of Tx antennas ($M = N$) for $\alpha = 4$.

- Superlinear scaling behavior of DPC with the number of antennas.
- Tightness of the upper bound depends on the pathloss exponent α and M (being tighter for α decreasing).
- Substantial gains appear even when only a few streams are transmitted.

ZF Performance Evaluation

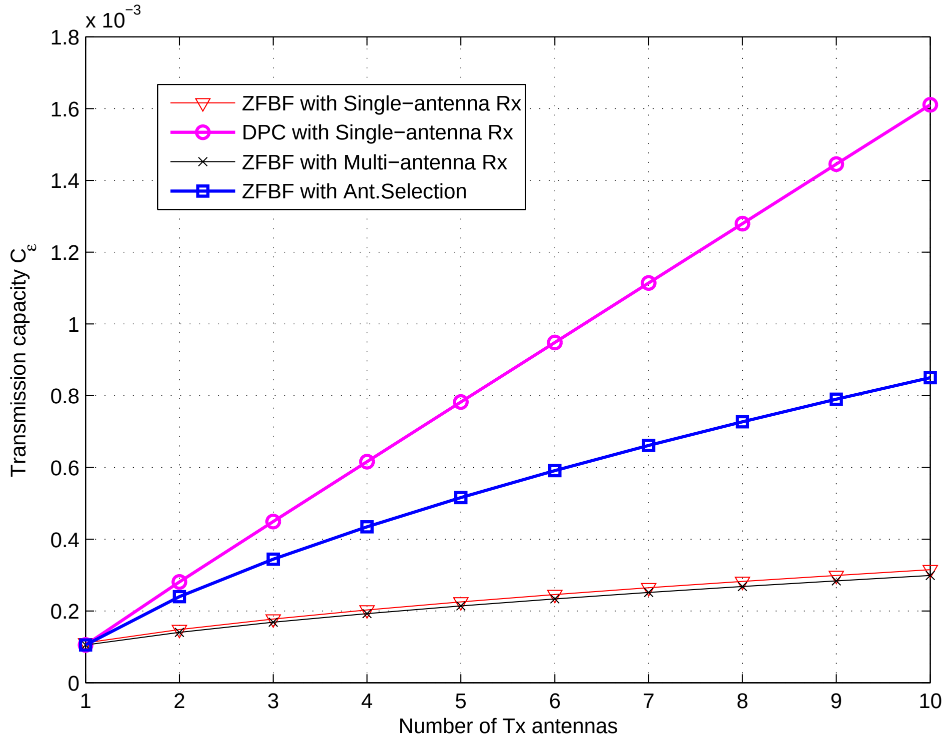

Figure 2: Capacity vs. nb. of Tx antennas for different SDMA precoding techniques ($\alpha = 4$)

- Linear scaling of MISO DPC and sublinear capacity behavior of linear precoding.
- Diversity-oriented receive processing combined with linear transmit processing is not sufficient to achieve linear scaling.

SDMA TC with imperfect CSIT (1/3)

Single-antenna Receivers

- Orthogonal Beamforming: The optimal contention density using OBF with partial CSIT (1 scalar) is given by

$$\lambda_\epsilon = \frac{-\log(1-\epsilon)}{\mathcal{I}_M d^2 \beta^{\frac{2}{\alpha}}} - \frac{\log(1+\beta d^{2\alpha})^{(M-1)}}{\mathcal{I}_M d^2 \beta^{\frac{2}{\alpha}}}$$

which results in transmission capacity loss of

$$\Delta_{\mathrm{TC}} = \frac{M(M-1)(1-\epsilon)\log(1+\beta d^{2\alpha})}{\mathcal{I}_M d^2 \beta^{\frac{2}{\alpha}}} = O(M^{2-\frac{2}{\alpha}})$$

- The number of Tx antennas (streams) for positive network contention density is upper bounded by

$$M < 1 + \frac{\log(1-\epsilon)^{-1}}{\log(1+\beta d^{2\alpha})}$$

SDMA TC with imperfect CSIT (2/3)

Multi-antenna Receivers

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Imperfect Channel Knowledge
- SDMA TC with imperfect CSIT (1/3)
- SDMA TC with imperfect CSIT (2/3)
- SDMA TC with imperfect CSIT (3/3)
- Performance Evaluation (1/2)
- Performance Evaluation (2/2)

The Way Forward

- Orthogonal Beamforming: If MRC Rx processing is employed with limited feedback OBF, the outage probability is given by

$$P_{\text{out}} = 1 - \left[\sum_{k=0}^{N-1} \frac{(-s)^k}{k!} \frac{\partial^k}{\partial s^k} \left(\mathcal{L}_{I_\Phi} \mathcal{L}_Y \right) \right]_{s=\beta d^\alpha}$$

with

$\mathcal{L}_{I_\Phi} = e^{-\lambda d^2 \beta^{\frac{2}{\alpha}} \mathcal{I}_M}$ Laplace transform of PPP interference

$\mathcal{L}_Y = (1 + s d^\alpha)^{(1-M)}$ Laplace transform of inter-stream/MU interference

- Inter-stream interference cancellation is required

SDMA TC with imperfect CSIT (3/3)

Key messages

- SDMA transmission capacity is **very sensitive** to CSIT inaccuracy!
- Single-stream transmission (with diversity) outperforms MISO-SDMA techniques
- For a target SIR = 0 dB, low ϵ is hard to be achieved with partial CSIT
- Interference cancellation combined with diversity-oriented receive processing should be envisaged for SDMA ad hoc networks with limited feedback
- Δ_{TC} grows M times faster than the MISO transmission capacity

Performance Evaluation (1/2)

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- SDMA TC with imperfect CSIT (1/3)
- SDMA TC with imperfect CSIT (2/3)
- SDMA TC with imperfect CSIT (3/3)
- Performance Evaluation (1/2)
- Performance Evaluation (2/2)

The Way Forward

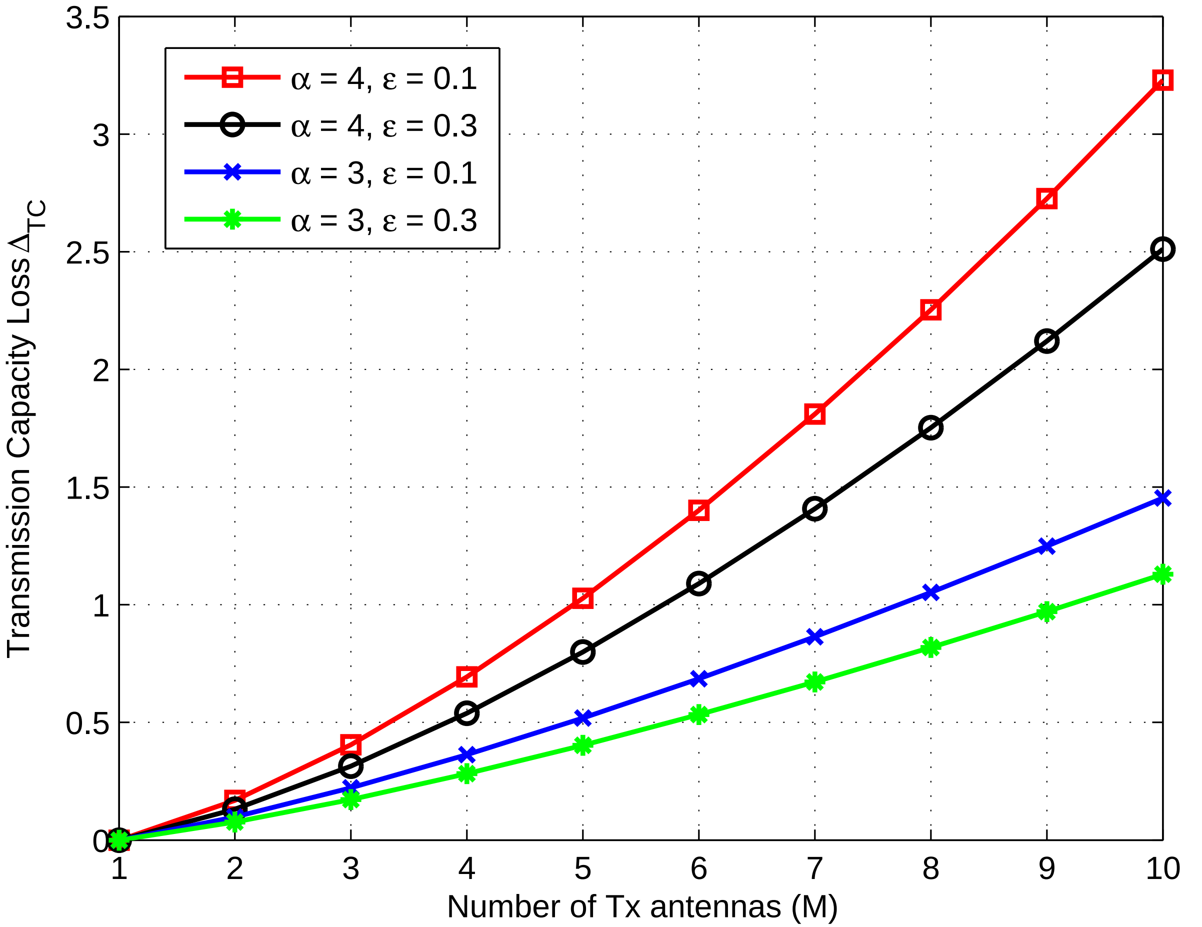

Figure 3: Trans. capacity loss Δ_{TC} vs. # of Tx antennas M

Performance Evaluation (2/2)

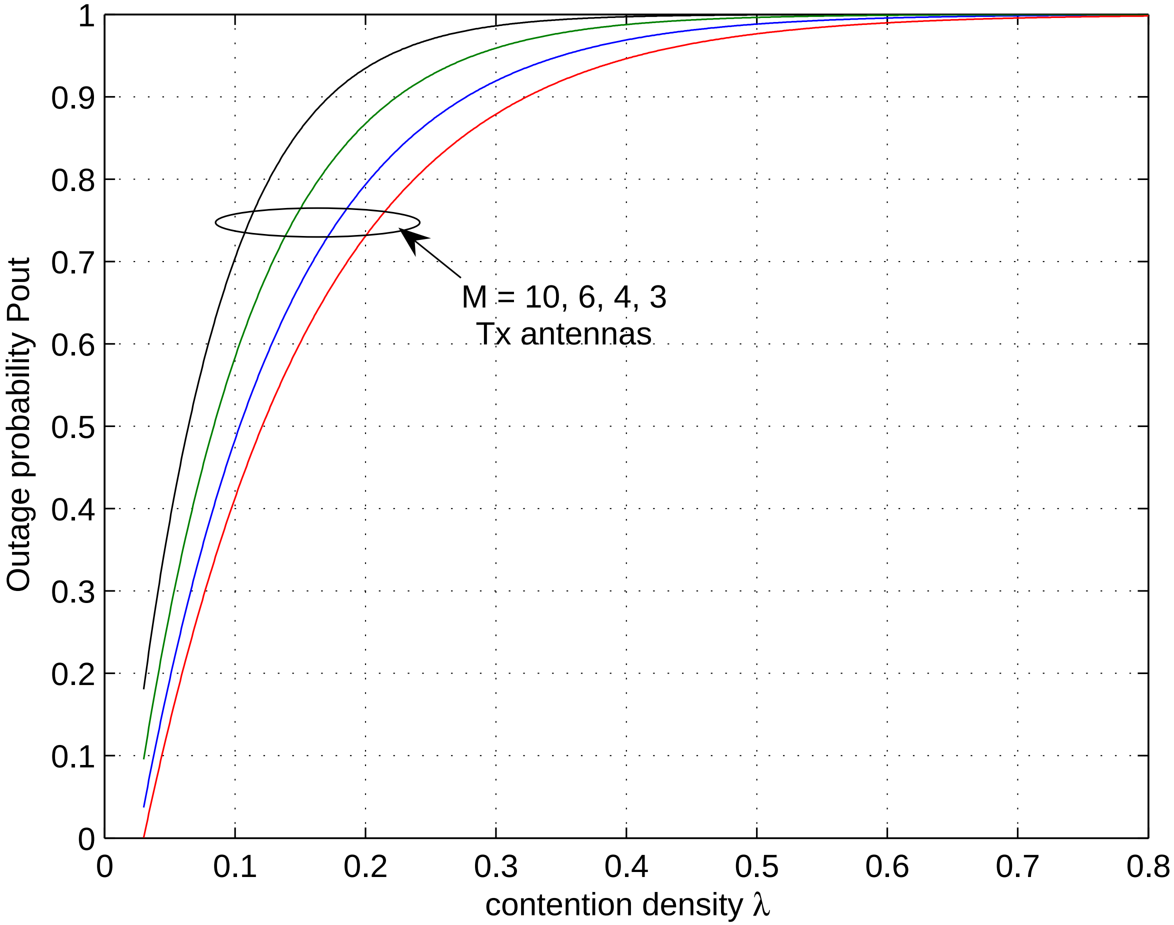

Figure 4: Outage Prob. vs. λ for SDMA with OBF + MRC (N=2)

The Way Forward

Concluding remarks & Perspectives

- MIMO allows for larger spatial packing of simultaneous transmissions
- Superlinear capacity increase is achieved with multiple antennas (with perfect CSIT)
- Limited feedback is very detrimental in SDMA ad hoc
- More results for codebook-based SDMA techniques - Interplay between capacity and CSIT resolution (# feedback bits)
- SDMA in ad hoc networks may result in delay/routing benefits
- Need for novel, MANET-tailored SDMA techniques (interference alignment with partial CSIT??)
- MAC layer coordination and scheduling to the rescue?

The Road Ahead

more interdisciplinary flavor required

- Extension to multihop MANETs (+ incorporate routing)
- Use of non-Poisson & heterogeneous point process (Cox, hardcore, clustered) to analyze sophisticated MAC strategies (scheduling, CSMA)
- A unifying analytical framework to account for overhead messaging is critical for a relevant network IT
- Extend TC framework with mobility (e.g. Lévy random walks, first passage processes)
- Understanding the unique spatial and temporal dynamics of ad hoc networks is essential

Thank you!