Universal Wyner-Ziv Coding For Gaussian Sources

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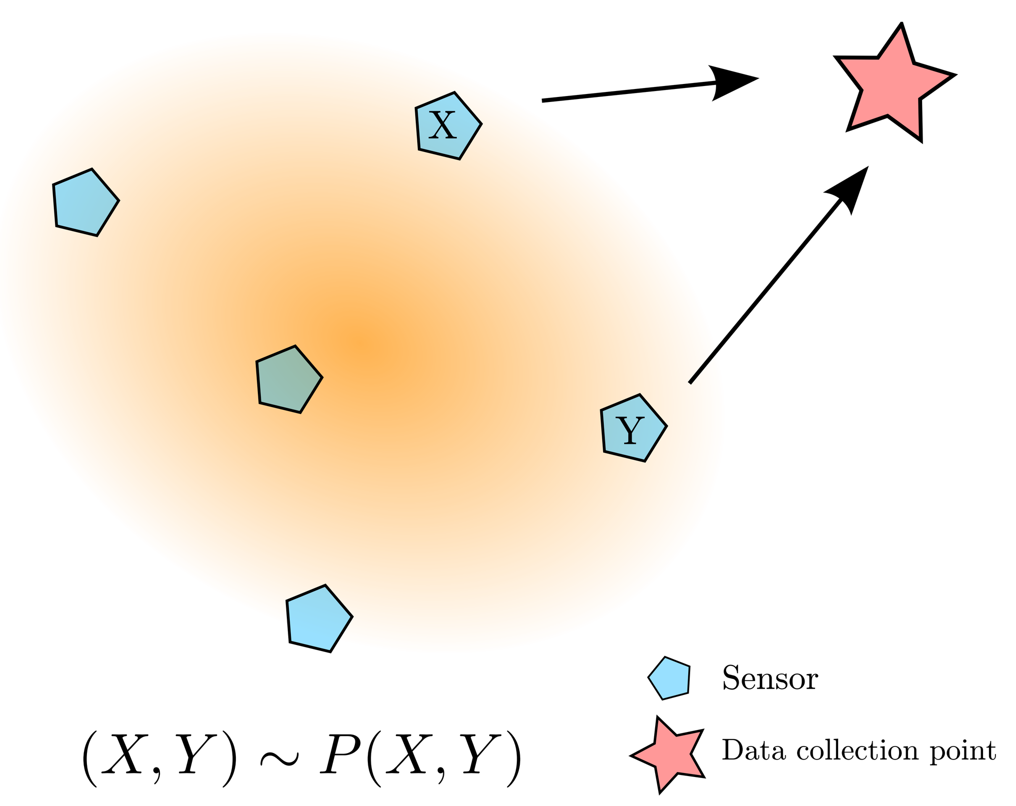
Context

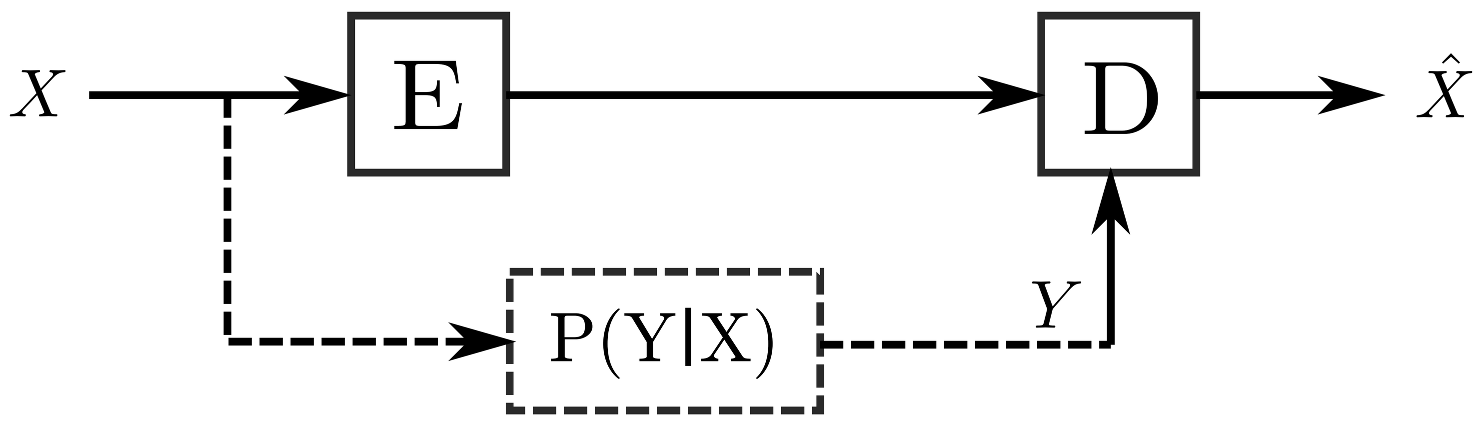
Lossy data compression in networks of sensors.



$P(X,Y)$ may be difficult to obtain

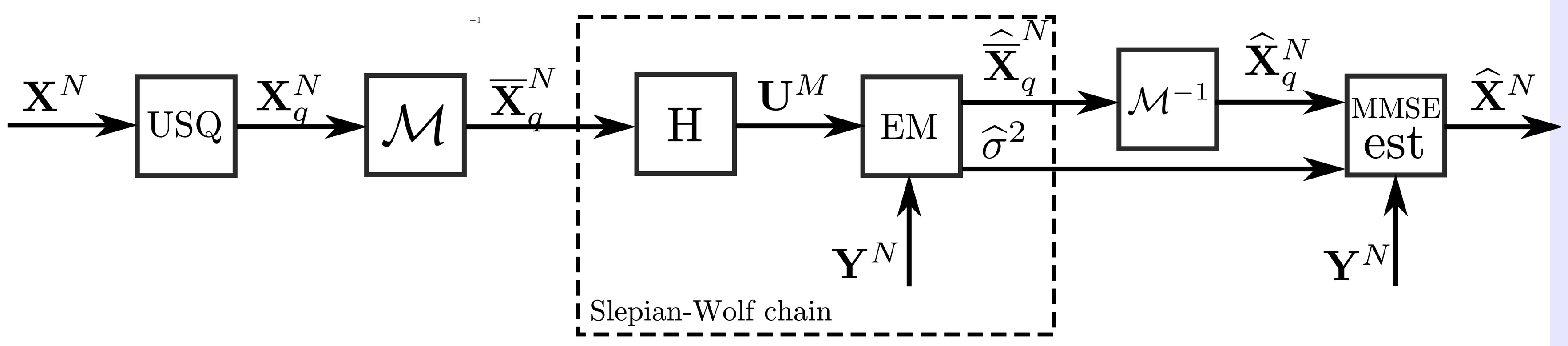
Signal Model

Source X and side information Y, with $Y = X + Z$.
$X \sim \mathcal{N}(0, \sigma_x^2)$,



$Z \sim \mathcal{N}(0, \sigma^2)$: correlation noise, σ^2 unknown but fixed for the sequence $\{(X_n, Y_n)\}_{n=1}^{+\infty}$, $\sigma^2 \in I_{\sigma^2}$

Universal Wyner-Ziv scheme



Quantization

USQ: Uniform Scalar Quantization, $q = 2^l$ levels, step Δ, $\mathcal{M}$: mapping to GF(q),

$$X = X_\mathrm{q} + B_\mathrm{q},\ B_\mathrm{q} \sim \mathcal{N}(0, \Delta^2/12)$$

Slepian-Wolf chain (lossless)

H: Coding of $\overline{\mathbf{X}}_q^n$ with LDPC code in GF(q)

$$\mathbf{U}^m = H^T \overline{\mathbf{X}}_q^n$$

Standard sum-product LDPC decoder requires σ^2
EM: for joint estimation of $\overline{\mathbf{X}}_q^n$ and σ^2

MMSE estimation

$$\widehat{x}_n = \alpha y_n + \beta x_{\mathrm{q},n}$$

where $\alpha = \left(\frac{\sigma^2}{\Delta^2/12} + \frac{\sigma_x^2+\sigma^2}{\sigma_x^2}\right)^{-1}$, $\beta = \frac{\sigma^2}{\Delta^2/12}\left(\frac{\sigma^2}{\Delta^2/12} + \frac{\sigma_x^2+\sigma^2}{\sigma_x^2}\right)^{-1}$.

EM algorithm

Initialization

First raw estimate $\widehat{\sigma}^{2,(0)}$ from $\mathbf{u}^m$ and $\mathbf{y}^n$ without using the LDPC decoder
Assumption: The U_m are independent.

$$\begin{aligned} L(\sigma^2) &= \log P(\mathbf{u}|\mathbf{y}, \sigma^2) + \log P(\mathbf{y}|\sigma^2) \\ &= \sum_{m=1}^{M} \log P(u_m | \{y_j^{(m)}\}_{j=1}^{\deg(m)}) + \sum_{n=1}^{N} \log P(y_n|\sigma^2) \ . \end{aligned}$$

$$P(u_m | \{y_j^{(m)}\}_{j=1}^{\deg(m)}) = \mathcal{F}_{u_m}^{-1}\Big(\prod_{j=1}^{\deg(m)} \mathcal{F}(W[h_j^{(m)}]\mathbf{p}_j)\Big)$$

($\mathcal{F}$: Fourier Transform, $h_j^{(m)}$: coefficients of H)
and

$$\widehat{\sigma}^{2,(0)} = \arg \max_{\sigma^2 \in I_{\sigma^2}} L(\sigma^2) \ .$$

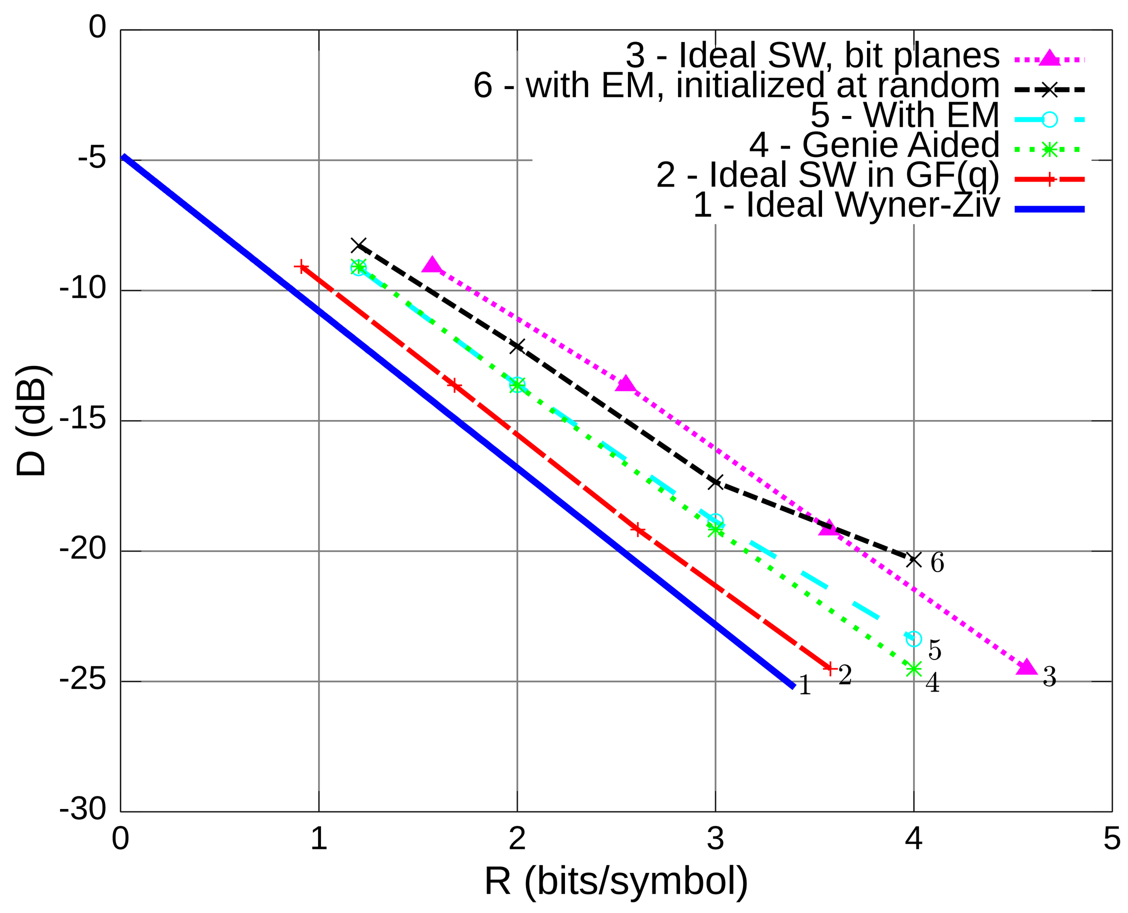
Iteration ℓ

$$\widehat{\sigma}^{2,(\ell+1)} = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=0}^{q-1} P_{k,n}^{(\ell)} (y_n - q_k)^2$$

where $P_{k,n}^{(\ell)} = P(X_{q,n} = k | y_n, \mathbf{u}, \widehat{\sigma}^{2,(\ell)})$, obtained from the LDPC decoder initialized with $\widehat{\sigma}^{2,(\ell)}$ and

$$\widehat{x}_n^{(\ell+1)} = \arg \max_{k \in \mathrm{GF}(q)} P_{k,n}^{(\ell+1)} \ .$$

Simulation results



$\sigma_x^2 = 1$, $N = 1000$, $\sigma^2 = 0.5$, $I_{\sigma^2} = [\underline{\sigma}^2, \overline{\sigma}^2] = [0, 0.55]$, scalar quantization over $[-3\sigma_x, 3\sigma_x]$, $q = 4, 8, 16$ and 32.

Conclusion and perspectives

- Universal coding scheme for the Wyner-Ziv coding of Gaussian sources with unknown variance of the correlation channel
- Construction of nested quantizer and design of good non-binary LDPC codes for Slepian-Wolf coding
- Generalization to other signal models, such as Laplacian, or Bernoulli-Gaussian.