A New Feedback Method for Dynamic Control of Manipulators

A new approach to the dynamic control of manipulators is developed from the viewpoint of mechanics. It is first shown that a linear feedback of generalized coordinates and their derivatives are effective for motion control in the large. Next, we propose a method for task-oriented coordinate control which can be easily implemented by a micro-computer and is suited to sensor feedback control. The proposed method is applicable even when holonomic constraints are added to the system. Effectiveness of the proposed method is verified by computer simulation.

1 Introduction

When we control the global motion of general manipulators, we are confronted with their nonlinear dynamics in many degrees of freedom. In much of the literature concerned with the dynamic control of manipulators, the complexity of nonlinear dynamics is emphasized and various methods that compensate all nonlinear terms in dynamics in real time are developed in order to reduce the complexity of control systems [1–3]. However, these methods require a large amount of complicated calculation so that it is difficult to implement these methods with low level controllers such as micro-computers. In addition, the reliability of these methods may be lost when a small error in computation or a small change in system’s parameters occurs, since these are not considered in the control. For most industrial robots, each joint of manipulator is independently controlled by simple linear feedback [4]. However, convergence to a target position has not been sufficiently investigated for general nonlinear mechanical systems.

In this paper, we develop a new approach to the motion control of mechanical manipulators. First, we propose a simple linear state feedback control for general mechanical systems from the viewpoint of mechanics and show that if we regard a generalized force as a control input then a linear feedback control makes the system attain to any configuration. It is also pointed out that this method is optimal in some sense.

Next, these results are extended to the case of task-oriented coordinate control which has been evolved by several authors [2, 5]. In conventional methods the data for the position described by task-oriented coordinates are transformed to a set of values for joint angles in the computer and these data are sent to each servo-system of the corresponding joint. Since in general this transformation contains an intricate nonlinear programming problem, it is very difficult to carry out the computation in real time. “Resolved Motion Rate Control” is applicable as a linear approximation method for this transformation [5], but it requires an inversion of Jacobian matrix whose calculation is quite troublesome in general.

In the proposed control system, the manipulator is directly controlled in the task-oriented coordinate space without the transformation and the stability of the system is easily assured. Moreover, it is shown that the proposed method is still applicable even when singularity or redundancy occurs in the relation between task-oriented coordinates and manipulator’s coordinates (joint angles) and some constraints which often appear in manipulator’s tasks are added to the system. These advantages of the method show its robustness for the change of environment. The proposed control law includes a computation of the Jacobian matrix which requires the longest time in this method. However, since appropriate real time algorithms for calculation of the Jacobian matrix have been developed in the literature [5], the proposed method can be easily implemented by a micro-computer. Effectiveness of the proposed method is verified by computer simulation.

2 Global Position Control in the Configuration Space

We consider a general mechanical system with \( n \) degrees of freedom. The motion of the system is described by generalized coordinates \( \mathbf{x} = (x_1, \ldots, x_n)^T \). In the case of manipulators, the variable \( x_i \) represents the angle of the \( i \)th joint. The kinetic energy \( T \) of the system is written as

\[
T = \frac{1}{2} \dot{\mathbf{x}}^T R(\mathbf{x}) \dot{\mathbf{x}}
\]

where the matrix \( R(\mathbf{x}) \) is symmetric and positive definite for all \( \mathbf{x} \) [6]. The generalized momentum is defined as [6]

\[
\mathbf{p} = \left( \frac{\partial T}{\partial x_1}, \ldots, \frac{\partial T}{\partial x_n} \right)^T = \left( \frac{\partial T}{\partial x} \right)^T = R(\mathbf{x}) \dot{\mathbf{x}}
\]

It is assumed that the potential function \( V(\mathbf{x}) \) of the system is twice differentiable with respect to \( \mathbf{x} \) and any entry of the Hessian of \( V(\mathbf{x}) \), \( \frac{\partial^2 V}{\partial x_i \partial x_j} \), is bounded for all \( \mathbf{x} \). This assumption is realized for general manipulators. The Hamiltonian \( H \) is expressed as \( H = T + V(\mathbf{x}) \) and the equation of motion is written as [6]

\[
\dot{\mathbf{x}} = \left( \frac{\partial H}{\partial \mathbf{p}} \right)^T = R^{-1}(\mathbf{x}) \mathbf{p}
\]
\[ \dot{p} = -\left( \frac{\partial H}{\partial x} \right)^T + Bu = -\left( \frac{\partial V}{\partial x} \right)^T + Bu \]  

(4)

where \( u = (u_1, \ldots, u_n)^T \) is a vector of generalized forces corresponding to \( x \) and \( B \) is a nonsingular \( n \times n \) matrix. Without loss of generality we assume \( B = I \) (identity matrix).

In this section, we consider a feedback stabilization which steers an arbitrary point in the configuration space, the space of variable \( x \), to the target point \( \tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)^T \). From the viewpoint of mechanics, position \( \tilde{x} \) is asymptotically stable, if the potential function of the system has a minimum at \( x = \tilde{x} \) and the system is completely damped in the sense that its positive definite dissipation function \[ [6] \]

is minimized. This is easily shown from well-known Lyapunov-Bellman’s equation of optimality \[ [8] \]. In fact, let us consider again the system equations (8) and (9), and define the functions \( h(x, w) \) and \( \lambda(t) \) as

\[ h(x, w) = \frac{1}{2} \left( x^T Q x + w^T Q^{-1} w \right) . \]

\[ \lambda(t) = \int_0^\infty h(x(\tau), w(\tau))d\tau \]

(12)

Then Lyapunov-Bellman’s equation (sufficient condition for optimality) can be written as

\[ \frac{\partial \lambda}{\partial t} = \min_w \left\{ h(x, w) + \left( \frac{\partial \lambda}{\partial x} , \frac{\partial \lambda}{\partial p} \right)^T \left( \frac{\partial h}{\partial x} \right) \right\} \]

\[ = \min_w \left\{ h(x, w) + \left( \frac{\partial \lambda}{\partial x} \right)^T \left( \frac{\partial h}{\partial x} \right) + \left( \frac{\partial \lambda}{\partial p} \right) w \right\} \]

(13)

The right-hand side of this can be written as

\[ \min_w \left\{ \frac{1}{2} w^T Q^{-1} w + \left( \frac{\partial \lambda}{\partial p} \right)^T w + g(\lambda, x, p) \right\} \]

\[ = \min_w \left\{ \frac{1}{2} \left( w^T Q^{-1} w + \left( \frac{\partial \lambda}{\partial p} \right)^T w + g(\lambda, x, p) \right) \right\} \]

(14)

\[ w^* = -Q^{-1} \frac{\partial \lambda}{\partial p} \]

(15)

To determine the optimal control \( w^* \), it is necessary to find the function \( \lambda \) which satisfies equation (13) \[ [8] \]. However, it is easily seen that \( \lambda = H \) satisfies equation (13). Therefore \( w^* \) is obtained from equation (15) as

\[ w^* = -Q^{-1} \frac{\partial H}{\partial p} \]

(16)

The performance index P.L. is related to smoothness of the motion, and in view of this, the linear velocity feedback \( -Qx \) is considered to be sufficiently effective as an optimal damping force.

Several types of functions are considered as a desired potential function \( V^0(x) \). Among these, we propose the simplest function described by

\[ V^0(x) = \frac{1}{2} (x^T W (x - \tilde{x})) \]

(17)

where the matrix \( W \) is \( n \times n \) symmetric and positive definite. This potential function is realized by linear position feedback plus constant bias, that is

\[ \left( \frac{\partial V^0}{\partial x} \right)^T = \left( \frac{\partial V}{\partial x} \right)^T (x) - W(x - \tilde{x}) \]

(18)
The positive definiteness of $V^0(x)$ is satisfied, if $V^0(x)$ is a convex function. To show this, we obtain, by differentiating $V^0(x)$ of equation (17) with respect to $x$,

$$
\left( \frac{\partial V^0}{\partial x} \right)^T = \left( \frac{\partial V}{\partial x} \right)^T (x) - \left( \frac{\partial V}{\partial x} \right)^T (\bar{x}) + W(x - \bar{x}) \tag{19}
$$

and

$$
V^0(\bar{x}) = 0, \quad \left( \frac{\partial V^0}{\partial x} \right)^T (\bar{x}) = 0.
$$

Furthermore, differentiating the equation (19) yields

$$
\left( \frac{\partial^2 V^0}{\partial x \partial x_i} \right)(x) = \left( \frac{\partial^2 V}{\partial x \partial x_i} \right)(x) + W
$$

Since each $\left( \frac{\partial^2 V}{\partial x \partial x_i} \right)(x)$ is assumed to be bounded, we can choose $W$ so that it satisfies

$$
\left( \frac{\partial^2 V}{\partial x \partial x_i} \right)(x) + W > 0 \tag{20}
$$

for all $x$. If we choose $W$ in this way, $V^0(x)$ becomes a strictly convex function due to (19) and (21), and has a global minimum $V^0(x) = 0$ at $x = \bar{x}$. Therefore, the position $(\bar{x}^T, O^T)$ is asymptotically stabilized in the large by linear state feedback (plus constant bias) $u$ described by

$$
u = \left( \frac{\partial V}{\partial x} \right)^T (\bar{x}) - W(x - \bar{x}) - Qx \tag{22}
$$

In particular, we can choose gain matrices $W$ and $Q$ as diagonal. This implies that, in spite of nonlinear coupled nature of the dynamics, global position control can be performed by a simple linear feedback with fixed gains for controlling each joint independently. However, it is not easy to analyze the transient behavior of motion. Even if the system is asymptotically stable, joint angles may make more than one revolution. Moreover, we tacitly assumed that the configuration space is an $n$-dimensional Euclidian space, but the configuration space should be the $n$-dimensional torus and the result mentioned above is not valid in this case because the function $V^0(x)$ of equation (17) is not continuous on the whole torus. However, we can estimate a region in the whole torus. Thus the convex set

$$
\{ x \in \mathbb{R}^n \mid \delta x = J^{-1}(x) \delta y \}
$$

and from this relation we obtain a vector of generalized forces for $y$ as follows,

$$
\left\{ \nu_i \right\} = \left( \frac{\partial ^T}{\partial y} \right) \left( J^T \right)^{-1} \left( \frac{\partial }{\partial y} \right) u \tag{29}
$$

Then, the equation of motion for $y$ becomes

$$
\dot{y} = \left( \frac{\partial T}{\partial q} \right)^T \tag{30}
$$

$$
\dot{q} = - \left( \frac{\partial T}{\partial y} \right)^T - \left( \frac{\partial V}{\partial y} \right)^T (J^T)^{-1} u \tag{31}
$$

In equations (30) and (31), only the term of generalized forces is different from that of equations (3) and (4). If we input $u$ defined by

$$
u = \left( \frac{\partial V}{\partial x} \right)^T - J^T \left( W(y - \bar{y}) + Qy \right) \tag{32}
$$

into equation (31), the Hamiltonian $\mathcal{H}$ of this system becomes

$$
\mathcal{H} = T + \frac{1}{2} \left( y - \bar{y} \right)^T W (y - \bar{y}) \tag{33}
$$

Differentiating $\mathcal{H}$ with respect to time, we obtain

$$
\dot{\mathcal{H}} = -\dot{y}^T Q \dot{y} \tag{34}
$$

This relation shows that the position $(y^T, \dot{y}^T) = (\bar{y}^T, \bar{y}^T)$ is
asymptotically stable by a similar argument as stated in the
previous section. Also, it can be shown that the control \( w = -J^T Q y \) is optimal in the sense that the performance index
\[
\int_0^\infty \left( \frac{1}{2} (y^T Q y + u^T J^{-1} Q (J^T)^{-1} u) \right) dt
\]
is minimized.

It should be noted that \( y \) may be partly measured by some
appropriate sensors and partly calculated by the value of \( x \) in
computer.

If the Jacobian matrix \( J(x) \) is singular at the target position
or \( m < n \), i.e., the system has redundancy for the task, the
equation of motion for \( y \) such as (30) and (31), cannot be
derived. In these cases, we introduce the slack variables for
convenience. To treat both cases simultaneously, assume that
the rank of Jacobian matrix \( J \) is \( m (\leq n) \), \( y_1, \ldots, y_m \)
are independent variables. The vector of slack variables \( y_s \) is
written as

\[
y_s = (y_{m+1}, \ldots, y_n)^T = f_s(x)
\]

which satisfies a condition,

\[
\det \left[ \frac{\partial f_s}{\partial x} \right] = \det [J_s] \neq 0 \text{ in } O
\]

where the vector \( f_s = (f^T, f_s^T) \) and matrix \( J_s \) is related with \( J \) as

\[
J_s = \begin{bmatrix}
    \vdots & \vdots \\
    \frac{\partial f_s}{\partial x} & \vdots
\end{bmatrix}
\]

Then the equation of motion for \( y_s^T = (y^T, y_s^T) \) is written as

\[
\frac{d}{dt} \begin{bmatrix}
y \\
y_s
\end{bmatrix} = \begin{bmatrix}
    \frac{\partial H}{\partial y} \\
    \frac{\partial H}{\partial y_s}
\end{bmatrix}^T + (J_s^T)^{-1} u
\]

(36)

(37)

where vector \( q_s \) is a generalized momentum for \( y_s \).
Now we consider an asymptotic stabilization with respect to \( y_s^T, q_s^T, q_s^T \).
In this case, the control \( u \) of equation (32) cannot damp
for the direction of \( y_s \) axis. Since we assume that \( y_s \) and \( y_s \)
are unknown vectors, the damping force such as \( -J_s^T Q y_s \)
cannot be realized. Therefore, we adopt \(-Q x \) as a damping
force instead of \(-J_s^T Q y_s\). and set the control \( u \) as

\[
u = \left( \frac{\partial V}{\partial x} \right)^T - J^T W (y - \hat{y}) - Q x
\]

(38)

where \( W \) is an \( m \times m \) positive definite matrix while \( Q \) is an
\( n \times n \) positive definite matrix. Since the term \(-Q x \) can be
written as \(- (J_s^T)^{-1} J_s^T y_s \), we can regard it as the
damping force which is derived from the dissipation function
\[\hat{y}_s^T (J_s^T)^{-1} Q (J_s^T)^T \hat{y}_s\]
in the space of \( y_s \). We consider the asymptotic stability of the
position by means of the linearized system of equations (36)
and (37). Substituting \( u \) of equation (38) into equation (37)
and linearizing equations (36) and (37) at \( (y^T, y_s^T, q^T, q_s^T) =
(y^T, y_s^T, O^T, O^T) \), we obtain

\[
\frac{d}{dt} \begin{bmatrix}
y \\
y_s \\
n \\
q_s
\end{bmatrix} = \begin{bmatrix}
    J_s R^{-1} J_s^T \\
    J_s R^{-1} J_s^T \\
    I \\
    I
\end{bmatrix} \begin{bmatrix}
y \\
y_s \\
n \\
q_s
\end{bmatrix} + \begin{bmatrix}
    \tilde{R}_{11} \\
    \tilde{R}_{12} \\
    0 \\
    0
\end{bmatrix} \begin{bmatrix}
y \\
y_s \\
n \\
q_s
\end{bmatrix}
\]

(39)

where \( (\tilde{y}_s^T, \tilde{y}_s^T) = (y^T - \hat{y}^T, y_s^T - \hat{y}_s^T) \), and \( \tilde{y}_s \) is an
appropriate vector. It may be easily seen that the vector \( \dot{y}_s \),
can be pulled out from the system of equations (39) and (40).
Consequently, we obtain the reduced system of equations (39)
and (40) as follows,

\[
\frac{d}{dt} \begin{bmatrix}
y \\
q_s
\end{bmatrix} = - \begin{bmatrix}
    W & 0 \\
    0 & 0
\end{bmatrix} \begin{bmatrix}
y \\
q_s
\end{bmatrix} - \begin{bmatrix}
    (J_s^T)^{-1} Q J_s^T \\
    (J_s^T)^{-1} Q J_s^T
\end{bmatrix} \begin{bmatrix}
y \\
q_s
\end{bmatrix}
\]

(40)

If we differentiate the function \( \tilde{H} \) of equation (33) with \( m \times m \)
matrix \( W \) along the solution trajectory of the system (46), we obtain

\[
\tilde{H} = -(q^T, q_s^T) J_s R^{-1} Q R^{-1} J_s^T \begin{bmatrix}
y \\
q_s
\end{bmatrix} = - \hat{y}_s^T (J_s^T)^{-1} Q J_s^T \hat{y}_s
\]

(42)

Since the set \( \{y_s^T, q_s^T, q_s^T\}; \tilde{H} = 0 \); i.e., \( \{q^T, q_s^T\} = O^T \) does
not contain the entire trajectory except \( \{\tilde{y}_s^T, q_s^T\} = O^T \),
we can conclude that the system (41) is asymptotically stable.

It should be noted that the vector \( y_s \) is introduced only for
convenience of the above argument. The vector \( y_s \) converges to
a certain position, with which we need not be concerned.

In this way, the control law of equation (38) is applicable for
position control even when the system has redundancy for
the task so far as we consider the linearized system (39) and
(40). This result also shows the validity of the feedback of
only part of the coordinates.

Though the asymptotic stability of the redundant system is
assured only for the linearized system in this section, global
asymptotic stability can be assured for the nonlinear system of
equations (36) and (37), provided that the boundedness of the
vector \( y_s \) can be assumed. It is rigorously proved by the result
of literature [10].

4 Constrained Dynamics

In many of a manipulator's tasks, the dynamics of the
system is attended by some constraints caused by a geometric
correspondence between manipulator and objects. Opening
a door, turning a crank, and some assembly tasks such as peg
in hole are given as such examples [11]. Since the structure of the
system is altered by these constraints, the stability of motion is
not always preserved [12]. Therefore we should design a
control system taking account of these constraints.

Let us consider the case that holonomic constraints are
added to the system in such a way that

\[
\Psi(x) = (\psi_1(x), \ldots, \psi_n(x))^T = O
\]

(43)

It is assumed that \( \psi_1(x), \ldots, \psi_n(x) \) are independent and
without loss of generality, \( x \) are partitioned as \( x^T = (x_1^T, x_2^T) \),
where \( x_1 \) is an \( n \times m \) dimensional vector of the independent part and
\( x_2 \) is an \( r \) dimensional vector of the dependent part. From the
equation (42), virtual displacement \( \delta x \) must satisfy

\[
\delta x = W \delta y
\]

(41)
\[
\frac{\partial \Psi}{\partial x} \delta x = \begin{pmatrix} \frac{\partial \Psi}{\partial x_1} & \ldots & \frac{\partial \Psi}{\partial x_D} \end{pmatrix} \delta x_D = 0 \quad (44)
\]

From the assumption, it follows that \( \det [\partial \Psi / \partial x_p] \neq 0 \) and \( x_D \) is determined by \( x_T \), i.e., \( x_D = x_D(x_T) \). We begin with calculating the term of generalized forces. Control \( u \) is partitioned as \( u^T = (u_f^T, u_D^T) \) corresponding to \( (x_f^T, x_D^T) \). From equation (44), \( \delta x_D \) is expressed by \( \delta x_I \) as
\[
\delta x_D = - \left( \frac{\partial \Psi}{\partial x_D} \right)^{-1} \left( \frac{\partial \Psi}{\partial x_I} \right) \delta x_I = D \delta x_I \quad (45)
\]
Calculating the virtual work for a virtual displacement of the form \( \delta x_I = \delta x' = (0, \ldots, \delta x_i, \ldots, 0)^T \) \((i = 1, \ldots, n-r)\), we obtain the generalized force vector \( u \), as
\[
u = u_f + D^T \delta x_I \quad (46)
\]
Then, the equation of motion with constraint (43) is derived by rewriting the Hamiltonian by only \( x_f \) as follows,
\[
\dot{x}_I = \left( \frac{\partial T_c}{\partial x_I} \right)^T \quad (47)
\]
\[
\dot{p}_I = - \left( \frac{\partial T_c}{\partial x_I} \right)^T - \left( \frac{\partial V_c}{\partial x_I} \right)^T + u_f + D^T(x_I)u_D \quad (48)
\]
where \( p_I \) is a generalized momentum for \( x_I, T_c, \) and \( V_c \) are defined as \( T(x, p) = T_c(x_I, p_I), V(x) = V_c(x_I) \). The term \( (\partial V_c / \partial x_I) \) is written as
\[
\left( \frac{\partial V_c}{\partial x_I} \right)^T = \left( \frac{\partial V_c}{\partial x_I} \right)^T + D^T \left( \frac{\partial V_c}{\partial x_D} \right)^T
\]
Therefore, to stabilize the position \( (x_f^T, p_f^T) = (\dot{x}_f^T, \dot{O})^T \) asymptotically, we set the control law as
\[
u = \begin{pmatrix} u_f \\ u_D \end{pmatrix} = \begin{pmatrix} \frac{\partial V_c}{\partial x_I} \\ (x_I - \dot{x}_I) \end{pmatrix} - \begin{pmatrix} W(x_I - \dot{x}_I) + Q \dot{x}_I \\ O \end{pmatrix} \quad (49)
\]
where \( W \) and \( Q \) are \((n-r) \times (n-r)\) positive definite matrices, respectively, and \( W \) satisfies
\[
\left( \frac{\partial^2 V_c}{\partial x_I^2} \right) + W > 0 \quad (50)
\]
Furthermore, we can easily see that the control law of equation (22) also assures the asymptotic stability of the position \( (x_f^T, O^T) \) in constrained case, if the target position \( \dot{x} \) satisfies the constraint condition (43). In fact, the potential function \( V_c^0 (x_I) = V_c^0 (x_I, x_D) \) and dissipation function \( x_f^T Q \dot{x}_I \) are positive definite as to \( x_I \) and \( x_I \), respectively. Here, the matrix \( Q \) is defined as
\[
\dot{Q} = (I, D^T) Q \begin{bmatrix} I \\ D \end{bmatrix} \quad (51)
\]
These results obviously hold for the case of task-oriented coordinate control system, i.e., the system of equation (30) and (31) and the control law (32) or (38). Moreover, combining the argument for the redundant case stated in the previous section, we can treat practically interesting tasks. For example, when we made the manipulator turn a crank as shown in Fig. 1, we should select \( r \) and \( \phi \) (in Fig. 1) as a coordinate for this task. To explain this in detail, suppose that the crank lies in the horizontal \((X - Y)\) plane (Fig. 1). If the hand grasps the handle of crank tightly, constraints \( r = \text{const} \) and \( Z = \text{const} \) are added to the system. Here, the variable \( Z \) indicates the vertical position of the hand. Therefore, to make the manipulator turn the crank, we must feedback only for \( \phi \) azimuth. If we use the control law of equation (38), \( y - \dot{y} = \phi - \dot{\phi}, W \) is scalar and \( J^T = (\partial \phi / \partial x)^T \) is an \( n \)-dimensional vector. Since the vector \((\phi, r)\) is related with \((X, Y)\) as
\[
\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} r \cos \phi & \sin \phi \\ r \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \phi \\ r \end{bmatrix} = J_1 \begin{bmatrix} \phi \\ r \end{bmatrix}
\]
we can easily obtain the relation
\[
\begin{bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial Z} \end{bmatrix} J_1 = J^{-1} J_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \cos \phi & -\sin \phi \\ r \sin \phi & r \cos \phi \end{bmatrix}
\]
and the Jacobian matrix \( J = (\partial \phi / \partial x) \) as follows,
\[
J = \begin{bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial Z} \end{bmatrix} = \begin{bmatrix} 1 & \cos \phi - r \sin \phi \\ 1 & r \sin \phi \end{bmatrix} \quad (51)
\]
Incrementing the value of \( \phi \) by degrees, the manipulator will turn the crank by degrees.

It should be noted that the information for the position of the center of rotation is not necessary to turn the crank by this control law. Moreover, the control law can be used when the value of \( r \) is unknown, since the value of \( r \) can be put into the term of feedback gain matrix \( W \).

5 Simulation Results

Effectiveness of the proposed control method was examined by computer simulation utilizing an arm model of Fig. 2. This arm has 4 degrees of freedom. Parameters of this arm are set in Table 1. We derived the equation of motion for this arm by Lagrange's method as follows,
Table 1 Parameters of manipulator

<table>
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<tr>
<th>Link</th>
<th>Mass (Kg)</th>
<th>$I_x$ (Kgm$^2$)</th>
<th>$I_y$ (Kgm$^2$)</th>
<th>$I_z$ (Kgm$^2$)</th>
<th>Length (m)</th>
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<td>0.04</td>
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<tr>
<td>4</td>
<td>1.0</td>
<td>0.01</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
x = R^{-1}(x)p
\]

\[
\dot{p}_i = \frac{1}{2} p^T R^{-1} R^{-1} R^{-1} p - \omega_i \mathbf{g} + u_i \quad (i = 1, \ldots, 4)
\]

Where

\[
x = (\theta_1, \theta_2, \theta_3, \theta_4)^T, \quad p = (p_1, p_2, p_3, p_4)^T
\]

The $R$ is a 4 x 4 symmetric and positive definite matrix whose elements $r_{ij}$ ($i, j = 1, \ldots, 4$) are written as

\[
r_{11} = (m_1 s_1^2 + m_2 l_2^2 + m_4 l_4^2 + I_3) (\sin \theta_2)^2
\]
\[
+ (m_1 l_3^2 + I_3) (\sin \theta_3)^2
\]
\[
+ (m_4 l_4^2 + I_4) (\sin \theta_4)^2
\]
\[
+ 2(m_1 l_1 s_3 + m_4 l_2 l_3) \sin \theta_2 \cdot \sin \theta_3
\]
\[
+ 2m_4 l_3 l_4 \sin \theta_3 \cdot \sin \theta_4
\]
\[
+ I_2 (\cos \theta_2)^2 + I_4 (\cos \theta_3)^2
\]
\[
+ I_4 (\cos \theta_4)^2 + I_1
\]
\[
r_{23} = (m_1 l_2 s_2 + m_4 l_2 l_4) \cos (\theta_2 - \theta_3) + I_3 + I_4
\]
\[
r_{24} = m_4 l_3 s_4 \cos (\theta_2 - \theta_4) + I_4,
\]
\[
r_{34} = m_4 l_4 s_4 \cos (\theta_3 - \theta_4) + I_4
\]
\[
r_{12} = r_{13} = r_{14} = 0
\]
\[
r_{22} = m_2 s_2^2 + (m_1 + m_2) l_2^2 + I_2 + I_3 + I_4
\]
\[
r_{33} = (m_3 l_3^2 + m_4 l_4^2 + I_3 + I_4 + r_{44}
\]
\[
= m_4 s_4^2 + I_4, \quad r_{ij} = r_{ji}
\]

where $m_i$ is the mass of the $i$th link and $l_i$ is the length from the $i$th joint to the $(i+1)$th joint, and $s_i$ is the length from the $i$th joint to the center of mass of the $i$th link. The $s_i$ is set as $s_1 = l_1/2$. $I_{ia}$ is the moment of inertia for $A$ axis of the $i$th link ($A = x, y, z$). The matrix $R^T$ is defined as $R^T = (\partial R/\partial \theta_j)$ ($i = 1, \ldots, 4$). Coefficients $g_i$ ($i = 1, \ldots, 4$) are

\[
g_1 = 0, \quad g_2 = (m_2 s_2 + m_3 l_3 + m_4 l_4) g, \quad g_3 = (m_3 s_3 + m_4 l_4) g, \quad g_4 = m_4 s_4 g
\]

where $g$ is a gravitational acceleration and is set as $g = 9.8 \text{ m/s}^2$. These feedback gains were chosen after several trials. Vector $y$ converges to the target smoothly in about 2 seconds. In this case, control $u$ is large at the beginning. If we set a trajectory which joins the initial position and the final position and change the intermediate target position $y$ along this trajectory, then the initial large magnitude of $u$ can be reduced. In Fig. 4

\[
\Delta x = (x, y, z, \theta_4). \quad \text{In Fig. 3 and Fig. 4 an initial position of joint angles is set as}
\]
\[
(\theta_1, \theta_2, \theta_3, \theta_4) = (0, -0.3, 1.2, 1.57) \text{ (rad)}
\]

A target position $\hat{y}$ is given as $\hat{y} = y(0) + \Delta y$. In Fig. 3 $\Delta y$ is set as $\Delta y = (0.4 \text{ m}, 0.3 \text{ m}, 0.2 \text{ m}, 0 \text{ rad})$. Feedback gain matrices $W$ and $Q$ of equation (32) are set as

\[
W = \text{diag}\{30, 30, 30, 5\}, \quad Q = \text{diag}\{15, 20, 15, 5\}
\]

These feedback gains were chosen after several trials. Vector $y$ converges to the target smoothly in about 2 seconds. In this case, control $u$ is large at the beginning. If we set a trajectory which joins the initial position and the final position and change the intermediate target position $y$ along this trajectory, then the initial large magnitude of $u$ can be reduced. In Fig. 4
fixed feedback gains as shown in sections 2 and 3, several trials for selecting gains are necessary in order to obtain better response. However, so far as these simulation results show, it seems that a single set of gains which shows a good response to a given standard task can be used successfully in a large region of work space.

6 Conclusion and Discussion

A theoretical consideration of the motion control of manipulators has been presented. Simple and useful control methods for nonlinear dynamics of general mechanical systems have been proposed from the view point of mechanics. This approach has shown its power in the task-oriented coordinate control and in the case of constrained dynamics. The proposed task-oriented coordinate control method seems suitable to sensor feedback control because the real data from each sensor can be directly used as a feedback input.

The change of environment around the manipulator may often produce mechanical constraints which may be described with much uncertainty. In many cases, the singularity of the Jacobian matrix is not avoidable in a wide range of motion and influenced by the constraints. In these circumstances, a feedback control of only part of coordinates which are attainable under the constraints seem to be an efficient method. In fact, our approach verifies the validity of this method.

In this paper, only the most basic studies on the motion control of general manipulators are carried out and improvement of response of the motion have not been sufficiently discussed. To obtain a highly refined motion, we should tune the feedback gains taking into account the matrix $R(x)$. Nevertheless, the simulation results show that a desirable motion can be obtained by choosing the gain matrices $W$ and $Q$ appropriately. More advanced control such as tracking a trajectory whose acceleration can not be neglected are under study for the system of task-oriented coordinate.

References


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