Average passivity for discrete-time and sampled-data linear systems

Fernando Tiefensee, Salvatore Monaco and Dorothée Normand-Cyrot

Abstract — Average passivity concepts are proposed for linear time-invariant discrete-time as well as sampled-data dynamics to remove the direct input-output link obstacle. Necessary and sufficient conditions for average passivity are given in terms of LMI’s. Direct discrete-time stabilizing and damping control strategies are proposed for average passive systems or Lyapunov stable dynamics. Linear port-controlled Hamiltonian - PCH systems are treated as an example to illustrate the stabilizing and damping performances through a simulated example.

I. INTRODUCTION

The notion of passivity inspired by electrical energy dissipation through resistors in circuits theory, is nowadays a recognized concept for systems stabilization at large (see [21], [6], [2], [3], [15]). While passivity is well defined for continuous-time systems, it is not so in the discrete-time case, where usual passivity definition requires a direct input/output link (D ≠ 0). It results that any continuous-time passive system without direct link loses such a property under sampling.

This problem motivates several approaches aiming to ensure passivity in discrete-time or sampled-data contexts [1], [9], [14]. More recently in [4], it was shown that passivity can be preserved under sampling with respect to some suitably defined sampled-data output; the same concept was generalized to nonlinear systems in [11] and the definition of average passivity has been introduced for discrete-time and sampled-data nonlinear systems. Using this new definition, in [12] it was shown that passivity and Hamiltonian conservation can be preserved by nonlinear Port Controlled Hamiltonian (PCH) systems under sampling. In [18], [19], [13], feedback controllers which stabilize nonlinear systems by damping improvement are inspired by the nonlinear average passivity definition.

In this paper we specialize those concepts to discrete-time linear time-invariant (LTI) systems. Average passivity conditions in the usual format of linear matrix inequalities are set in Section II where stabilizing and damping controllers are also proposed. Section III deals with passivity under sampling; we show that the sampled system is average passive with respect to a modified output and possibly with respect to the output itself. The resulting stabilizing and damping controllers are given. The example of a PCH system is treated in Section IV; it is shown that Hamiltonian conservativity is preserved under sampling and a direct sampled-data damping controller is described. Simulation results are discussed.

II. DISCRETE-TIME SYSTEMS

A. Preliminaries and problem settlement

Consider a discrete-time LTI system $\Sigma_d$

$$x_{k+1} = A_dx_k + B_d u_k$$
$$y_k = C_dx_k + D_d u_k$$

with $A_d$, $B_d$, $C_d$ and $D_d$ matrices of appropriate dimensions, $x_k \in \mathbb{R}^n$, $y_k$ and $u_k = [u_{k1} \ldots u_{km}]^T$.

Definition 2.1: $\Sigma_d$ is passive with quadratic storage function $V = \frac{1}{2}x^TPx$, $P = P^T > 0$ if for all $x_k$ and $u_k$ the stored energy does not exceed the supply rate, $s(y,u)$, defined as the inner product $u_T^k y_k$; i.e.

$$V(x_{k+1}) - V(x_k) \leq u_T^k y_k$$

where $T$ indicates the matrix transposition.

Equivalently, $\Sigma_d$ is passive if and only there exists a matrix $P = P^T > 0$ verifying the linear matrix inequality (LMI) below

$$\begin{bmatrix}
A_d^T P A_d - P & A_d^T P B_d - C_d^T \\
B_d^T P A_d - C_d & B_d^T P B_d - (D_d + D_d^T)
\end{bmatrix} \leq 0.$$

LMI’s solvability conditions are discussed in [7], and can be interpreted as the linear discrete-time version of Kalman-Yakubovich-Popov (KYP) properties.

Proposition 2.1: [7] $\Sigma_d$ is passive with quadratic storage $V(x) = \frac{1}{2}x^TPx$ if and only if there exist real matrices $P = P^T > 0$, $L$ and $W$ s.t.

(i) $A_d^T P A_d - P = -L^T L$  \hspace{1cm} (4)

(ii) $A_d^T P B_d - C_d^T = -LW$  \hspace{1cm} (5)

(iii) $B_d^T P B_d - (D_d + D_d^T) = -W^T W$.  \hspace{1cm} (6)

From (4), one deduces that $\Sigma_d$ is passive only if there are no eigenvalues outside the unitary disk of the complex plane and those on the boundary are simple ones (stability). From (6), one concludes that $D_d$ cannot be equal to zero so that any discrete-time LTI system without direct input/output link cannot be passive according to Definition 2.1.

Remark 2.1: To avoid the necessity of a direct input-output link, dissipativity can be related to different supply rates set as $s(y,u) = y^T Q y + 2 u^T S y + u^T R u$, with $Q = Q^T$, $R = R^T$ and $S$, matrices of appropriate dimensions [14].
B. Average passivity

Following [11] where a new concept of average passivity is proposed for nonlinear discrete-time systems adopting the Differential/Difference Representation - DDR [10], one sets equivalently to (1)

\[ x^+: = A_d x; \quad \frac{dx^+(u)}{du} : = B_d \quad \text{with} \quad x^+(0) = x^+. \]  

More precisely, \( x^+(u) \) represents a curve in \( \mathbb{R}^n \) parameterized by \( u \) and (1) is described as two differential and difference equations modeling respectively the free and controlled evolutions. From \( x_k \), one computes \( x_k^+(0) = A_d x_k \) and one integrates the differential equation between 0 and \( u_k \) to recover (1); i.e.

\[ x_k^+(u_k) := x_{k+1} = A_d x_k + \int_0^{u_k} B_d dv = A_d x_k + B_d u_k. \]  

In such a framework, it is natural to associate with (7) the output \( y^+(x_k, u) = C_d x_k^+(u) \) and to define the \( u \)-average output as follows

\[ y_{av}^+(x_k, u_k) := \sum_{i=1}^{m} \frac{1}{u_i} \int_0^{u_i} y^+(x_k, v) dv_i \]

\[ = \sum_{i=1}^{m} \frac{1}{u_i} \int_0^{u_i} C_d x_k^+(v) dv_i \]

\[ = C_d A_d x_k + C_d B_d \frac{u_k}{2}. \]

Accordingly, one defines average dissipativity.

Definition 2.2: The discrete-time LTI system \( \Sigma_d \) without direct input/output link \( (D_d = 0) \), is said average passive with quadratic storage \( V(x) = \frac{1}{2} x^T P x \ (P = P^T > 0) \) if for all \( x_k \) and \( u_k \), one verifies the inequality

\[ V(x_{k+1}) - V(x_k) \leq u_k^T y_{av}^+(x_k, u_k) \]

where the \( u \)-average output \( y_{av}^+(x_k, u_k) \) is defined in (9), \( \Sigma_d \) is said average lossless when \( V(x_{k+1}) - V(x_k) = u_k^T y_{av}^+(x_k, u_k) \).

Rewriting (10) in terms of the LMI

\[ \begin{bmatrix} A_d^T P A_d - P & A_d^T (P B_d - C_d^T) \\ (B_d^T P - C_d) A_d & (B_d^T P - C_d) B_d \end{bmatrix} \leq 0 \]

necessary and sufficient conditions are easily deduced.

Proposition 2.2: -Average passivity without direct input-output link- The discrete-time LTI system \( \Sigma_d \), with \( (D_d = 0) \) is average passive with quadratic storage \( V(x) = \frac{1}{2} x^T P x \) if and only if there exist real matrices \( P = P^T > 0, L \) and \( W \) verifying

\[ \begin{align*}
(i) & \quad A_d^T P A_d - P = -L^T L \\
(ii) & \quad A_d^T (P B_d - C_d^T) = -LW \\
(iii) & \quad (B_d^T P - C_d) B_d = -W^T W.
\end{align*} \]

\( \Sigma_d \) is average lossless iff \( (i)-(iii) \) hold true with \( L = W = 0 \).

Remark 2.2: Comparing Proposition 2.1 and Proposition 2.2, we note that average passivity is made possible provided \( C_d B_d \) is different from zero.

Assuming \( D_d \neq 0 \) in \( \Sigma_d \), the \( u \)-average output is computed according to

\[ y_{av}^+(x_k, u_k) := \sum_{i=1}^{m} \frac{1}{u_i} \int_0^{u_i} (C_d(x^+(v_i)) + D_d v) dv_i = C_d A_d x_k + \frac{1}{2} (C_d B_d + D_d) u_k. \]

Proposition 2.3: -Average passivity with direct input-output link- The discrete-time LTI system \( \Sigma_d \), with a direct input/output link \( (D_d \neq 0) \) is average passive with quadratic storage function \( V(x) = \frac{1}{2} x^T P x \) if and only if there exists a real matrix \( P = P^T > 0 \) verifying the LMI:

\[ \begin{bmatrix} A_d^T P A_d - P & A_d^T (P B_d - C_d^T) \\ (B_d^T P - C_d) A_d & (B_d^T P - C_d) B_d - D_d - C_d B_d \end{bmatrix} \leq 0. \]

The solvability of such a LMI is conditioned to the existence of real matrices \( P = P^T > 0, L \) and \( W \) verifying (11), (12) and

\[ (i^D) \quad B_d^T P D_d - D_d - C_d B_d = -W^T W. \]

An analogy between the sampled-data time-average output proposed in [4] (which preserve passivity under sampling) and the discrete-time \( u \)-average output \( y_{av}^+ \), described in (9), can be set in the sense that the first one is calculated by a temporal averaging of the continuous-time output during one sampled period, while the second one is computed by an output average with respect to the control input through one step (in discrete-time the output evolution stay constant during one step and thus a temporal average makes no sense).

C. Average-passivity based control

The design of stabilizing control laws based on passivity criteria is well known. Roughly speaking, if \( \Sigma_d \) is detectable (i.e. the non-observable subsystem is asymptotically stable) and \( u_k \rightarrow y_k \) passive according to the usual definition, then the negative output control law \( u_k = -K y_k \) with \( K > 0 \) ensures \( V(x_{k+1}) - V(x_k) \leq -K^2 x_k^2 \) for all \( x_k \neq 0 \) and asymptotic stabilization is guaranteed by the Lasalle’s principle (see [6] and [2]). The goal of this section is to describe stabilizing controllers for systems without direct input-output link assumed average passive.

Theorem 2.1: Given the discrete-time LTI system \( \Sigma_d \) without direct input/output link, suppose it is average passive with quadratic storage \( \frac{1}{2} x^T P x \ (P = P^T > 0) \) and the output \( y^+(x,0) = C_d A_d x \) detectable, then the discrete-time feedback

\[ u_k = -K \left( I + \frac{K}{2} C_d B_d \right)^{-1} C_d A_d x_k, \]

with \( K > 0 \), asymptotically stabilizes \( \Sigma_d \).

Proof: It is a matter of computations to show that \( u_k \) in (15), computed according to \( u_k = -K y_{av}^+(x_k, u_k) \) satisfies

\[ (K + 1) - V(x_k) \leq -K y_{av}^+(x_k, u_k) y_{av}^+(x_k, u_k) \leq 0. \]

Asymptotic stability follows from detectability and the Lasalle’s principle.
For a system $\Sigma_d$ with direct input-output link, the controller (15) generalizes as follows
\[
uk = -K \left( I + \frac{K}{2} (C_d B_d + D_d) \right)^{-1} C_d A_d x_k. \tag{16}
\]

We note that even though the so defined controllers can be formally seen as negative average output feedbacks, their computations require state measurements. Work is progressing to complete the design with observers schemes.

\[\text{III. SAMPLED-DATA SYSTEMS}\]

As well known, passivity is lost under sampling for systems without direct input-output link (see [3], [5], [11]). The solution proposed in [4] consists in looking forward in time to define a new output mapping with respect to which usual passivity can be preserved. Hereafter, we specialize the average passivity concepts introduced for purely discrete-time dynamics to sampled-data dynamics which inherit some properties of the continuous-time dynamics.

\[\text{A. PASSIVITY PROPERTIES UNDER SAMPLING}\]

Given a continuous-time LTI system $\Sigma_c$
\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) \tag{18} \\
y(t) &= C x(t) \tag{19}
\end{align*}
\]
with $A$, $B$ and $C$ matrices with appropriate dimensions, $\Sigma_c$ is said passive with quadratic storage $V = \frac{1}{2} x^T P x$, if there exists a real matrix $P = P^T > 0$ s.t.
\[
\dot{V}(x(t)) \leq u^T(t) y(t) \tag{20}
\]
for all $t \geq 0, u \in \mathbb{R}^m$, or equivalently if there exists a real matrix $P = P^T > 0$ verifying the continuous-time KYP-properties:
\[
\begin{align*}
(i) & \quad PA + A^T P \leq 0 \tag{21} \\
(ii) & \quad C = B^T P. \tag{22}
\end{align*}
\]

The equivalent sampled-data system is given by $\Sigma^\delta$
\[
\begin{align*}
x_{k+1} &= A^\delta x_k + B^\delta u_k \tag{23} \\
y_k &= C x_k \tag{24}
\end{align*}
\]
with $A^\delta := e^{\delta A}$, $B^\delta := \int_0^\delta e^{\tau A} B d \tau$, $\delta \in [0,T^\tau]$ a finite time interval so that the passivity criteria (3) is transformed into the LMI below
\[
\begin{bmatrix}
A^\delta^T P A^\delta - P & A^\delta^T P B^\delta - C^T \\
B^\delta^T P A^\delta - C & B^\delta^T P B^\delta
\end{bmatrix} \leq 0. \tag{25}
\]

Even though $\Sigma_c$ satisfies (20), $\Sigma^\delta$ cannot verify (25) because the term $B^\delta^T P B^\delta$ is strictly positive. Consequently, passivity is lost under sampling for any system without direct input-output link.

\[\text{B. SAMPLED-DATA \delta u-AVERAGE PASSIVITY}\]

We refer in this section to the DDR of the sampled-data dynamics, equivalent to a given continuous-time dynamics (18); i.e. from (23)
\[
x^\tau = A^\delta x; \quad \frac{d x^\tau(\delta \nu)}{d \delta \nu} = B^\delta \frac{d}{d \delta} \quad \text{with} \quad x^\tau(0) = x^\tau(0) \tag{26}
\]
Given a passive system $\Sigma_c$, is its sampled-data equivalent average passive? Unfortunately such property does not hold in general. However, as shown in Theorem 3.1 below, it turns out that average passivity holds with reference to the output
\[
y^\delta(x(\nu)) := B^\delta x^\tau(\nu). \tag{27}
\]

\[\text{Theorem 3.1: Let the continuous-time dynamics (18) assumed stable with Lyapunov function $V = \frac{1}{2} x^T P x$, $P = P^T > 0$, then its sampled-data equivalent dynamics with output}
\]
\[
y^\delta(x, u) = B^\delta^T P x^\tau(u) \tag{27}
\]
is average passive with quadratic storage $V$.

\[\text{Proof: Specializing (3) on (26) one has}
\]
\[
\begin{align*}
V(x_{k+1}) - V(x_k) &= \frac{1}{2} x^T_k \left( A^\delta^T P A^\delta - P \right) x_k \\
&\quad + u_k^T B^\delta^T P A^\delta x_k + \frac{1}{2} u_k^T B^\delta^T P B^\delta u_k. \tag{28}
\end{align*}
\]
The stability condition of the continuous-time dynamics
\((AT + PA \leq 0)\) ensures negativity of the first term on the
right hand side of (28) (A stable implies \(A^\delta\) stable); i.e.
\[
\frac{1}{2} \frac{d}{dt} \left( A^T P A^\delta - P \right) x_k = \frac{1}{2} \int_{0}^{\delta t} x(t)^T (AT + PA)x(t) dt \leq 0
\]
and thus
\[
V(x_{k+1}) - V(x_k) \leq u_k^T B^T P A^\delta x_k + \frac{1}{2} u_k^T B^T P B^\delta u_k
\]
so verifying average passivity of the link \(u \rightarrow B^T P x^+ (u)\); i.e.
\[
V(x_{k+1}) - V(x_k) \leq u_k^T y_{av}(x_k, u_k)
\]
(29)
with
\[
y_{av}(x_k, u_k) := \frac{1}{\delta t} \int_{0}^{\delta t} y_\delta(x, v) dv := C_{av}^\delta x_k + D_{av}^\delta u_k
\]
and \(C_{av}^\delta := B^T P A^\delta\), \(D_{av}^\delta := \frac{1}{2} B^T P B^\delta\).

On these bases the following result can be stated for
sampled-data stabilization with damping improvement.

**Theorem 3.2:** Given a continuous-time LTI dynamics, suppose it is stable with Lyapunov function \(V = \frac{1}{2} x^T P x\), \(P = P^T > 0\), let the output \(y_\delta(x, 0) = B^T P A^\delta x\) suppos detectable for its sampled-data equivalent dynamics, then the sampled-data control law
\[
u_k = u^\delta_k = -K(I + KD_{av}^\delta)^{-1} C_{av}^\delta x_k \quad \text{with} \quad K > 0 \quad (31)
\]
ensures asymptotical stabilization and damping improvement at
the sampling instants.

**Proof:** By construction, the controller (31) makes \(V(x_{k+1}) - V(x_k)\) not positive because of (29). Asymptotic stabilization follows from detectability of \(y_\delta(x, 0) = B^T P A^\delta x\).

IV. PORT-CONTROLLED HAMILTONIAN AS AN EXAMPLE

From average passivity concepts we illustrate in this sec-
tion how Hamiltonian conservativeness and Port Controlled
Hamiltonian - PCH passivity (see [20]) are preserved under
sampling. The \(\delta u\)-average output is characterized in both
cases.

A. LTI-PCH systems

A continuous-time PCH dynamics \(\mathcal{H}_c\), with a quadratic
Hamiltonian \(H(x) = \frac{1}{2} x^T Q x\), \(Q = Q^T > 0\) is defined by
\[
x(t) = (J - R)Qx(t) + Bu(t)
\]
with \(x \in \mathbb{R}^n\), \(u \in \mathbb{R}\). The interconnection matrix
\(J \in \mathbb{R}^{n \times n}\) is constant and skew-symmetric, i.e. \(J = -J^T\) and
\(J^T J x = 0\); the dissipativeness matrix \(R \in \mathbb{R}^{n \times n}\), is also constant,
positive definite and symmetric, i.e. \(R = R^T \geq 0\). \(B \in \mathbb{R}^{n \times m}\) is constant.

Adopting the Hamiltonian as system storage function and
specializing condition (21) on \(\mathcal{H}_c\), one verifies that its free
evolution is stable \((J - R)Q^2 + Q^T (J - R)^T = -2RQ^2 \leq 0\).

**Remark 4.1:** In [16], it is shown that any LTI stable (resp.
asymptotically stable) system can be rewritten in the form
of \(\mathcal{H}_c\), with output \(y_\delta = B^T Q x\), \(Q = Q^T > 0\), \(J = -J^T = \frac{1}{2} (AQ^{-1} - Q^{-1} A^T)\) and \(R = R^T = \frac{1}{2} (AQ^{-1} + Q^{-1} A^T)\) semi-
positive (resp. positive).

Under sampling \(\mathcal{H}_c\) can be rewritten in the DDR form (26)
with \(P = Q\), \(A^\delta = e^{\delta (J - R) Q}\) and \(B^\delta = \int_{0}^{\delta} e^{(J - R) Q} B dt\).

**Proposition 4.1:** Given a continuous time LTI-PCH system
with \(Q = Q^T > 0\) and \(R = R^T > 0\) (resp. \(R = 0\)), then for
all \(\delta \in ]0, T^*]\) its sampled-data equivalent is average passive
(resp. average lossless) with respect to
\[
y_{av}^\delta (x, u) = B^T P A^\delta x + \frac{1}{2} \delta u^T P B^\delta u \quad (32)
\]
The \(\delta u\)-average output is computed as follows:
\[
y_{av}^\delta (x, u) = B^T P A^\delta x + \frac{1}{2} \delta u^T P B^\delta u \quad (32)
\]
with
\[
y_{av}^\delta (x, u) = C_{av}^\delta x_k + D_{av}^\delta u_k \quad (30)
\]
and \(C_{av}^\delta := B^T P A^\delta\), \(D_{av}^\delta := \frac{1}{2} B^T P B^\delta\).

V. EXAMPLE - THE MASS SPRING SYSTEM

The performances of the proposed sampled-data stabilizing
controller by damping improvement is illustrated by the
mass-spring system. Simulations are worked out comparing
\(u_k\) in (31) with a sampled-data control law designed in
[4]. The studied system consists in three masses connected
through two springs modeled as a 6-dimensional PCH system
(see [16]):
\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3
\end{bmatrix} =
\begin{bmatrix}
-Q_1 & I_3 & -R_p \\
-I_3 & -R_p & 0 \\
-R_p & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\partial H/\partial q_1 \\
\partial H/\partial q_2 \\
\partial H/\partial q_3
\end{bmatrix} + \begin{bmatrix}
0 \\
B_p \\
0
\end{bmatrix} u
\]
(33)
where \(q = (q_1, q_2, q_3)^T\) and \(p = (p_1, p_2, p_3)^T\) denote the
generalized coordinates vector and the generalized momenta
vector with respect to \(m_1\), \(m_2\) and \(m_3\), respectively. \(I_3\) is the
3-dimensional identity matrix. Supposing that the system is
actuated by an horizontal force \(u\) applied on \(m_1\) and
that the output \(y\) is the displacement velocity of \(m_1\) on
the horizontal plan, one has \(B_p = [1 \ 0 \ 0]^T\). The system losses
are represented by the dissipation matrices \(R_q = R_q^T > 0\) and
\(R_p = R_p^T > 0\). The Hamiltonian function is defined by the
sum of kinetic and potential energies
\[
H = \frac{1}{2} \begin{bmatrix}
p_1^2 \\
p_2^2 \\
p_3^2
\end{bmatrix}
\begin{bmatrix}
m_1 \\
m_2 \\
m_3
\end{bmatrix}
+ \frac{1}{2} \begin{bmatrix}
(k_1(q_1 - q_2)^2 + k_2(q_2 - q_3)^2)
\end{bmatrix}
\]
with \(k_1, k_2 > 0\) the spring elastic constants, then \(H = \frac{1}{2} x^T Q x\)
with
\[
Q = \begin{bmatrix}
Q_1 & 0 & 0 \\
0 & Q_2 & 0 \\
0 & 0 & Q_3
\end{bmatrix}
\]
\[
M = \begin{bmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{bmatrix}
\]

It is easy to see that the system is passive since its free
evolution is stable, i.e.
\[
H(t) = - \begin{bmatrix}
\partial H/\partial q^T \\
\partial H/\partial q
\end{bmatrix} R_q \begin{bmatrix}
\partial H/\partial q \\
\partial H/\partial p
\end{bmatrix} R_p < 0
\]
A static-state feedback
\[
u_c(t) = -Ky(t) = -K \frac{p_1}{m_1}
\]
injects additional damping to the system and its convergence
speed is controlled by the gain \(K\).

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A. Sampled-data design

The sampled-data model \((A^\delta, B^\delta, C)\) is written in the form of (23)-(24), with
\[
A = \begin{bmatrix} R_p \kappa & M^{-1} \\ -\kappa & R_p M^{-1} \end{bmatrix}, \quad B^T = [0 \ B^T_p] \quad \text{and} \quad C = [0 \ B^T_p M^{-1}].
\]

Since the free-evolution of the system is stable, the sampled-data stabilizing control \(u^\delta_k\) in (31) asymptotically stabilizes the origin of (33)-(34).

B. Alternate sampled-data strategy

Another approach is presented in [4]. It is shown that the system (23)-(24) is passive under sampling with respect to the output
\[
y^*(x,u) = \frac{1}{\delta} \int_{k\delta}^{(k+1)\delta} y(\tau) d\tau = \frac{C^* x + D^* u}{\delta}
\]
with \(C^* = C \int_0^\delta e^{A \tau} d\tau\) and \(D^* = C \int_0^\delta B^2 d\tau\). Then, a stabilizing controller is given by
\[
\delta u^* = -K(I + KD^*)^{-1} C^* x_k \quad \text{with} \quad K > 0.
\]

C. Simulation Results

Figure 1(a)-1(b) compare the storage function and the output evolution of the closed-loop continuous-time system with the sampled-data ones under emulated \(u_{d0} = -K \frac{p_k}{m_k},\ u_k^\delta\) and \(u^*\) controllers for \(\delta = 150ms\). Even if the emulated control consists in an output feedback, its performances are degraded for increasing \(\delta\), since \(u_k \rightarrow y_k\) passivity of (33)-(34) is lost under sampling and \(u_{d0}\) ensures strict negativity of \(H(x_{k+1}) - H(x_k)\) only to approximations in the order \(\delta\) (with an error in \(O(\delta^2)\)). It results that the inter-sampling \(H\) evolution (figure 1(a)) presents increasing regions, some oscillations appear on the output \(y\) and in \(q\) trajectory (figures 1(b) and 1(c)); as a consequence, \(u_{d0}\) assumes high amplitudes to ensure suitable damping assignment (figure 1(d)). Even if the alternate solution \(u^*\) slightly improves the closed-loop system response, softly reducing control-input amplitudes and oscillations on \(y\) and \(q\), these improvements are not so convincing. In the other hand \(u_k^\delta\) ensures a good damping assignment under sampling reducing control amplitudes and oscillations on \(y\) and \(q\). \(u_k^\delta\) robustness with respect of \(\delta\) is put in light in figures 2(a)-2(d). In fact the damping performances are maintained for a \(\delta = 150ms\).

One concludes that in practical applications, when a sampled-data passivity based controller can be performed using small \(\delta\), the emulated solution can be implemented, since it consists in an output feedback and ensures acceptable performances. However, when larger sampling periods and higher control performances are required, the proposed sampled-data damping assignment (which consists in a state feedback) should be prioritized.

VI. CONCLUSION

Average passivity concepts are specified for discrete-time and sampled-data LTI systems. It is shown that using such a concept, the direct input/output link obstacle is removed and necessary and sufficient conditions of average passivity are set. Feedback control laws constructed from the \(u\)-average output and ensuring asymptotic stability through damping improvement are described. Their performances are illustrated and compared with a previous solution proposed in [4]. A port-controlled Hamiltonian example illustrates the results putting in light the controller robustness to larger sampling lengths.
Working is progressing and a interesting theoretical expansion should be the interpretation of average passivity concepts in a frequency domain. In a practical way, these concepts can be suitable applied to design feedback controllers to linearized models of VTOLS [8] and for the energetic management of a hybrid fuel cell/supercapacitor supply system [17].

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