ON THE MINIMUM TIME PROBLEM FOR DRIFTLESS LEFT-ININVARIANT CONTROL SYSTEMS ON $SO(3)$

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Abstract. In this paper, we investigate the structure of time-optimal trajectories for a driftless control system on $SO(3)$ of the type $\dot{x} = x(u_1 f_1 + u_2 f_2)$, $|u_1|, |u_2| \leq 1$, where $f_1, f_2 \in so(3)$ define two linearly independent left-invariant vector fields on $SO(3)$. We show that every time-optimal trajectory is a finite concatenation of at most five (bang or singular) arcs. More precisely, a time-optimal trajectory is, on the one hand, bang-bang with at most either two consecutive switchings relative to the same input or three switchings alternating between two inputs, or, on the other hand, a concatenation of at most two bangs followed by a singular arc and then two other bangs. We end up finding a finite number of three-parameters trajectory types that are sufficient for time-optimality.

1. Introduction and Statement of the Main Results. We consider the time-optimal problem of the control system $\Sigma$ given by:

$$\dot{x} = x(u_1 f_1 + u_2 f_2),$$  

where $x \in SO(3)$, $|u_i| \leq 1$ and $f_i \in so(3)$ give rise to left invariant vector fields (acting on the right) on $SO(3)$. The control system $\Sigma$ can be seen as a model for reorienting a satellite: the vector fields $f_i$ represent infinitesimal rotations of the satellite about two non collinear fixed axis and the inputs $u_i$ are scale multiples of the angular velocities of the rotors rotations. The problem consists of orienting the satellite from a given position to another one in minimum time.

The control system $\Sigma$ is given as $\left(\{SO(3), [-1, 1]^2, Ad(\Sigma), (f_1, f_2)\}\right)$ where $SO(3)$ is the state space, $[-1, 1]^2$ is the control set, $Ad(\Sigma)$ is the control space (defined next) and $f_1, f_2$ are $3 \times 3$ skew-symmetric matrices, defining the dynamics of $\Sigma$ by (1). The control space $Ad(\Sigma)$ is the set of admissible controls $u = (u_1, u_2)$ i.e. the set of measurable functions $u = (u_1, u_2): [a, b] \to [-1, 1]^2$, where $a, b$ depend on $u$ in general (see Jurdjevic [1] for a more general definition of control systems). A trajectory $\gamma$ of $\Sigma$ is an absolutely continuous curve $\gamma: J \to SO(3)$ with $J = [a, b]$.

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