Tilt estimator for 3D non-rigid pendulum based on a tri-axial accelerometer and gyrometer

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***Abstract*—The paper presents a new observer for tilt estimation of a 3-D non-rigid pendulum. The system can be seen as a multibody robot attached to the environment with a ball joint. There is no sensor for the joint position of the sensor. The estimation of tilt, i.e. roll and pitch angles, is mandatory for balance control for a humanoid robot and all tasks requiring verticality. Our method obtains tilt estimations using encoders on other joints and inertial measurements given by an IMU equipped with tri-axial accelerometer and gyrometer mounted in any body of the robot. The estimator takes profit from the kinematic coupling resulting from the pivot constraint and uses the entire signal of accelerometer including linear accelerations. Almost Global Asymptotic convergence of the estimation errors is proven together with local exponential stability. The performance of the proposed observer is illustrated by simulations.**

I. Introduction

One predominant goal of robotics is to be able to perform versatile tasks involving the application of contact forces on the environment. In some cases, these contact point constitute the only link between the floating base of the robot and the world reference frame. The most prominent example of these tasks is legged locomotion, but it may also include aerial or marine robots performing environment-related tasks such as torquing, drilling or maintaining position using an anchor fixed on the environment. Most of these tasks require the contact point to remain at a precise position and not to detach or slip. The observance of such a constraint generates a kinematic coupling allowing to model the robot as a kinematic chain attached to the environment with an unactuated 3D spherical joint that we will call pivot. This can be simply summarized as a pendulum with the contact as the 3D pivot point. This model is sometimes used to control the motion generation of the robot, such as for humanoid locomotion [**?**] or reaction wheel cube that balances on edges and corners [**?**].

One main issue regarding this class of systems is that beside the unactuation, there is usually no direct measurement of the configuration of this pivot. Of course, properly estimating this configuration is of crucial importance in most tasks. Nevertheless, several kinds of sensors are sensitive to this configuration, and may be used to estimate it. The most broadly used ones are tri-axial accelerometer and gyrometer. These sensors are usually cheap, compact and reliable. An accelerometer provides the sum of the gravitational field and the linear acceleration, and the gyrometer provides the angular velocity of the sensor, both of these measurements are expressed in the frame of the sensor. It is straightforward to notice that this set of sensors provides invariant signals relative to different rotations around the gravitational field direction

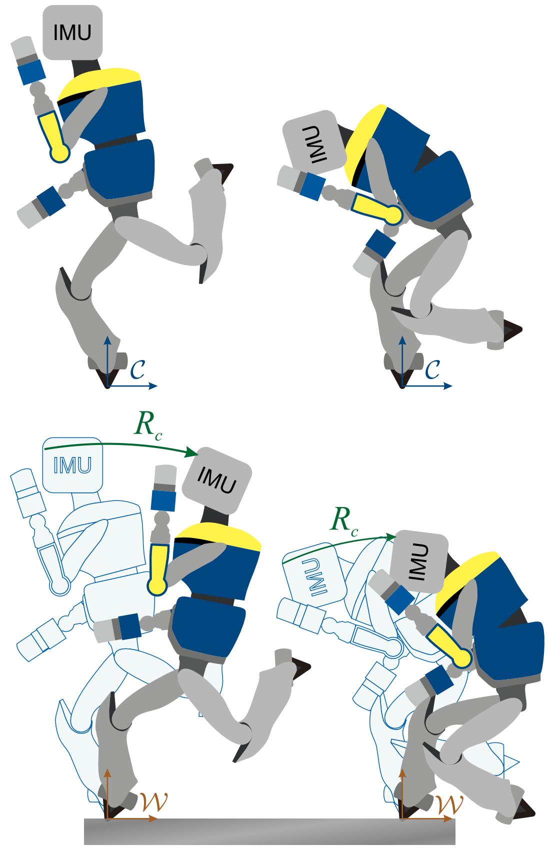

Figure 1. Top figures: the robot is attached to the environment through a 3D pivot joint. The encoders only provide the configuration in a local frame called control frame and represented by $\mathcal{C}$. Bottom figures: In the world frame $\mathcal{W}$ the configuration of the real robot is rotated compared to its value in the control frame (light wireframe robot). Only the IMU can provide data about this rotation, R_c. We need to account for encoders in this estimation, to distinguish local kinematics from pivot positions. For example the rotation R_c is here identical in the right and the left lower figures with different IMU configurations.

which is mostly vertical to the ground. This means that this orientation, usually called yaw angle, is not observable using this sensing system without additional knowledge of the nature of the contact.

Nevertheless, in an important number of these scenarios there is a specific need for a precise estimation of the two other degrees of freedom, which can be referred to as roll and pitch angles, or simply tilt. These two degrees of freedom describe the configuration of the pivot relative to the gravitational field. They are then essential for maintaining balance for legged robots [**?**], and may also allow drones and submarine robots to achieve tasks requiring precise orientation with respect to verticality such as construction or digging [?], [?].

In the context of these tasks, tilt estimation has been the topic of an important amount of research. But the vast majority of these works do not take into account the kinematic coupling provided by the contact point and these methods resort to consider that the accelerometer provides only in average the direction of the gravitational field. This approximation creates a systemic discrepancy between the measurements and the model and leads to delays and misestimations when this assumption is not verified. Few works take into account the kinematic constraint of the contact in the estimation of the tilt, but most of them only consider the case where the robot can be modeled as a rigid body, specifically regarding the position and orientation of the sensor. However, the motion of the robot modifies the relative position and orientation of the sensor to the pivot point and modifies the measurements provided by the sensors. This motion is usually well-known because it is not only provided by joint encoders but also is the outcome of a motion controller with a known dynamics. To our best knowledge, only two developed works deal with state estimation for non-rigid pendulum-like structures. However, both works emphasize on the modeling of this kinematics and resorted to the use of Kalman filtering techniques, which only provide an optimality guarantee with regard to the linearized dynamics around the estimated state. Therefore, in despite of the overall good performances of these estimators there is no real proof of the stability of the estimation error, especially if the estimator is intended to provide state feedback for closed loop control.

In this paper we provide a state estimator which aims to address all these shortcomings by providing a state estimator for the tilt of a pendulum which uses an accelerometer and a gyrometer, (i) without neglecting the linear accelerations compared to gravity, (ii) well suited to the case of an articulated robot, and (iii) with a proven Lyapunov stability. Furthermore, this estimator reaches local exponential stability performances, which makes it particularly suitable for the use as a state feedback for closed-loop control.

PRESENTATION DES SECTIONS

II. Problem statement

The system we address is a robot linked to the environment through a ball joint that we call pivot. Without loss of generality we may consider that the pivot is located at the origin of the inertial global frame. The configuration of the pivot is a pure 3D rotation describing a transformation between the global frame and the local frame of the robot, also called control frame ($\mathcal{C}$). Therefore we represent this rotation by the special orthogonal matrix $R_c \in \mathbb{R}^{3\times3}$. For instance, the sensor s located at position ${}^cp_s \in \mathbb{R}^3$ and orientation ${}^cR_s \in \mathbb{R}^{3\times3}$ in the control frame ($\mathcal{C}$) is actually at $p_s = R_c{}^cp_s$ and has the orientation $R_s = R_c{}^cR_s$ in the global frame ($\mathcal{W}$). This problem is sketched in Figure 1.

There is no sensor providing the pivot configuration. Instead, the robot is equipped with an IMU consisting in an accelerometer and a gyrometer, both of them are on three axes. The position of this IMU may be not rigidly linked to the ball joint, and can be located in another body of the robot. Since the robot can modify its actuated joint kinematics the IMU may move in the control frame. Therefore, we have to consider its position ${}^cp_s \in \mathbb{R}^3$, its orientation represented by the orthogonal matrix ${}^cR_s \in \mathbb{R}^{3\times3}$, together with their respective first-order time-derivatives ${}^c\dot{p}_s \in \mathbb{R}^3$ and ${}^c\omega_s \in \mathbb{R}^3$ such that ${}^c\dot{R}_s = S({}^c\omega_s)\,{}^cR_s$, where S is the skew-symmetric operator, i.e.

$$S\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \begin{pmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{pmatrix}.$$

The values of ${}^c\dot{p}_s$, ${}^cR_s^T$, ${}^c\dot{p}_s$ and ${}^c\omega_s$ can be obtained through the positions and velocities of the joint encoders and are often the outcome of a motion controller. Therefore, these values are considered to be perfectly known.

The accelerometer provides the sum of the gravitational field and the linear acceleration of the sensor, expressed in the sensor frame. In other words

$$y_a = R_s^T\left(g_0 e_z + \ddot{p}_s\right),$$

where y_a, p_s, R_s, g_0 and e_z are the accelerometer measurements, the position and the orientation of the IMU, standard gravity constant and a unit vector along the gravitational field respectively.

The gyrometer provides the angular velocity of the IMU, expressed in the sensor frame. In other words

$$y_g = R_s^T\omega_s,$$

where ω_s is the angular velocity vector of the sensor in the global frame such that $\dot{R}_s = S(\omega_s)\,R_s$.

We can see from these equations that the measurements are invariant regarding rotations around the gravitational field vector e_z. For instance, if we replace the pivot configuration R_c by R_zR_c where R_z is a rotation around e_z

$$R_z = I + \sin(\theta)S(e_z) + (1 - cos\theta)S^2(e_z),$$

where θ is any angle in $[-\pi, \pi]$, we obtain precisely the same measurements y_a and y_g. Therefore, the orientation that can be estimated through this sensing system is incomplete. Nevertheless, we show here that one partial information is observable and consists in $R_c^Te_z$, the direction of the gravitational field in the local frame of the robot. This data is the most important variable required to control balance and may be considered as a measure of "verticality" in general.

By replacing R_s and p_s by $R_c{}^cR_s$ and $R_c{}^cp_s$ respectively and performing time-derivations, we obtain

$$\begin{aligned} y_g &= {}^cR_s^T\,{}^c\omega_s + {}^cR_s^TR_c^T\omega_c, && (1)\\ y_a &= {}^cR_s^TR_c^T\left(\left(S(\dot{\omega}_c) + S^2(\omega_c)\right)R_c\,{}^cp_s + 2S(\omega_c)R_c\,{}^c\dot{p}_s\right)\\ &\quad + {}^cR_s^T{}^c\ddot{p}_s + g_0\,{}^cR_s^TR_c^Te_z. && (2)\end{aligned}$$

where ω_c is the angular velocity vector of the pendulum such that $\dot{R}_c = S(\omega_c)\,R_c$.

In the following section we develop the state observer for the estimation of $R_c^T e_z$.

III. State estimator

A. State definition

By using properties of skew-symmetric matrices, we may rewrite the equations (1) and (2) as

$$y_g = {}^cR_s^T\, {}^c\omega_s + {}^cR_s^T R_c^T \omega_c, \tag{3}$$

$$y_a = {}^cR_s^T \left(\left(S(R_c^T \dot{\omega}_c) + S^2(R_c^T\omega_c) \right) {}^cp_s + 2S(R_c^T\omega_c)\, {}^c\dot{p}_s \right) + {}^cR_s^T {}^c\ddot{p}_s + g_0\, {}^cR_s^T R_c^T e_z. \tag{4}$$

The first variable we define is the pivot angular velocity expressed in the control frame $y_1 = R_c^T \omega_c$. We have

$$y_1 = {}^cR_s \left(y_g - {}^cR_s^T\, {}^c\omega_s \right), \tag{5}$$

and since all the rightmost variables are known we may consider y_1 as known.

Let's define also the following state variables:

$$\begin{aligned} x_1 &= S({}^cp_s) y_1 - {}^c\dot{p}_s, \\ x_2 &= R_c^T e_z, \end{aligned}$$

with $x_1 \in \mathbb{R}^3$ and $x_2 \in \mathbb{S}^2$, with the set $\mathbb{S}^2 \subset \mathbb{R}^3$ is the unit sphere centered at the origin, and defined as

$$\mathbb{S}^2 = \left\{ x \in \mathbb{R}^3 / \, \|x\| = 1 \right\}.$$

The variable x_1 is also considered known since we know ${}^cp_s$, ${}^c\dot{p}_s$ and y_1. On the contrary, x_2 is the state we aim at estimating and cannot be obtained algebraically.

By left-multiplying Equation (4) by ${}^cR_s$ and replacing the expression of y_1 of equation (5) we get

$$S({}^cp_s) R_c^T \dot{\omega}_c + S({}^c\dot{p}_s) y_1 - {}^c\ddot{p}_s = - S(y_1) \left(S({}^cp_s) y_1 - {}^c\dot{p}_s \right) + g_0 R_c^T e_z - {}^cR_s y_a. \tag{6}$$

We notice that the left member of equation (6) is the first order time-derivative of x_1. This, together with the time-differentiation of x_2, provide us with the following dynamic equations

$$\begin{cases} \dot{x}_1 &= -S(y_1)x_1 + g_0 x_2 - {}^cR_s y_a, \\ \dot{x}_2 &= -S(y_1) x_2. \end{cases} \tag{7}$$

The system ((7)) is suitable for the observer synthesis.

B. State-observer and error dynamics:

In order to estimate $x_2 = R_c^T e_z$, we propose the following state-observer

$$\begin{cases} \dot{\hat{x}}_1 &= -S(y_1)\hat{x}_1 + g_0 \hat{x}_2 - {}^cR_s y_a + \alpha(x_1 - \hat{x}_1), \\ \dot{\hat{x}}_2 &= -S(y_1 - \beta S(\hat{x}_2)(x_1 - \hat{x}_1))\hat{x}_2, \end{cases} \tag{8}$$

where α, β are positive scalar gains which verify the condition $\beta g_0 < \alpha^2$ and $\hat{x}_1$ and $\hat{x}_2$ are the estimations of x_1 and x_2 respectively.

The initial value of $\hat{x}_2$ should be in $\mathbb{S}^2$. Then the dynamics of the last equation ensures that the norm of this vector remains constant in time. The initial value for $\hat{x}_1$ on its side could be anywhere in $\mathbb{R}^3$.

We define the following estimation errors $\tilde{x}_1 = x_1 - \hat{x}_1$ and $\tilde{x}_2 = x_2 - \hat{x}_2$, a time-differentiation of these expressions provide us with the following error dynamics:

$$\begin{cases} \dot{\tilde{x}}_1 &= -S(y_1)\tilde{x}_1 - \alpha\tilde{x}_1 + g_0\tilde{x}_2, \\ \dot{\tilde{x}}_2 &= -S(y_1)\tilde{x}_2 + \beta S^2(\hat{x}_2)\tilde{x}_1. \end{cases} \tag{9}$$

To run the analysis of errors, we set $z_i = R\tilde{x}_i$. We notice also that $\dot{R}_c = R_c S(R_c^T\omega_c) = R_c S(y_1)$ and $R_c(\tilde{x}_2 + \hat{x}_2) = e_z$, we obtain this new error dynamics

$$\begin{cases} \dot{z}_1 &= -\alpha z_1 + g_0 z_2, \\ \dot{z}_2 &= \beta S^2(e_z - z_2) z_1. \end{cases} \tag{10}$$

The nice property of this new dynamics is that it is autonomous and defines a time-invariant ordinary differential equation (ODE) which simplifies drastically the stability analysis. In fact, if one define the state $\xi := (z_1, z_2)$ and the state space $\Upsilon := \mathbb{R}^3 \times \mathbb{S}_{e_z}$ with $\mathbb{S}_{e_z} = \left\{ z \in \mathbb{R}^3 | \, (e_z - z) \in \mathbb{S}^2 \right\}$, one can write (10) as $\dot{\xi} = F(\xi)$ where F gathers the right-hand side of (10) and defines a smooth vector field on Υ.

IV. Stability analysis

A. Asymptotic stability

Define $G_0 = \frac{\beta g_0}{\alpha^2}$ and consider the following positive-definite differentiable function $V : \Upsilon \to \mathbb{R}^+$

$$V = \frac{\|\alpha z_1 - g_0 z_2\|^2}{2} + g_0^2 \frac{\|z_2\|^2}{2}, \tag{11}$$

which is radially unbounded over Υ.

Theorem 1. *The time-invariant ODE defined by (10) verifies the following*

1) *It admits two equilibrium points namely the origin $(0,0)$ and $(\frac{2g_0}{\alpha} e_z, 2e_z)$.*
2) *All trajectories of (10) converge to one of the equilibrium points defined in item 1.*
3) *The equilibrium $(0,0)$ is locally asymptotically stable with a domain of attraction containing the set*

$$V_c := \left\{ \xi = (z_1, z_2) \in \Upsilon \mid V(\xi) < 2g_0^2 \right\}.$$

4) *The equilibrium $(\frac{2g_0}{\alpha} e_z, 2e_z)$ is not stable (i.e., at least one the eigenvalues of the corresponding linear system has positive real part). This implies that the system (10) is almost globally stable with respect to the origin in the following sense: there exists an open dense subset $\Upsilon_0 \subset \Upsilon$ such that, for every initial condition $\xi_0 \in \Upsilon$, the corresponding trajectory converges asymptotically to $(0,0)$.*

Proof: Let's prove the four items of the theorem

1) The equilibria are calculated by solving the equation $F(\xi) = 0$, where F is the nonlinear function describing (10), we get the following

$$\begin{cases} 0 &= -\alpha z_1 + g_0 z_2, \\ 0 &= \beta S^2(e_z - z_2) z_1. \end{cases}$$

The trivial solution is $(0,0)$ and the second solution is calculated if we consider that $(z_1, z_2) \neq (0,0)$, so we can write

$$z_1 = \frac{g_0}{\alpha} z_2, \qquad (12)$$
$$0 = \beta \frac{g_0}{\alpha} S(e_z - z_2) S(e_z) z_2. \qquad (13)$$

We know that $z_2 \in \mathbb{S}_{e_z}$, so the only solution of (13) is $z_2 = 2e_z$, which gives from (12) that $z_1 = \frac{2g_0}{\alpha} e_z$. This complete the proof of item 1.

2) The time derivative of (11) in view of (10) yields

$$\dot{V} = -\alpha (1 - G_0) \|\alpha z_1 - g_0 z_2\|^2 + \alpha g_0^2 G_0 z_2^T S^2(e_z) z_2 - \alpha G_0 \left((\alpha z_1 - g_0 z_2)^T (e_z - z_2) \right)^2. \qquad (14)$$

One easily verifies that $\dot{V} < 0$ if (z_1, z_2) is not an equilibrium. Since (10) is autonomous and V is radially unbounded, one can use Lasalle's invariance theorem. Therefore, every trajectory converges to a trajectory along which $\dot{V} \equiv 0$.

3) Since V is non-increasing, $V(\xi) < 2g_0^2$ at $t = 0$, implies that $\|z_2(t)\| < 2$ for every $t \geq 0$. Since the trajectory converges to one of the two equilibrium points, it must be $(0,0)$ because this is the only one contained in V_c.

4) The linearized system around the equilibrium $(\frac{2g_0}{\alpha} e_z, 2e_z)$ is given by the following dynamics

$$\dot{X} = AX,$$

with $X = \begin{pmatrix} (z_1 - \frac{2g_0}{\alpha} e_z)^T & (z_2 - 2e_z)^T \end{pmatrix}^T$ and A is a constant matrix having the form

$$A = \begin{bmatrix} -\alpha I & g_0 I \\ \beta S^2(e_z) & -2\alpha G_0 S^2(e_z) \end{bmatrix}$$

The characteristic polynomial of the matrix matrix A is given by

$$P(\lambda) = \lambda (\lambda + \alpha) \left(\lambda^2 + \alpha (1 - 2G_0) \lambda - g_0 \beta \right)^2.$$

We find that this polynomial has two real positive roots, which are given by

$$\lambda = \alpha \frac{\sqrt{(1 + 4G_0^2)} - (1 - 2G_0)}{2} > 0,$$

which means that the equilibrium $(\frac{2g_0}{\alpha} e_z, 2e_z)$ is unstable. This completes the proof of the theorem. ■

B. Local exponential convergence

From equation (14) we can write the following

$$\dot{V} \leq -\alpha (1 - G_0) \|\alpha z_1 - g_0 z_2\|^2 + \alpha g_0^2 G_0 z_2^T S^2(e_z) z_2$$

In order to find the conditions of exponential convergence, let's observe the following relations

$$z_2^T S^2(e_z) z_2 = - \|z_2\|^2 + \frac{1}{4} \left(\|z_2\|^2 \right)^2$$

$$\dot{V} \leq -\alpha (1 - G_0) \|\alpha z_1 - g_0 z_2\|^2 - \alpha g_0^2 G_0 \left(1 - \frac{1}{4} \|z_2\|^2 \right) \|z_2\|^2$$

In the case of $\|V(\xi)\| < 2g_0^2$ at $t = 0$, we can say it exists a fixed $\epsilon > 0$ such that $1 - \frac{1}{4} \|z_2(t)\|^2 > \epsilon$, since the equilibrium which correspond to $\|z_2\| = 2$ is non attractive, so we can write the following

$$\dot{V} \leq -\alpha (1 - G_0) \|\alpha z_1 - g_0 z_2\|^2 - \alpha g_0^2 G_0 \epsilon \|z_2\|^2$$

$$\dot{V} \leq -min((1 - G_0, G_0 \epsilon) \alpha \left(\|\alpha z_1 - g_0 z_2\|^2 + g_0^2 \|z_2\|^2 \right)$$

which can be written as

$$\dot{V} \leq -2min (1 - G_0, G_0 \epsilon) \alpha V$$

which gives the following inequality

$$V(t) \leq V(0) e^{-2min(1 - G_0, G_0 \epsilon) \alpha t}$$

This leads to the local exponential convergence of the errors to the equilibrium $(0,0)$.

V. SIMULATIONS

In this section, we present simulation results showing the effectiveness of the proposed estimator. We generated the signal ${}^c\omega_s$ with trigonometric functions and generated the trajectory of ${}^cR_s$ by integration. Figure 2 shows time plot of ${}^cR_s$ represented by roll, pitch and yaw angles. We generated the trajectory of ${}^cp_s$ by integrating the signal ${}^c\dot{p}_s$ which is the sum of filtered noise and a linear feedback loop to maintain ${}^cp_s$ around the value $(0, 0, 1.3)$. Finally we generated $\dot{\omega}_c$ signals using trigonometric functions and obtained ω_c and R_c trajectories by integration. Afterwards we generated the measurement signals y_a for accelerometer and y_g for gyrometer using equations (1) and (2).

We have considered for the simulations the initial conditions for the estimator which correspond to the initial errors $\tilde{x}_1(0) = 0$ and $\tilde{x}_2(0) = \begin{pmatrix} -1.87 & 0.28 & 0.39 \end{pmatrix}^T$. The parameters of the estimator have been chosen as $\alpha = 19.8$ and $\beta = 10$, so the condition $(G_0 = \frac{g_0 \beta}{\alpha^2} < 1)$ is verified. We performed two simulation tests, one without considering noise and one with white centered Gaussian noise with standard deviation of 0.04 (normalized) added to the three elements of vector measurements y_g and with standard deviation of 0.2 (normalized) added to the three elements of vector measurements y_a.

Figure 3 on top and bottom shows the evolution of the estimation errors $\tilde{x}_1$ without noise and with noise, respectively. Figure 4 and Figure 5 show the estimation tracking of the variable x_2 and the estimation errors $\tilde{x}_2$ with respect to time, without and with noise respectively. We can see that the estimation error converges to zero in about one second. For the noisy case, even if the estimation error $\tilde{x}_1$ shows some sensitivity, we see that the error $\tilde{x}_2$ filters this noise in a relatively efficient way.

figures/Euler_angles-eps-converted-to.pdf

Figure 2. Plot showing the values of the orientation of the IMU in the control frame $(\mathcal{C})$ expressed using Euler angles.

VI. Discussion and conclusion

The estimation of tilt and attitude in general is a topic of active research, especially when IMU signals are used. Accelerometers are at the core of this problem mainly because their signal contains the value of the gravitational field in the frame of the sensor. In static cases, this property allows for an algebraically accessible tilt measurement. However, in the dynamic cases, this measurement is mixed with the linear acceleration in an algebraically indistinguishable way. In many works the acceleration is considered negligible compared to gravity field [**?**], and is therefore considered as a noise. Filtering approaches are commonly used to remove this signal [**?**].

Accelerometers are also commonly used together with gyrometers. Gyrometers provide rotation velocities in the local reference frame. Their signals are commonly merged with accelerometers using Kalman Filtering [**?**], but are often exploited to correct the filtered accelerometer signals using complementary filtering [**?**].

Several other works rely on the presence of additional data to reconstruct the attitude. For instance, magnetometers [**?**] or vision [**?**] can be used to retrieve redundant attitude signals allowing to reduce the effect of accelerometer errors. Finally, a fusion with external measurements such as GPS [**?**] or landmark relative position [**?**] allow to better distinguish the linear acceleration from gravitational field measurements and allows to observe the linear part of the kinematics.

We see through this brief summary that the translational component of the motion of the IMU is commonly considered either as a noise that requires to be deleted or as an independent dynamics which needs to be observed. However, in the specific case of the pendulum, this linear part of the kinematics is coupled with the angular motion which explains the presence of the angular velocity and event angular acceleration in the signals of the accelerometer (see Equation (2)). This enables us to use this signal without any need of filtering and to still be able to reconstruct tilt despite a high level noise level. The translation-rotation coupling is entirely due to the presence of the anchor point of pivot. However, in several works addressing cases similar to pivot link position

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figures/x1_noisy-eps-converted-to.pdf

Figure 3. Plot showing the estimation error for x_1. On the top, the case where there is no noise, and on the bottom the noisy case.

figures/x2-eps-converted-to.pdf

figures/x2_error-eps-converted-to.pdf

Figure 4. Estimation of x_2 in the case without noise. On the top a plot showing a comparison between values of x_2 and its estimation $\hat{x}_2$. On the bottom we see the evolution of the estimation error.

figures/x2_noisy-eps-converted-to.pdf

figures/x2_error_noisy-eps-converted-to.pdf

Figure 5. Estimation of x_2 in the noisy case. On the top a plot showing values of x_2 and its estimation $\hat{x}_2$. On the bottom we see the estimation error for x_2.

estimation are still resorting to classical methods where the IMU is considered as an unconstrained floating object, even if the reconstructed attitude are merged with encoder data afterwards [?]. It is worth to note that in addition to orientation, the orientation estimation a pendulum provides also data on the position of the limbs of the robot, because of the pivot constraint. This relationship allows also to design position controllers on the base of attitude estimators, similarly to hand position compensation presented in [?].

Only few works dealt with attitude estimation taking into account the pivot constraints. One example is the tilt estimation for rigid pendulum around the pivot using multiple accelerometers [?]. This observer was used especially for balancing the reaction wheel cube on edges and corners [?]. In addition to the requirement of multiple accelerometers at different locations is only limited to rigid pendulum cases. Another work from legged robotics community considers also contact information [?]. This estimator considers the case of multiple contacts and uses an extended Kalman Filter. The contact information is introduced in the model kinematics but only at the prediction step rather than as a constraint. Their model is intended to take into account the cases of contact slippage, but this variable is not observable using inertial sensors. Another work uses also extended Kalman Filtering for a humanoid robot having flexible contacts with the environment [?]. The contact information was introduced as pseudo measurements in order to allow the pivot constraint to be slightly violated. This observer was extended to take into account the dynamical model of the flexibility [?]. However the use of extended Kalman filtering only provides the guarantee of optimality around the linearized dynamics around the predicted state and gives no proof of convergence.

To our best knowledge, our estimator is the only one providing almost globally convergent estimation of tilt for the case of non-rigid pendulum system. The only orientation data missing in the estimator is the orientation around the gravitational field direction, or yaw angle. This orientation is proven to be out of reach of this measurement system. Therefore, the addition of other sensors such as magnetometers are necessary to obtain this estimate. The introduction of this kind of sensors is the topic of a possible improvement of the presented method.

Finally, the introduction of a model for the dynamics of the pivot could also increase the quality of the observation, specifically by creating coupling between the measurement data of the IMU and other values which are non-observable otherwise. These values include yaw angle without needing additional data, but may go to the estimation of contact forces with the environment [?]. This is also the topic of next developments regarding this kind of systems.

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