Wireless Networks with Imperfect Side-Information

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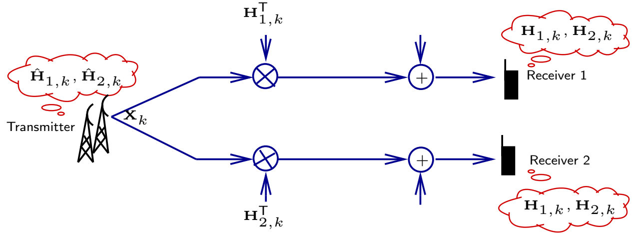
Based on collaborations with Amos Lapidoth and Shlomo Shamai (Shitz)

Imperfect Side-Information on Channel State

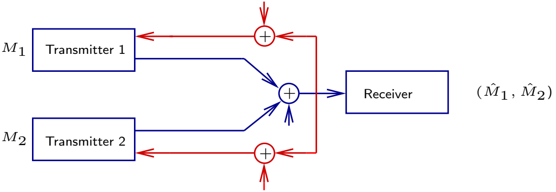


- Fading MIMO broadcast channel
- Fading known perfectly to receivers, but only imperfectly to transmitter
- E.g.: Slowly fading channels with separate fb-link

Lapidoth/Shamai/Wigger'05

Imperfection causes MIMO capacity-gain to collapse!

Side-Information via Imperfect Output Feedback

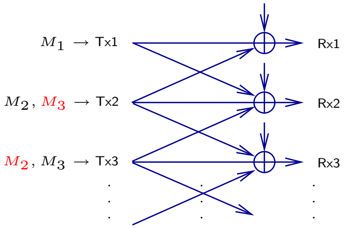


- Gaussian multiple-access channel with *noisy* output feedback
- E.g.: Up-link/down-link scenarios with much stronger down-link
- IT-wisdom: Feedback cannot hurt! *But does it help?*

Lapidoth&Wigger'06

- Even if noisy, feedback is *always* beneficial!
- Noisy feedback is almost as good as perfect feedback!

Imperfect Side-Information on Message Sets



- Cognitive Interference Networks with $\mathcal{N}$ tx/rx-pairs
- Each tx knows *some* of other txs' messages
- E.g.: Co-located txs, cognitive radio systems

Lapidoth/Shamai/Wigger'07

Depending on specific topology of network:

- # of "interference-free channels" collapses with incomplete SI
- $\mathcal{N}$ "interference-free channels" achievable even with little SI

Robustness of Capacity w.r.t. Imperfect Side-Information

"Almost Perfect" $\ll$ "Perfect"

or

"Almost Perfect" $\approx$ "Perfect" ?

Preliminaries: Gaussian Networks

- Independent messages:

$$M_\nu \sim \mathcal{U}\left\{1,\ldots,\lfloor e^{nR_\nu}\rfloor\right\}, \quad \nu \in \{1,\ldots,\mathcal{N}\}$$

- Outputs corrupted by independent, additive noise sequences:

$$\{Z_{\nu,k}\}\ \text{IID}\ \sim \mathcal{N}(0,N)$$

- Input power constraints:

$$\frac{1}{n}\mathsf{E}\left[\sum_{k=1}^{n}\mathbf{X}_{\nu,k}^{2}\right] \le P_\nu$$

- $P_\nu = P \quad \forall \nu \in \{1,\ldots,\mathcal{N}\} \qquad \Longrightarrow \qquad \text{SNR} \triangleq \frac{P}{N}$

Preliminaries: Capacity

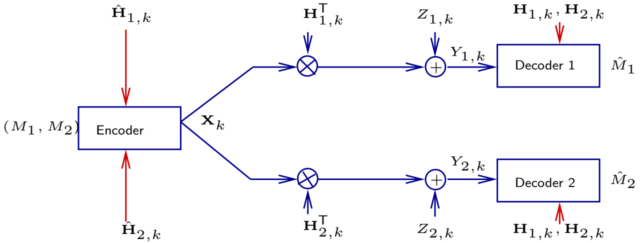
- Capacity region $\mathcal{C}$: closure of set of all rate-tuples $(R_1, \ldots, R_{\mathcal{N}}) \in \mathbb{R}^{\mathcal{N}}$ for which $\exists$ block-length n schemes such that

$$p(\text{error}) \to 0 \qquad \text{as } n \to \infty$$

- Sum-rate capacity C_{sum}: supremum of $R_{\text{sum}} = \sum_{\nu=1}^{\mathcal{N}} R_\nu$ over all $(R_1, \ldots, R_{\mathcal{N}}) \in \mathcal{C}$

Part 1

Fading MIMO Broadcast Channel



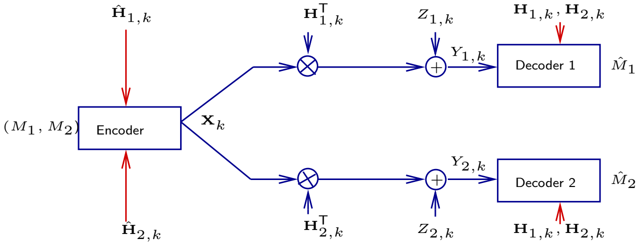
Side-information: Channel state

Imperfection: Transmitter knows fading sequences only approximately!
(modified Caire/Shamai'03 setting)

Lapidoth/Shamai/Wigger'05

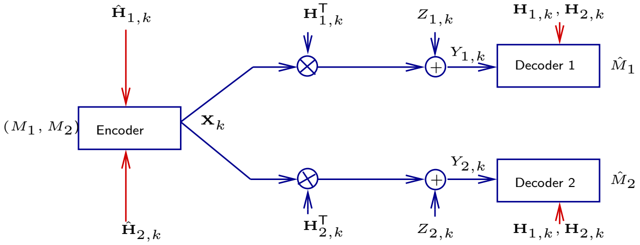
Imperfection causes MIMO capacity-gain to collapse!

Fading MIMO Broadcast Channel



- Transmitter has 2 antennas
- Each of the two receivers has 1 antenna
- $Y_{\nu,k} = \mathbf{H}_{\nu,k}^{\mathsf{T}} \mathbf{X}_k + Z_{\nu,k}, \quad \nu \in \{1,2\}$
- $\mathsf{E}\left[\|\mathbf{H}_{\nu,k}\|^2\right]$ bounded

Fading MIMO Broadcast Channel



- $\mathbf{H}_{\nu,k} = \hat{\mathbf{H}}_{\nu,k} + \tilde{\mathbf{H}}_{\nu,k}$
- Transmitter knows a-causally *realizations* of $\{\hat{\mathbf{H}}_{1,k}\}, \{\hat{\mathbf{H}}_{2,k}\}$
- Receivers know *realizations* of $\{\tilde{\mathbf{H}}_{1,k}\}, \{\tilde{\mathbf{H}}_{2,k}\}, \{\hat{\mathbf{H}}_{1,k}\}, \{\hat{\mathbf{H}}_{2,k}\}$ (optimistic)
- Law of $\{\tilde{\mathbf{H}}_{1,k}\}, \{\tilde{\mathbf{H}}_{2,k}\}, \{\hat{\mathbf{H}}_{1,k}\}, \{\hat{\mathbf{H}}_{2,k}\}$ determines precision of transmitter state-information

Degrees of Freedom

- Degrees of freedom:

$$\overline{\lim}_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})}$$

- Characterizes maximum high SNR throughput of system

Perfect Channel State Information for Transmitter

- If $\{\mathbf{H}_{1,k}\}$, $\{\mathbf{H}_{2,k}\}$ known to receivers and transmitter [Caire&Shamai'03]:

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log \mathsf{SNR}} = 2$$

- For single transmit-antenna:

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = 1$$

- $\Longrightarrow$ Capacity-gain thanks to 2 transmit antennas!

No Channel State Information for Transmitter

- In general still open!
- If laws of $\{\mathbf{H}_{1,k}\}$ and $\{\mathbf{H}_{2,k}\}$ identical

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{Sum}}(\mathsf{SNR})}{\frac{1}{2}\log \mathsf{SNR}} = 1$$

- With only a single transmit-antenna:

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{Sum}}(\mathsf{SNR})}{\frac{1}{2}\log \mathsf{SNR}} = 1$$

Approximate Channel State Information for Transmitter

- Obviously:

$$1 \leq \overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{Sum}}(\mathsf{SNR})}{\frac{1}{2}\log \mathsf{SNR}} \leq 2$$

- If $\{\hat{\mathbf{H}}_1\} = \{\hat{\mathbf{H}}_2\}$ with probability 1 and if conditional laws of $\{\tilde{\mathbf{H}}_1\}$ and $\{\tilde{\mathbf{H}}_2\}$ given $(\{\hat{\mathbf{H}}_1\}, \{\hat{\mathbf{H}}_2\})$ identical

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{Sum}}(\mathsf{SNR})}{\frac{1}{2}\log \mathsf{SNR}} = 1$$

Main Result for Fading MIMO BC

- High SNR throughput of fading MIMO BC extremely sensitive to imperfect transmitter CSI

Theorem 1 (Lapidoth,Shamai,Wigger'05)

If

$$\begin{aligned} \varliminf_{n\to\infty} \frac{1}{n} h(\tilde{\mathbf{H}}_{1,1}^n | \hat{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{2,1}^n) &> -\infty \\ \varliminf_{n\to\infty} \frac{1}{n} h(\tilde{\mathbf{H}}_{2,1}^n | \hat{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{2,1}^n) &> -\infty \end{aligned}$$

then

$$\varlimsup_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{Sum}}(\mathsf{SNR})}{\frac{1}{2}\log \mathsf{SNR}} \leq \frac{4}{3}$$

Note: Precision fixed and not improved as SNR $\to \infty$

Key-Tool in Proof

Lemma 1 (Lapidoth/Shamai/Wigger'05)

Let (X_1, X_2) *be of finite-variance and independent of* $Z \sim \mathcal{N}(0, N)$*. Let*

$$\mathcal{H}(\theta) \triangleq h\big(X_1 \cos\theta + X_2 \sin\theta + Z\big), \quad -\pi \leq \theta < \pi.$$

Then for any two bounded density functions $f_{\Theta_1}(\cdot)$ *and* $f_{\Theta_2}(\cdot)$ *on* $[-\pi, \pi)$*,*

$$\int_{-\pi}^{\pi} \mathcal{H}(\theta) f_{\Theta_1}(\theta)\, \mathrm{d}\theta \geq \frac{1}{2} \int_{-\pi}^{\pi} \mathcal{H}(\theta) f_{\Theta_2}(\theta)\, \mathrm{d}\theta + \frac{1}{4} \log N + \frac{9}{2} h(\Theta_1) - \gamma,$$

where γ *is some universal constant.*

Proof Outline I

- Reveal M_1 to Receiver 2
- Use Fano's inequality
- Use max-entropy theorem

$$R_1 + R_2 \leq \underbrace{\frac{1}{n}\left[h(Y_{1,1}^n|\tilde{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{2,1}^n) - h(Y_{2,1}^n|M_1, M_2, \tilde{\mathbf{H}}_{2,1}^n, \hat{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{2,1}^n)\right]}_{\leq \frac{1}{2}\log(1+\mathsf{SNR})+O(1)} + \underbrace{\frac{1}{n}\left[h(Y_{2,1}^n|M_1, \tilde{\mathbf{H}}_{2,1}^n, \hat{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{2,1}^n) - h(Y_{1,1}^n|M_1, \tilde{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{1,1}^n, \hat{\mathbf{H}}_{2,1}^n)\right]}_{=?}$$

Proof Outline II

- With "Marton-like" expansion:

$$
\begin{aligned}
? \quad = \quad & \frac{1}{n}\sum_{k=1}^{n}\Big[h(\mathbf{H}_{2,k}^{\intercal}\mathbf{X}_k + Z_{2,k}|\ldots, \tilde{\mathbf{H}}_{2,k}, \hat{\mathbf{H}}_{2,k}) \\
& \qquad\qquad -h(\mathbf{H}_{1,k}^{\intercal}\mathbf{X}_k + Z_{1,k}|\ldots, \tilde{\mathbf{H}}_{1,k}, \hat{\mathbf{H}}_{1,k})\Big]
\end{aligned}
$$

- With Lemma 1 and taking care of magnitude

$$
\begin{aligned}
? \quad \leq \quad & \frac{1}{n}\sum_{k=1}^{n}\left[\frac{1}{2}h(\mathbf{H}_{2,k}^{\intercal}\mathbf{X}_k + Z_{2,k}|\ldots, \tilde{\mathbf{H}}_{2,k}, \hat{\mathbf{H}}_{2,k})\right] + O(1) \\
\leq \quad & \frac{1}{4}\log(1+\mathsf{SNR}) + O(1)
\end{aligned}
$$

- $\Longrightarrow \qquad R_1 + R_2 < \frac{3}{2}\cdot\frac{1}{2}\log\mathsf{SNR} + O(1)$

Proof Outline III

- But we can do a bit better

$$R_1 + \frac{1}{2}R_2 < \frac{1}{2}\log \mathsf{SNR} + O(1) \quad (1)$$

- Replacing role of Rx 1 and Rx 2

$$R_2 + \frac{1}{2}R_1 < \frac{1}{2}\log \mathsf{SNR} + O(1) \quad (2)$$

- Adding (1) and (2) and dividing by $\frac{3}{2}$ leads to

$$R_1 + R_2 < \frac{4}{3} \cdot \frac{1}{2}\log \mathsf{SNR} + O(1)$$

Summary/Conjecture for Fading MIMO BC

- Degrees of freedom of fading MIMO BC collapse from 2 [Caire&Shamai'03] to at most 4/3 with imprecisions in transmitter side-information!

Summary: (Lapidoth/Shamai/Wigger'05)

$$\overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} \leq \frac{4}{3}$$

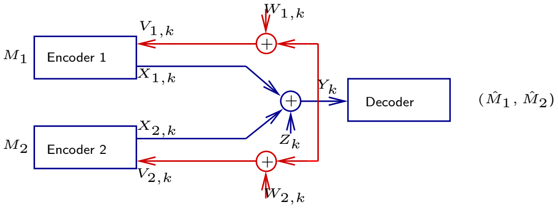
- Conjecture: For imprecise transmitter channel state information, degrees of freedom collapse completely! $\Rightarrow$ "No gain" from having 2 tx-antennas!

Conjecture:

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = 1$$

Part 2

Two-User Gaussian MAC with Imperfect Feedback



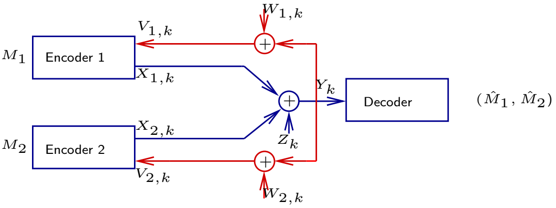
Side-information: Output feedback

Imperfection: Feedback links are noisy

Lapidoth&Wigger'06

- Feedback noise hardly influences perfect-feedback capacity!
- Noisy feedback links are *always* beneficial!

Two-User Gaussian MAC with Imperfect Feedback



- $Y_k = x_{1,k} + x_{2,k} + Z_k \qquad V_{\nu,k} = Y_k + W_{\nu,k}$
- $\{(W_{1,k}, W_{2,k})\} \sim$ IID $\mathcal{N}(\mathbf{0}, \mathsf{K}_{W_1 W_2})$; $\qquad \mathsf{K}_{W_1 W_2} = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\varrho \\ \sigma_1\sigma_2\varrho & \sigma_2^2 \end{pmatrix}$
- Noisy feedback: $\qquad X_{\nu,k} = f_{\nu,k}\left(M_\nu, \mathbf{V}_{\nu,1}^{k-1}\right)$

No Feedback; $\sigma_1^2 = \sigma_2^2 = \infty$

- For Gaussian two-user MAC without feedback:

$$C_{\mathsf{NoFB}} = \left\{ (R_1, R_2) : \begin{array}{ccc} R_1 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1}{N}\right) \\ R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_2}{N}\right) \\ R_1 + R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1+P_2}{N}\right) \end{array} \right\}$$

General MAC with Perfect Feedback

- Even for *memoryless* MACs perfect feedback can increase capacity [Gaarder&Wolf'73]
- Capacity of general memoryless MACs with perfect feedback open!
- Cover&Leung'81: $\mathcal{R}_{\text{CL}}$ achievable rate region with perfect feedback
- For some channels $\mathcal{R}_{\text{CL}}$ is capacity! [Willems'82]

Perfect Feedback; $\sigma_1^2 = \sigma_2^2 = 0$

- For Gaussian two-user MAC with perfect feedback [Ozarow'84]:

$$C_{\mathsf{PerfectFB}} = \bigcup_{\rho \in [0,1]} \left\{ (R_1, R_2) : \begin{array}{rcl} R_1 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1(1-\rho^2)}{N}\right) \\ R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_2(1-\rho^2)}{N}\right) \\ R_1 + R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2}\rho}{N}\right) \end{array} \right\}$$

$$C_{\mathsf{NoFB}} \subset C_{\mathsf{PerfectFB}} \qquad \text{(strictly)}$$

One-Sided Perfect Feedback to User 2; $\sigma_1^2 = \infty, \sigma_2^2 = 0$

- Ozarow's scheme doesn't work
- Cover&Leung's scheme still does

$$\mathcal{R}_{\mathsf{CL}} = \bigcup_{\substack{\rho_1,\rho_2\in \\ [0,1]}} \left\{ (R_1,R_2): \begin{array}{rcl} R_1 & \leq & \frac{1}{2}\log\left(1+\frac{P_1(1-\rho_1^2)}{N}\right) \\ R_2 & \leq & \frac{1}{2}\log\left(1+\frac{P_2(1-\rho_2^2)}{N}\right) \\ R_1+R_2 & \leq & \frac{1}{2}\log\left(1+\frac{P_1+P_2+2\sqrt{P_1P_2}\rho_1\rho_2}{N}\right) \end{array} \right\}$$

- Van der Meulen'87:

$$C_{\mathsf{OneSided}} = \mathcal{R}_{\mathsf{CL}} \ ?$$

Answer to van der Meulen (Lapidoth&Wigger'06)

$$\mathcal{R}_{\mathsf{CL}} \subset C_{\mathsf{OneSided}} \qquad \text{(can be strict)}$$

Noisy Feedback; $0 \leq \sigma_1^2, \sigma_2^2 \leq \infty$

- Carleial'82, Willems/van der Meulen/Schalkwijk'83:
  - Collapse to C_{NoFB} for σ_1^2, σ_2^2 above some threshold depending on P_1, P_2, N
  - Don't converge to $C_{\text{PerfectFB}}$ when $\sigma_1^2, \sigma_2^2 \downarrow 0$

Just shortcomings of schemes or inherent in problem?

- Gastpar'05

- Ozarow's scheme doesn't work

Main Results: Noisy Feedback (Lapidoth&Wigger'06)

- Feedback is always beneficial!

Theorem 2

For $0 < P_1, P_2, N < \infty$*:*

$$\mathsf{C}_{\mathsf{NoFB}}(P_1, P_2, N) \subset \mathsf{C}_{\mathsf{NoisyFB}}(P_1, P_2, N, \mathsf{K}_{W_1 W_2}) \qquad \text{(strictly)}$$

whenever $\min\{\sigma_1^2, \sigma_2^2\} < \infty$

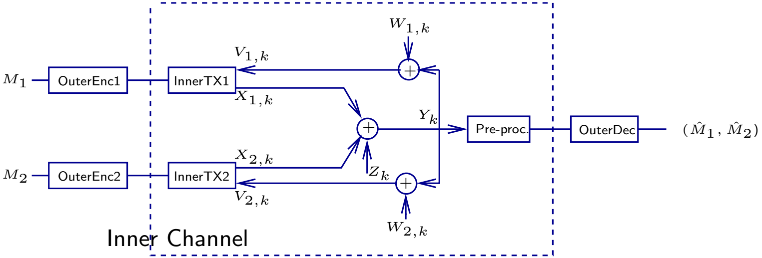
- Almost-perfect feedback is like perfect feedback

Theorem 3

For $0 < P_1, P_2, N < \infty$*:*

$$\lim_{(\sigma_1^2, \sigma_2^2) \downarrow \mathbf{0}} \mathsf{C}_{\mathsf{NoisyFB}}(P_1, P_2, N, \mathsf{K}_{W_1 W_2}) = \mathsf{C}_{\mathsf{PerfectFB}}(P_1, P_2, N)$$

Imperfect Feedback Scheme: Concatenated Structure



- Inner channel:
  - One input symbol to each inner transmitter every κ channel uses
  - One pre-processor output every κ channel uses
  - Inner txs & pre-proc. generalize Ozarow's perfect feedback scheme
- Outer code ignores feedback, codes to achieve capacity of inner channel
- Outer code & generalized inner scheme & good finite κ make Ozarow's scheme robust to noisy feedback

Summary for MAC with Noisy Feedback (Lapidoth&Wigger'06)

Encoding scheme

- We propose "robustification" of Ozarow's perfect feedback scheme for imperfect feedback

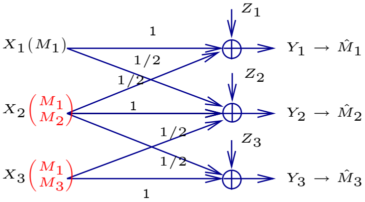
Unlike previous schemes, our scheme leads to

- Feedback always increases capacity if σ_1^2, σ_2^2 not both ∞
- Noisy feedback capacity tends to perfect feedback capacity when $\sigma_1^2, \sigma_2^2 \downarrow 0$
- Answer to van der Meulen:
  One-sided feedback capacity can strictly include Cover&Leung region

Part 3

Cognitive Interference networks

Example:



Side-information: Knowledge about other transmitters' messages

Imperfection: Transmitters know only some of the messages

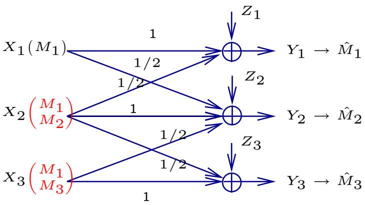
Lapidoth/Shamai/Wigger'07

Depending on specific topology of network:

- Degrees of freedom collapse with incomplete side-information
- $\mathcal{N}$ degrees of freedom achievable even with little side-information
- Degrees of freedom can be a non-integer number

Cognitive Interference networks

Example:



- $\mathcal{N}$ transmitters and receivers, each with a single-antenna
- $\mathsf{H} = (h_{j,\ell}) \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}$ of full rank; *constant*
- $\mathbf{Y}_k = \mathsf{H}\mathbf{X}_k + \mathbf{Z}_k$
- $\mathcal{S}_\nu \subseteq \{1, \ldots, \mathcal{N}\}$
- $i \in \mathcal{S}_\nu \quad \Longleftrightarrow \quad$ Transmitter ν knows M_i

Degrees of Freedom

- Degrees of freedom:

$$\eta\left(\mathsf{H}, \{\mathcal{S}_\nu\}\right) \triangleq \overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})}$$

- Characterizes maximum high SNR throughput of system

- For parallel Gaussian channels:

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = \mathcal{N}$$

- Time-sharing:

$$\overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} \geq 1$$

Perfect Side-Information; $\mathcal{S}_\nu = \{1, \ldots, \mathcal{N}\} \quad \forall \nu$

- All $\mathcal{N}$ transmitters know all $\mathcal{N}$ messages!
- Setting corresponds to a MIMO Broadcast Channel
- [Vishwanath/Tse'03, Viswanath/Tse'03, Yu/Cioffi'04]:

$$\lim_{\mathsf{SNR} \to \infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = \mathcal{N}$$

No Side-Information; $\mathcal{S}_\nu = \{\nu\}$

- In general still an open problem!
- Høst-Madsen/Nosratinian: Fully connected networks

$$1 \leq \overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} \leq \frac{\mathcal{N}}{2}$$

Lapidoth/Shamai/Wigger'07

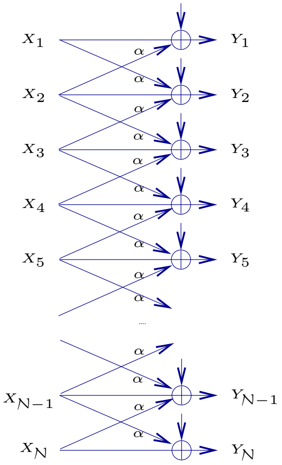
For *certain* fully connected networks with *full-rank* matrix H:

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = 1$$

$\Longrightarrow$ At high SNR can't do much better than time-sharing!

Partial Side-Information

Example: Wyner's linear model for cellular systems (down-link)



- $\alpha > 0$
- No SI:

Lapidoth/Shamai/Wigger'07

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = \mathcal{N} - \lfloor \mathcal{N}/2 \rfloor$$

- Partial SI: Next and previous $J > 0$ msgs:

Lapidoth/Shamai/Wigger'07

$$\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = \mathcal{N} - \left\lfloor \frac{\mathcal{N}}{J+2} \right\rfloor$$

- Partial SI better than No SI, can even lead to $\mathcal{N}$ degrees of freedom

Main Questions for Partial Side-Information

- Degrees of freedom for general cognitive networks → don't know!

Main questions we shall answer:

- For which H's is there "strictly partial SI" s.t. $\lim_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} = \mathcal{N}$?
- For which H's is there "strictly partial SI" which outperforms "No SI"?
- Must degrees of freedom $\overline{\lim}_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})}$ be integer?

Partial Interference Cancelation (Lapidoth/Shamai/Wigger'07)

- With precoding/silencing transmitters construct parallel Gaussian channels
- Possible precoding matrices depend on side-information
- $p^*\left(\mathsf{H}, \{\mathcal{S}_\nu\}\right)$: maximum number of parallel channels using partial interference cancelation
- $$\overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})} \geq p^*\left(\mathsf{H}, \{\mathcal{S}_\nu\}\right)$$

Results I (Lapidoth,Shamai,Wigger'07)

- Partial interference cancelation is optimal when $p^* = \mathcal{N}$ or $p^* = \mathcal{N} - 1$

Theorem 5

Given $(\mathsf{H}, \{\mathcal{S}_\nu\})$*:*

$$p^* = \mathcal{N} \implies \eta = \mathcal{N}$$
$$p^* = \mathcal{N} - 1 \implies \eta = \mathcal{N} - 1$$
$$p^* < \mathcal{N} - 1 \implies \eta < \mathcal{N} - 1$$

- p^* characterizes networks where $\eta = \mathcal{N}$ and networks where $\eta = \mathcal{N} - 1$

Corollary 1

$$\eta(\mathsf{H}, \{\mathcal{S}_\nu\}) = \mathcal{N} \iff p^*(\mathsf{H}, \{\mathcal{S}_\nu\}) = \mathcal{N}$$
$$\eta(\mathsf{H}, \{\mathcal{S}_\nu\}) = \mathcal{N} - 1 \iff p^*(\mathsf{H}, \{\mathcal{S}_\nu\}) = \mathcal{N} - 1$$

η *not in open interval* $(\mathcal{N} - 1, \mathcal{N})$

Reminder: $\eta\left(\mathsf{H}, \{\mathcal{S}_\nu\}\right) \triangleq \overline{\lim}_{\mathsf{SNR}\to\infty} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})}$

Results II (Lapidoth,Shamai,Wigger'07)

- Given H: characterization of side-information required for $\eta = \mathcal{N}$

Theorem 6

Given H*:*

$$\eta(\mathsf{H}, \{\mathcal{S}_\nu\}) = \mathcal{N} \quad \Longleftrightarrow \quad \left(\mathsf{rank}\left(\mathsf{H}_{(i)}^{(j)}\right) = \mathcal{N} - 1 \Longrightarrow i \in \mathcal{S}_j\right), \ \forall i, j$$

- Characterization of H's where $\eta = \mathcal{N}$ requires full side-information

Corollary 2

Given H*:* *"Perfect side-information" required for* $\eta = \mathcal{N}$

$$\Longleftrightarrow$$

$$\mathsf{rank}\left(\mathsf{H}_{(i)}^{(j)}\right) = \mathcal{N} - 1, \qquad \forall i \neq j$$

- $\mathsf{H}_{(i)}^{(j)}$: H without j-th column, i-th row

Results III (Lapidoth,Shamai,Wigger'07)

- Characterization of H's where all strictly partial SI are like no SI

Theorem 7

No "strictly partial side-information" can increase degrees of freedom whenever

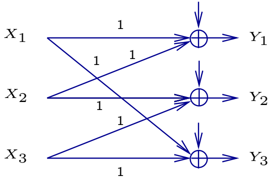
$$\mathsf{H} \textit{ is diagonal} \quad \textit{or} \quad \mathsf{H} = \begin{pmatrix} \times & 0 & 0 & \cdots & 0 & \times & 0 & \cdots & 0 & 0 \\ 0 & \times & 0 & \cdots & 0 & \times & 0 & \cdots & 0 & 0 \\ & & & \cdots & & & \cdots & & & \\ & & & \cdots & & & \cdots & & & \\ 0 & 0 & 0 & \cdots & \times & \times & 0 & \cdots & 0 & 0 \\ \times & \times & \times & \cdots & \times & ? & \times & \cdots & \times & \times \\ 0 & 0 & 0 & \cdots & 0 & \times & \times & \cdots & 0 & 0 \\ & & & \cdots & & & \cdots & & & \\ & & & \cdots & & & \cdots & & & \\ 0 & 0 & 0 & \cdots & 0 & \times & 0 & \cdots & \times & 0 \\ 0 & 0 & 0 & \cdots & 0 & \times & 0 & \cdots & 0 & \times \end{pmatrix}$$

x: *non-zero entry,* $?$: *arbitrary entry*

Extension of Partial Interference Cancelation Scheme

- Code over multiple channel uses (inspired by Weingarten et al.'07)

Example:



- Extend scheme over 2 channel uses
- No SI: $\rightarrow \eta \geq 3/2$

Lapidoth/Shamai/Wigger'07

For above example: $\eta = 3/2$

$\Longrightarrow$ Degrees of freedom need not be an integer!

Summary for Cognitive Interference Networks

Lapidoth/Shamai/Wigger'07

- Characterized networks where $\eta = \mathcal{N}$ and those where $\eta = \mathcal{N} - 1$
- Characterized H's where "full side-information" necessary for $\eta = \mathcal{N}$
- Characterized H's where "partial side-information" *never* increases d.o.f.
- Degrees of freedom (of single-antenna networks) can be non-integer
- *Fully-connected* networks with full-rank H can have only 1 d.o.f.

Reminder: $\eta \triangleq \overline{\lim_{\mathsf{SNR}\to\infty}} \frac{C_{\mathsf{sum}}(\mathsf{SNR})}{\frac{1}{2}\log(\mathsf{SNR})}$, $\mathcal{N}$: # transmitters/receivers

Summary of Talk

Fading MIMO BC with CSI @ Tx/Rxs (Lapidoth/Shamai/Wigger'05)

- Imprecisions in CSI @ tx $\Rightarrow$ degrees of freedom collapse from 2 to $\leq \frac{4}{3}$

2-User Gaussian MAC with Output Feedback (Lapidoth&Wigger'06)

- Almost noise-free feedback $\approx$ noise-free feedback
- Even noisy feedback is always beneficial
- Answer van der Meulen's question

Cognitive Interference Networks (Lapidoth/Shamai/Wigger'07)

- Characterized networks with $\mathcal{N}$ and $\mathcal{N}-1$ degrees of freedom
- Characterized H's where partial SI never increases degrees of freedom
- Characterized H's where *some* partial SI achieves $\mathcal{N}$ degrees of freedom
- Degrees of freedom can be non-integer!
- *Fully-connected* networks with full-rank H can have 1 degree of freedom

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See also: http://people.ethz.ch/∼wiggerm