Operating Point Selection in Multiple Access Rate Regions

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Outlines

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3. Selection Criteria
4. Operating Point Selection in Polytope Rate Regions
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6. Operating Point Selection in non-Convex Rate Regions
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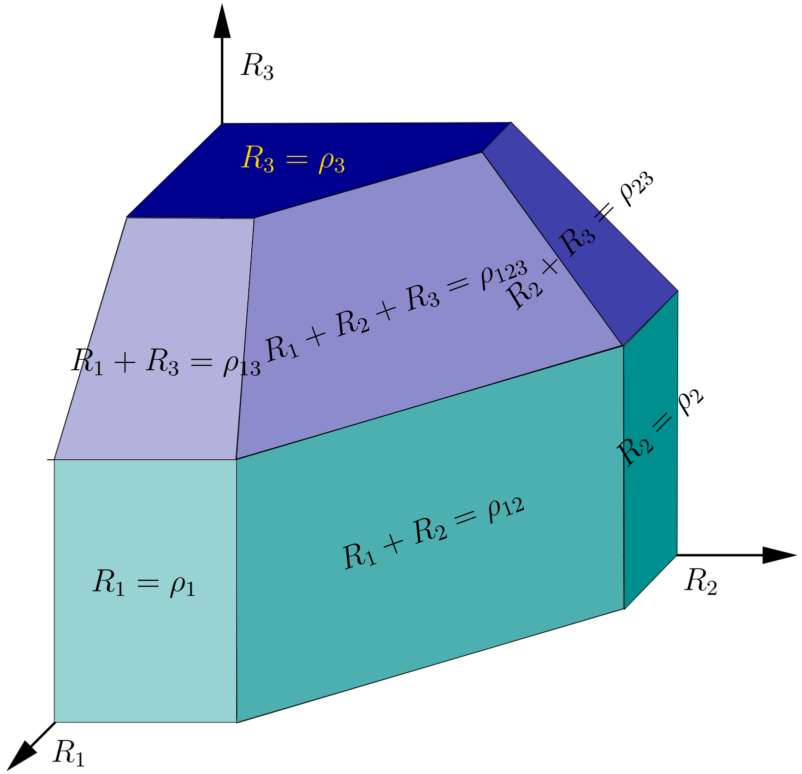
Rate Regions for MAC: Polytope Regions



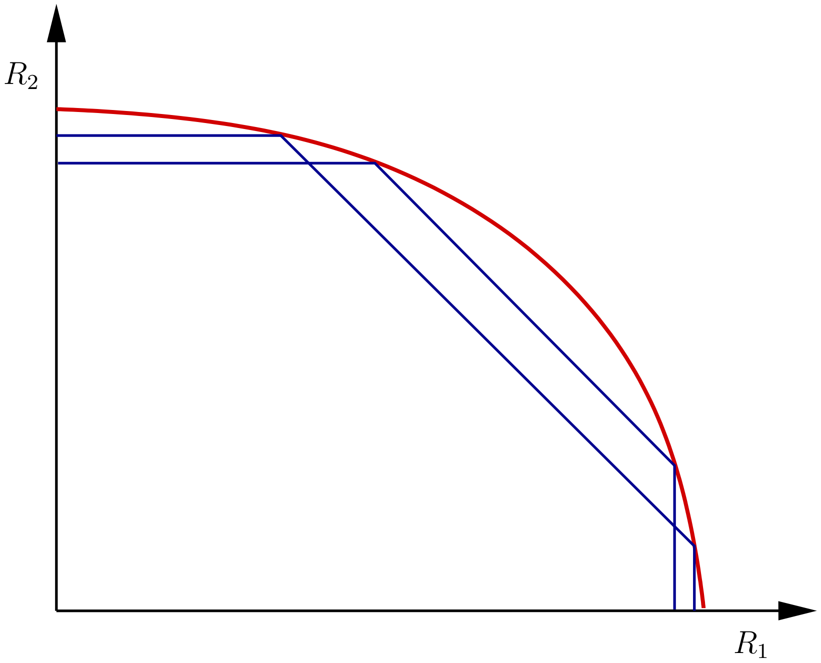
$$R_1 + R_2 + R_3 \leq \rho_{123}$$
$$R_1 + R_2 \leq \rho_{12} \quad R_1 + R_3 \leq \rho_{13}$$
$$R_2 + R_3 \leq \rho_{23}$$
$$R_1 \leq \rho_1 \qquad R_2 \leq \rho_2 \qquad R_3 \leq \rho_3$$

General expression:

$$\sum_{i=1}^{n} R_i \leq \rho(\mathcal{S}) \qquad \mathcal{S} \subseteq \{1, 2, \ldots n\}$$

- **MAC with single tx and rx antennas and time invariant channel;**
- **MAC with single tx and rx antennas and flat fading channels known at rx but not at tx;**
- **Multiple tx and rx antennas with unbiased flat fading known at rx but not at tx.**

Rate Regions for MAC: non-Polytope Convex Regions



$$\mathcal{C} = \bigcup_{\mathcal{P} \in F} \mathcal{C}_{\mathcal{P}}$$ F set of power policies $\mathcal{P}$

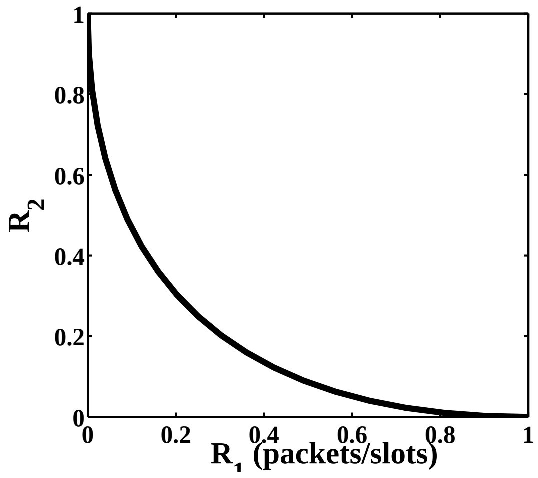
satisfying power constraints,

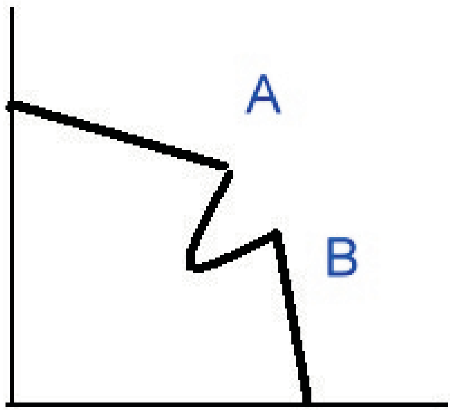
$$\mathcal{C}_{\mathcal{P}} \left\{ \sum_{i=1}^{n} R_i \le \rho(\mathcal{S}, \mathcal{P}) \qquad \mathcal{S} \subseteq \{1, 2, \dots n\} \right.$$

The convex rate region is the union of polytope rate regions, when time-sharing of coding schemes is possible.

- **MAC with single tx and rx antennas and time invariant channels with ISI;**
- **MAC with multiple tx and rx antennas and time invariant channel (with and without ISI);**
- **Single or multiple tx and rx antennas with flat fading known at rx and tx.**

Rate Regions for MAC: non-Convex Regions





When time-sharing of coding schemes is not possible, rate regions may be non-convex.

Elements preventing time-sharing

- **No common time reference is shared by transmitters**
- **Absence of feedbacks.**

MAC with non-convex rate region

- **Collision channel without feedback;**
- **Recovery channel.**

Selection Criteria

Fairness Criteria ⇓ **Optimum Points**

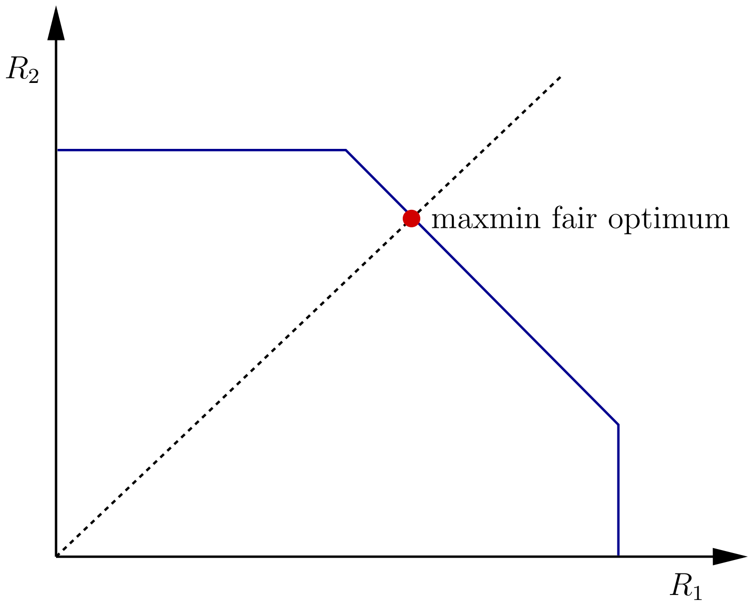
- **Maxmin farness**
- **Generalized α-fairness**
- **Weighted generalized α-fairness**

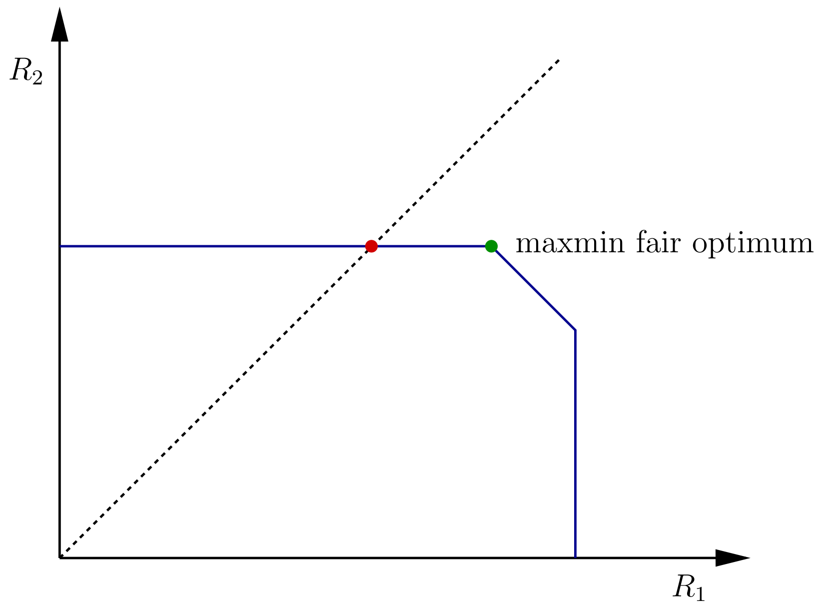
Games ⇓ **Equilibrium points**

- **Social games with common constraints**
- **Normal equilibrium**

Fairness Criteria: Maxmin Fairness

A rate allocation is said to be maxmin fair if the rate allotted to a user can not be improved without decreasing the one of any other user having equal or less rate.



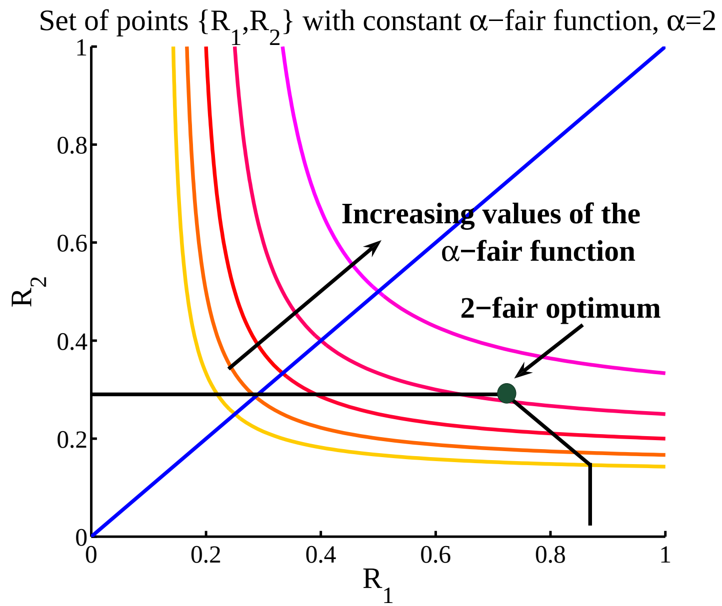


Fairness Criteria: Generalized α-Fairness

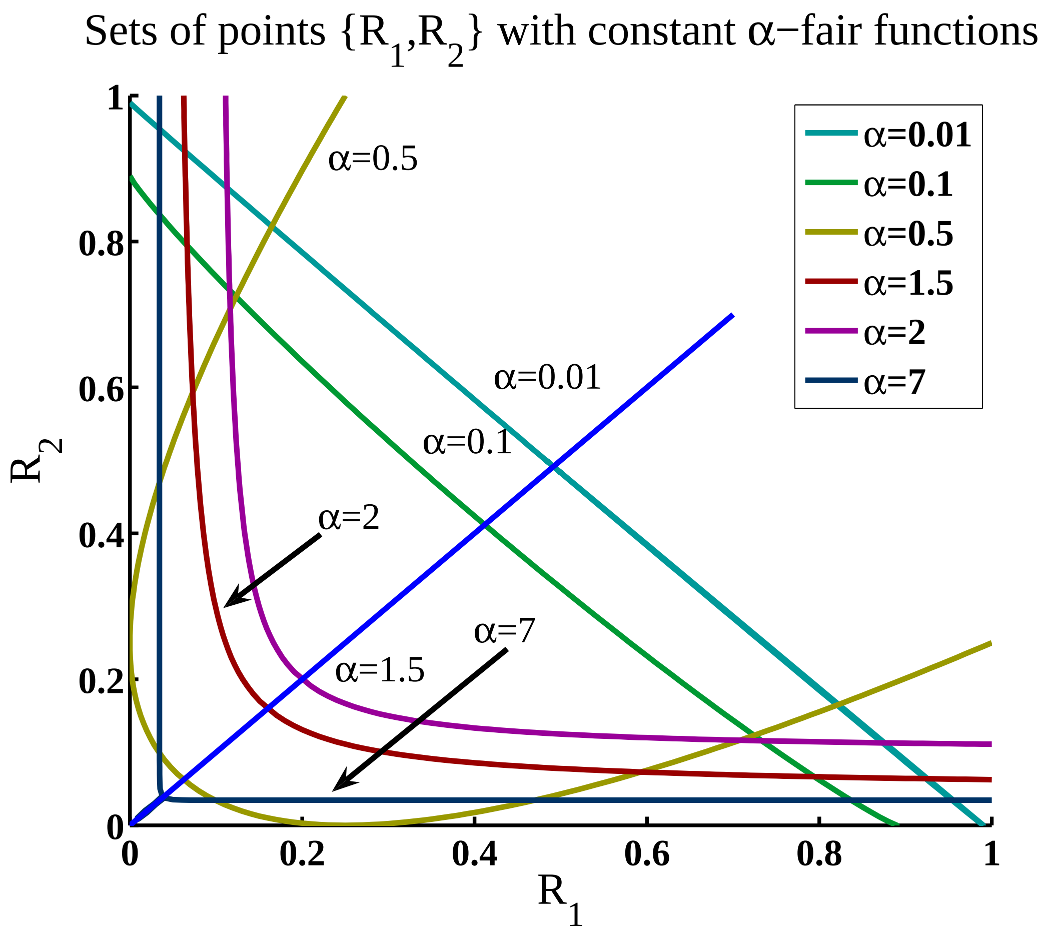
Let $U(R)$ a utility function, typically strictly increasing and concave, a generalized α-fair assignment maximizes the utility function

$$\sum_i \frac{U(R_i)^{1-\alpha}}{1-\alpha} \qquad \text{for } \alpha \neq 1$$

The utility function $U(R) = R$ leads to conventional α-fair assignment



Fairness Criteria: α-Fairness (2)



- $\alpha = 0 \Rightarrow$ **globally optimum assignment (sum rate);**
- $\alpha \rightarrow 1 \Rightarrow$ **proportionally fair assignment;**
- $\alpha = 2 \Rightarrow$ **harmonic fairness;**
- $\alpha = +\infty \Rightarrow$ **maxmin fairness.**

Rate Allocation as Social Games with Common Constraints

A social game with common constraints in the MAC consists in the maximization of the rate R_i, or, more generally, of a strictly concave increasing function of the rate $U(R_i)$ under the constraint that no user can tolerate the losses that would occur if transmission rates were chosen outside the achievable rate region.

All the rate allocations corresponding to points of the dominant face of a polytope rate region are Nash equilibrium and Pareto efficient. Any other point is not an equilibrium.

Constrained Games, Pricing, and Normal Equilibrium

Define the payoff function for user i as

$$L_i^{\boldsymbol{\lambda}}(R_1, R_2 \dots R_n) = U(R_i) + \lambda_i \left(\sum_i R_i - \rho(\{1, 2, \dots n\}) \right)$$

- λ_i is the unit price user i pays for a deviation of the equilibrium point from the dominant face.
- A specific Nash equilibrium on the game with common constraint is obtained if we consider the following game
  - Utility functions: $L_i^{\boldsymbol{\lambda}}(R_1, R_2 \dots R_n)$;
  - Common constraint: the rate region obtained from the original problem by suppressing the constraint on the dominant face.

Constrained Games, Pricing, and Normal Equilibrium (2)

If the unit price is equal for all users, i.e. $\boldsymbol{\lambda} = (\lambda, \lambda \ldots \lambda)$, the equilibrium point is the normalized equilibrium.

A normalized equilibrium leads to scalable distributed pricing (billing per packet and independent of the tx)

Motivations and Objects

- **How do the optimum fair points depend on the shape of the rate region?**
- **Are the optimum fairness points unique?**
- **Can they be attained by distributed algorithms?**
- **How are they related each other?**
- **How do the optimum fair points change by changing the utilities functions?**
- **How do the optimum fair points change for different classes of services?**
- **Are distributed scalable pricing applicable? i.e. there exist a normalized equilibrium?**

Fairness for Polytope Rate Regions

For all $\alpha \geq 0$ the α-fair assignment coincides with the unique maxmin fair.

For all $\alpha \geq 0$ and any strictly concave increasing utility function, the generalized α-fair assignment coincides with the unique maxmin fair.

The α-fair optimum point belongs to the dominant face and is Shur-majoried by any other point on the dominant face.

There exists an algorithm (Shum et Sung '06) with complexity $O(n^2)$ to determine the maxmin fair rate allocation.

Normalized Equilibrium for Polytope Rate Regions

The normalized equilibrium exists and is unique. It coincides with the unique maxmin fair rate allocation.

Decentralized, scalable pricing policies are feasible!

Decentralized rate allocation are possible!

The global optimization problem $\sum_i U(R_i)$ is maximized at the maxmin fairness rate allocation.

α-Fairness and Convex Non-Polytope Rate Regions

The α-fair rate allocation for the orthogonal time invariant MAC (FDMA/TDMA)

$$R_i \leq \theta_i \ln\left(1+\frac{P_i h_i}{\theta_i \sigma^2}\right) \qquad \forall S \subseteq \{1,\dots n\} \qquad 0 \leq \sum_i \theta_i \leq 1$$

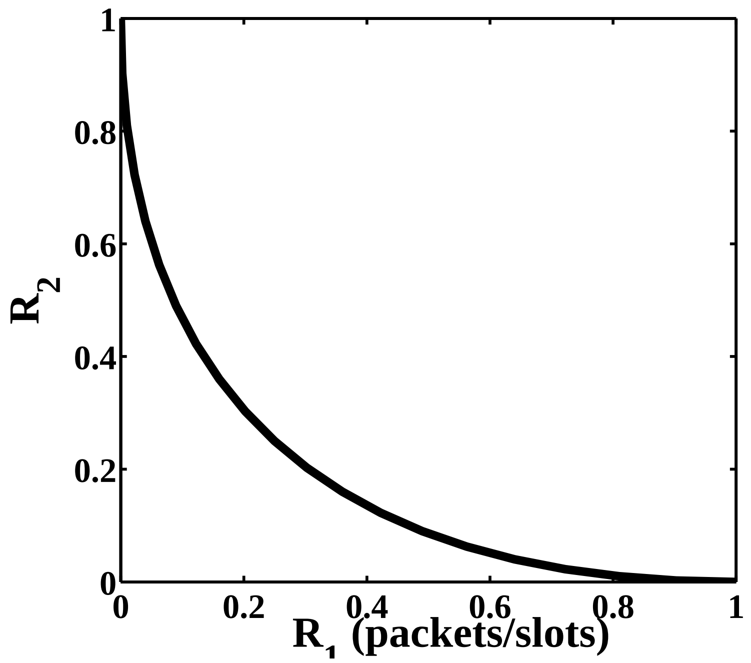
exists and is unique. It is obtained for the set $\{\theta_i\}$ satisfying the system

$$\left[\theta_i \ln(1+\frac{P_i h_i}{\theta_i \sigma^2})\right]^{-\alpha}\left[\ln(1+\frac{P_i h_i}{\theta_i \sigma^2}) - \frac{P_i h_i}{\theta_i \sigma^2 + P_i h_i}\right] = \lambda_0,$$

$$\sum_{i=1}^{n} \theta_i = 1.$$

For strictly convex rate regions the α-fair allocation is unique !

Non-Convex Rate Regions: Collision Channel



$$C_i = p_i \prod_{\substack{j=1 \\ j \neq i}}^{n} (1 - p_j) \text{ and } \sum_{j=1}^{n} p_j = 1$$

There exists a threshold α^* such that

- $\alpha > \alpha^*$**: the α-fair rate allocation is unique and coincide with the maxmin fair.**
- $\alpha = \alpha^*$**: there exist several optimum points. One is the maxmin fair rate allocation and the others correspond to the total rate optimization. If $n = 2$ any point on the boundary is α^*-fair.**
- $\alpha < \alpha^*$**: the α-fair rate allocations correspond to the total rate optimum points. They are the n points $R_i = 1$, and $R_j = 0$.**

Generalized Collision Channel

$$R_i \leq p_i \prod_{\substack{j=1 \\ j \neq i}}^{n} (1 - p_j) + q p_i \left(1 - \prod_{\substack{j=1 \\ j \neq i}}^{n} (1 - p_j) \right) \text{ and } 0 \leq p_i \leq 1$$

There exists a threshold α^* such that

- **$\alpha > \alpha^*$: the α-fair rate allocation is unique and coincide with the maxmin fair $R_1 = \ldots = R_n = q$.**
- **$\alpha < \alpha^*$: there are n fair rate allocations corresponding to the total rate optimization. More specifically, $R_i = 1$, and $R_j = 0$.**
- **$\alpha = \alpha^*$: there exist $n + 1$ optimum points. One is the maxmin fair rate allocation and the others correspond to the total rate optimum points.**

Conclusions 1

- How do the optimum fair points depend on the shape of the rate region?

  For polytope rate regions, all the investigated fair criteria lead to the same optimum. The optimum is also the unique normalized equilibrium achievable with games. For non-polytope convex rate regions, different fair criteria lead to the different unique optima. For non-convex rate regions, the fair rate allocations depend in general of the fair criteria and are not unique.

- Are the optimum fairness points unique?

  Uniqueness is a general property of the convex rate region. In general, the fair rate allocation points are not unique for non-convex rate regions and they may be a discrete finite or even a continuous finite set.

- Can they be attained by distributed algorithms?

  In case of polytope rate regions the fair points coincide also with the normalized equilibrium. $\Longrightarrow$ Distributed and scalable pricing and allocation algorithms.

Conclusions 2

- How do the optimum fair points change by changing the utilities functions?
  For polytope rate regions, the fair optimum does not change by changing the fairness function except for the weighted α-fairness
- How do the optimum fair points change for different classes of services?
  If the weighted α-fair optimum is on the dominant face it depends on α.
- Are distributed scalable pricing applicable? i.e. there exist a normalized equilibrium?
  In polytope rate regions, do exist unique normalized equilibrium. Rate allocation and billing can be performed in a distributed manner.