Channel State Feedback over the MIMO-MAC

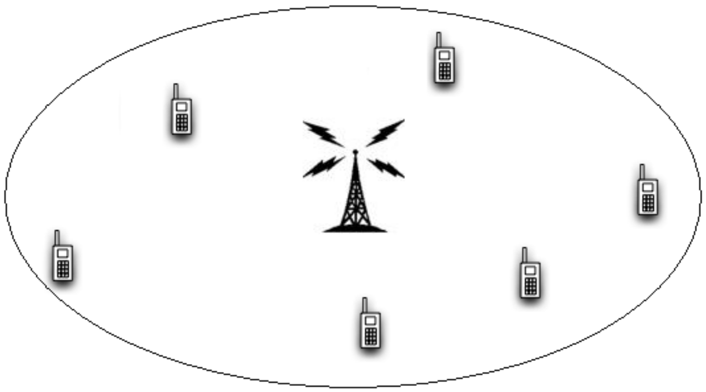
K. Raj Kumar
University of Southern California

Joint work with
Giuseppe Caire, University of Southern California

17 March 2009

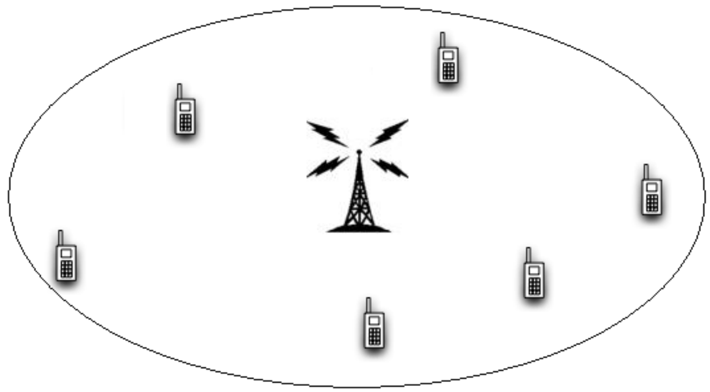
- Scenario: FDD Cellular system
- Problem of feedback of downlink channel coefficients from mobile terminals to base station
- Relevance of average distortion
- Performance of separated source-channel coding and joint source-channel coding
- Simulation results

The FDD Cellular Setting



- Frequency division duplex (FDD) system
- Sufficient spacing between uplink and downlink channels (i.i.d.)
- BS serves multiple users using some MIMO broadcast channel precoding technique (DPC, linear beamforming, ...)
- BS needs to know the downlink channels
- User terminals (UT) know downlink channel perfectly through training
- UTs need to feedback channel coefficients to the BS (over a MIMO-MAC)

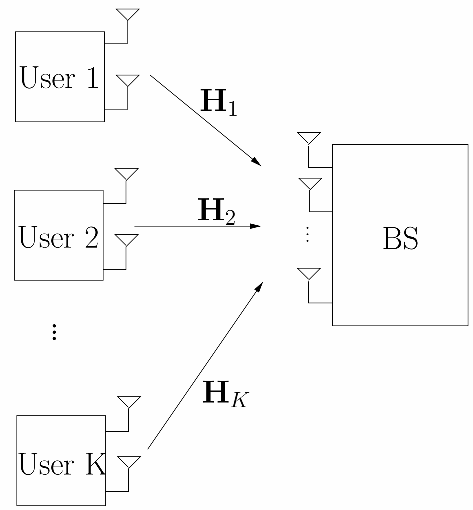
The FDD Cellular Setting



- Previous literature:
  - ◇ Assume perfect feedback at fixed rate
  - ◇ Assume feedback info is piggybacked "somehow" onto uplink transmissions
  - ◇ Analog feedback (send unquantized channel coefficients)
- Some comments:
  - ◇ Low delays, Short block-lengths
  - ◇ Have fixed time-frame allocated for feedback (*not* fixed number of bits)
    - ∗ Users with different SNRs feedback different number of bits, over the same time-frame

Goal: Design low-latency, low-complexity CSI feedback schemes for the uplink MIMO-MAC

MIMO-MAC model

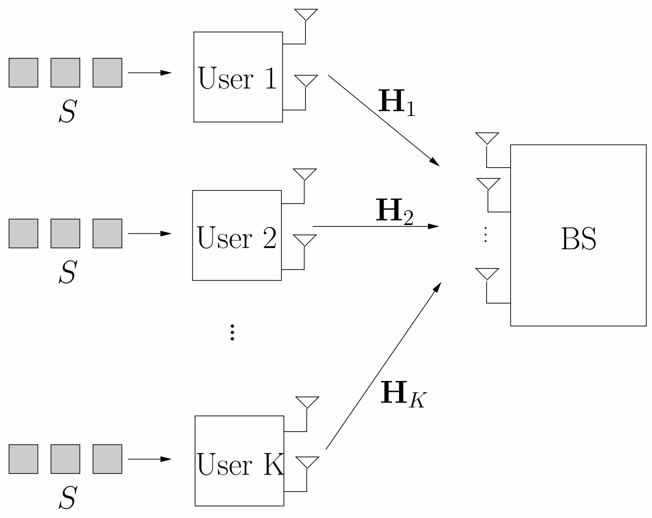


- K users, each with N_t transmit antennas, M antennas at the BS
- Channel model:

$$\underbrace{\mathbf{Y}}_{M \times T} = \sqrt{\rho} \sum_{i=1}^{K} \mathbf{H}_i \mathbf{X}_i + \mathbf{W}$$

  - ◇ Block-length T
  - ◇ $\mathbf{H}_i \in \mathbb{C}^{M \times N_t}$, $\mathbf{X}_i \in \mathbb{C}^{N_t \times T}$
- Quasi-static Rayleigh fading $\mathbf{H}_i \sim$ i.i.d. $\mathcal{CN}(0,1)$, $\mathbf{W} \sim$ i.i.d. $\mathcal{CN}(0,1)$
- Per-user power constraint $\mathbb{E}[\|\mathbf{X}_i\|_F^2] \leq T$
- $\rho \longrightarrow$ Uplink transmit SNR

Some Notation



- Information source: channel coefficients, $\sim \mathcal{CN}(0,1)$
- Each user transmits S samples from this source in T channel uses
- Bandwidth efficiency $b = T/S$
- $D(\rho) \rightarrow$ Average squared error distortion between the true downlink channel and the reconstruction at the BS
- Define

$$\delta(b) \triangleq -\lim_{\rho\to\infty} \frac{\log D(\rho)}{\log \rho},$$

  i.e., $D(\rho) \doteq \rho^{-\delta(b)}$
- $\delta^*(b) \rightarrow$ best possible $\delta(b)$ for *any* scheme

Relevance of Average Distortion

- Downlink rate gap $\Delta R(\rho)$: Difference between downlink rate achievable with perfect CSIT and the rate achievable with imperfect CSIT
- Relevance of distortion: [Jindal 2006], [Caire et al 2007]

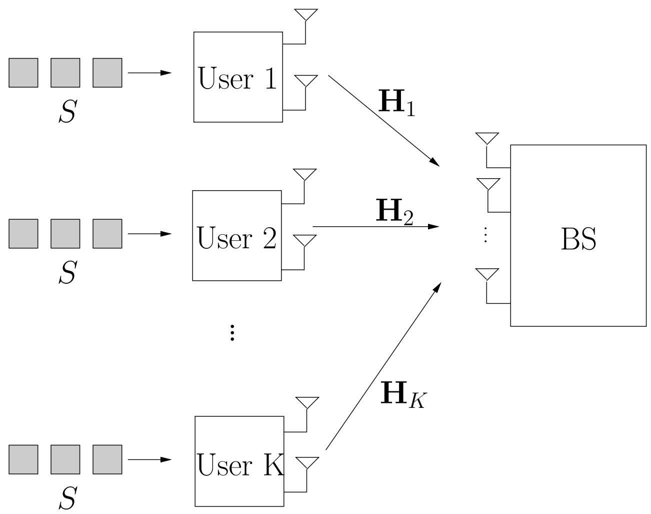
$$\begin{aligned} \Delta R(\rho) &\propto \log(1 + \rho D(\rho)) \\ &\doteq \log(1 + \rho \cdot \rho^{-\delta(b)}) \end{aligned}$$

  ◇ $\delta(b) < 1 \Rightarrow \Delta R(\rho) \to \infty$ with ρ
  ◇ $\delta(b) = 1 \Rightarrow$ constant rate gap
  ◇ $\delta(b) > 1 \Rightarrow$ rate gap vanishes with ρ

- Analog feedback achieves $\delta(b) = 1$ (constant rate gap)

Can we design schemes with $\delta(b) > 1$?

Upper bound on the distortion exponent $\delta^*(b)$



- Equal rates $R_i = R = r \log \rho$, bandwidth expansion b for each user
- Informed transmitter upper bound: Provide all channels $\{\mathbf{H}_i\}_{i=1}^K$ to all users.
- Each user transmits at the maximum R such that

$$(R, R, \ldots, R) \in \mathcal{C}_{MAC}(\{\mathbf{H}_i\}, \rho)$$

- From the expression for the capacity region for the MIMO-MAC,

$$R \leq \frac{1}{|\mathcal{K}|} \log\det\left(\mathbf{I} + \rho \sum_{k \in \mathcal{K}} \mathbf{H}_k \mathbf{H}_k^{\mathsf{H}}\right)$$

for all subsets $\mathcal{K} \subseteq \{1, \ldots, K\}$

Upper bound on the distortion exponent $\delta^*(b)$

- Channel known at transmitter: optimal to use separated source channel schemes in this case
  - ◇ Source coding rate $R_s = bR$

$$
\begin{aligned}
D(\rho) &\geq \mathbb{E}[\exp(-bR)] \\
&\geq \mathbb{E}\left[\det\left(\mathbf{I} + \rho \sum_{k \in \mathcal{K}} \mathbf{H}_k \mathbf{H}_k^{\mathsf{H}}\right)^{-\frac{b}{\mathcal{K}}}\right]
\end{aligned}
$$

- Let $\mathbf{H}_{\mathcal{K}}$ be the equivalent $M \times |\mathcal{K}| N_t$ channel comprising of the channels of the users in $\mathcal{K}$ stacked next to each other
- Let $\lambda_1 \leq \cdots \leq \lambda_{m_{\mathcal{K}}}$ to be the $m_{\mathcal{K}}$ ordered non-zero eigenvalues of $\mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^{\mathsf{H}}$, where $m_{\mathcal{K}} \triangleq \min\{M, |\mathcal{K}| N_t\}$
- Obtain

$$
\begin{aligned}
D(\rho) &\geq \mathbb{E}\left[\exp(-bR(\mathbf{H}))\right] \\
&\geq \mathbb{E}\left[\prod_{i=1}^{m_{\mathcal{K}}} (1 + \rho \lambda_i)^{-\frac{b}{|\mathcal{K}|}}\right]
\end{aligned}
$$

Upper bound on the distortion exponent $\delta^*(b)$

- Set $\lambda_i \doteq \rho^{-\alpha_i}$. Joint pdf of the α_i:

$$p(\boldsymbol{\alpha}) \doteq \begin{cases} \rho^{-\sum_{i=1}^{m_\mathcal{K}}(2i-1+n_\mathcal{K}-m_\mathcal{K})\alpha_i}, & \alpha_1 \geq \cdots \geq \alpha_{m_\mathcal{K}} \geq 0 \\ \rho^{-\infty}, & \text{otherwise} \end{cases}.$$

- Compute average distortion and obtain

$$\delta^*(b) \leq \min\left\{\delta_1(b), \ldots, \delta_K(b)\right\}, \;\; \delta_k(b) = \sum_{i=1}^{\min\{M,kN_t\}} \min\left\{\frac{b}{k}, 2i-1+|M-kN_t|\right\}$$

Can we achieve this upper bound? Need to analyze particular schemes to obtain achievability results.

The Diversity-Multiplexing Tradeoff (DMT) formulation

- Asymptotic characterization of the tradeoff between rate and reliability for outage limited channels [Zheng and Tse, 2003]

Rate

- Multiplexing gain r

$$R = r \log_2 \rho$$

Reliability

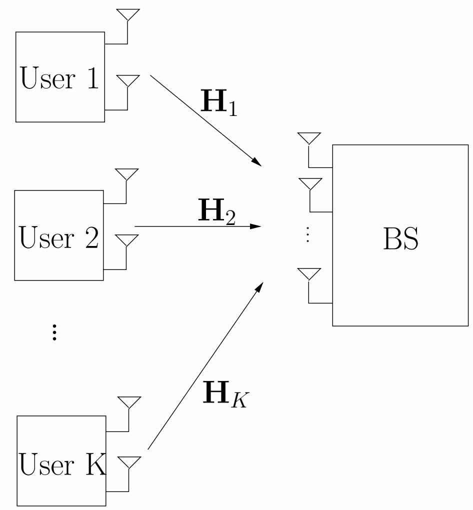
- Diversity gain $d(r)$: $P_e \doteq \rho^{-d(r)}$

$$\lim_{\text{SNR}\to\infty} \frac{\log P_e(\rho)}{\log \rho} = -d(r)$$

DMT $d^*(r) \to$ best possible diversity for multiplexing gain r

- Turns out: $d^*(r)$ is the negative SNR exponent of outage probability

DMT of the MAC



- Channel model:

$$\mathbf{Y} = \sqrt{\rho} \sum_{i=1}^{K} \mathbf{H}_i \mathbf{X}_i + \mathbf{W}$$

- Per-user power constraint

$$\mathbb{E}[\|X_i\|_F^2] \leq T$$

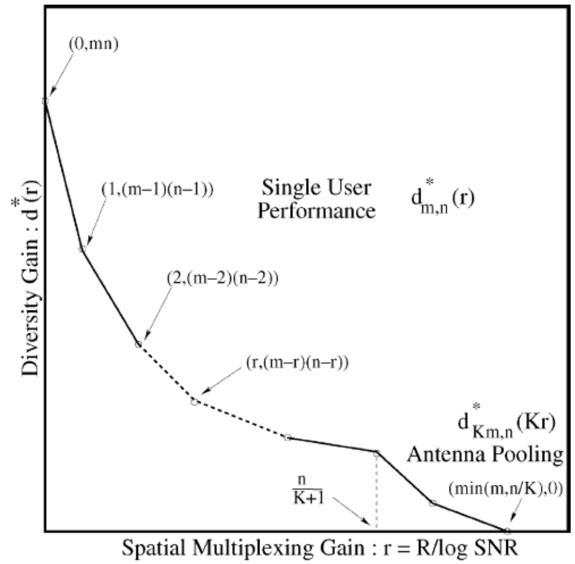
- Common diversity requirement

$$P_e^{(i)} \dot{\leq} \rho^{-d}, \quad i = 1, \ldots, K$$

- Symmetric rates

$$R_i = R = r \log \rho, \quad i = 1, \ldots, K$$

DMT of the MAC



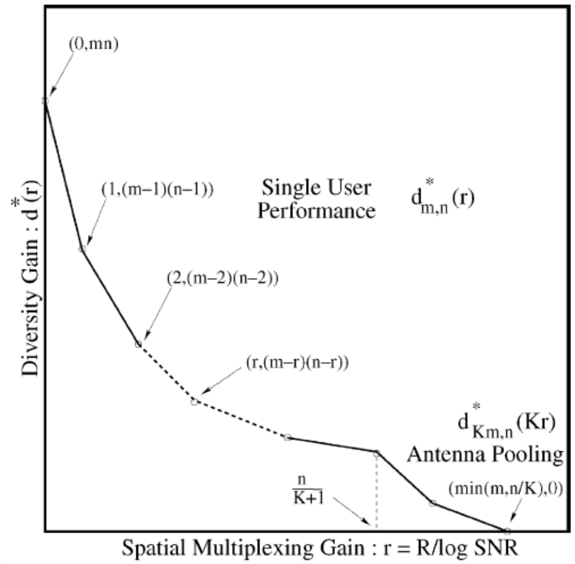
- Symmetric DMT of the MAC [Tse-Viswanath-Zheng, 2004]

$$d^*(r) = \begin{cases} d^*_{M,N}(r), & r \le \min\left\{M, \frac{N}{K+1}\right\} \\ d^*_{KM,N}(Kr), & r \ge \min\left\{M, \frac{N}{K+1}\right\} \end{cases},$$

where $d^*_{M,N}(r)$ is the piecewise linear function that interpolates $(M-r)(N-r)$ for $r = 0, 1, \ldots, \min\{M, N\}$.

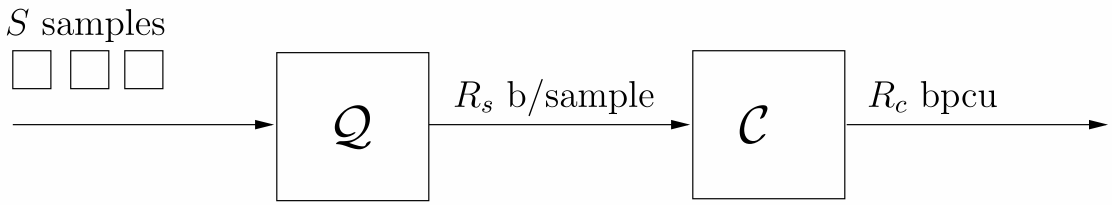
- Achievable for $T \ge KM + N - 1$ with random Gaussian codes

DMT of the MAC$_2$



- Typical error-events:
  - $\diamond$ $r \leq \min\left\{M, \frac{N}{K+1}\right\}$: just one of the user's message is decoded incorrectly
  - $\diamond$ $r \geq \min\left\{M, \frac{N}{K+1}\right\}$: all user messages are decoded incorrectly
- Joint decoding is necessary: successive interference cancellation, time-sharing are suboptimal

Separated source-channel coding



- $R_s = bR_c$
- Can show:

$$D_{sep}(\rho) \leq D_Q(R_s) + \kappa P_e(\rho)$$

- Rates $R_c = r_c \log \rho$, $R_s = r_s \log \rho \Rightarrow (r_s = br_c)$
- Rate distortion achieving quantizer: $D_Q(R_s) \doteq \rho^{-br_c}$
- DMT optimal channel code: $P_e(\rho) \doteq d^*(r_c)$
- Notice: DMT formulation is especially relevant - might have large SNR imbalances between users, feedback bits required scale as $\log \rho$ [Jindal 2006]
- Obtain

$$D_{sep}(\rho) \dot{\leq} \rho^{-br_c} + \kappa \rho^{-d^*(r_c)}.$$

- Choose multiplexing gains r_c and r_s to minimize $D_{sep}(\rho)$: Solve

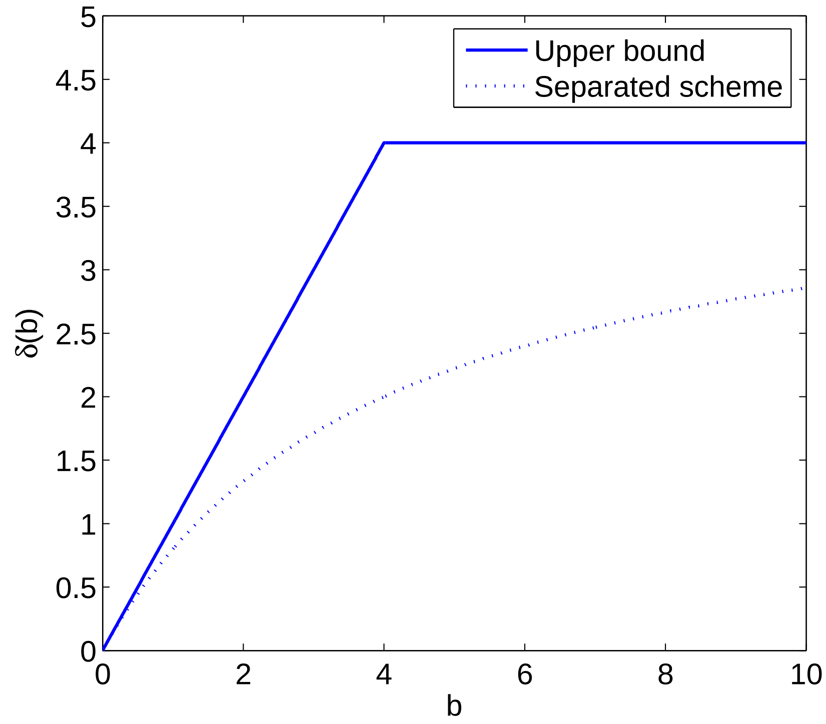
$$br_c = d^*(r_c)$$

Separated source-channel coding

- When $N_t = 1$, $M = K$,

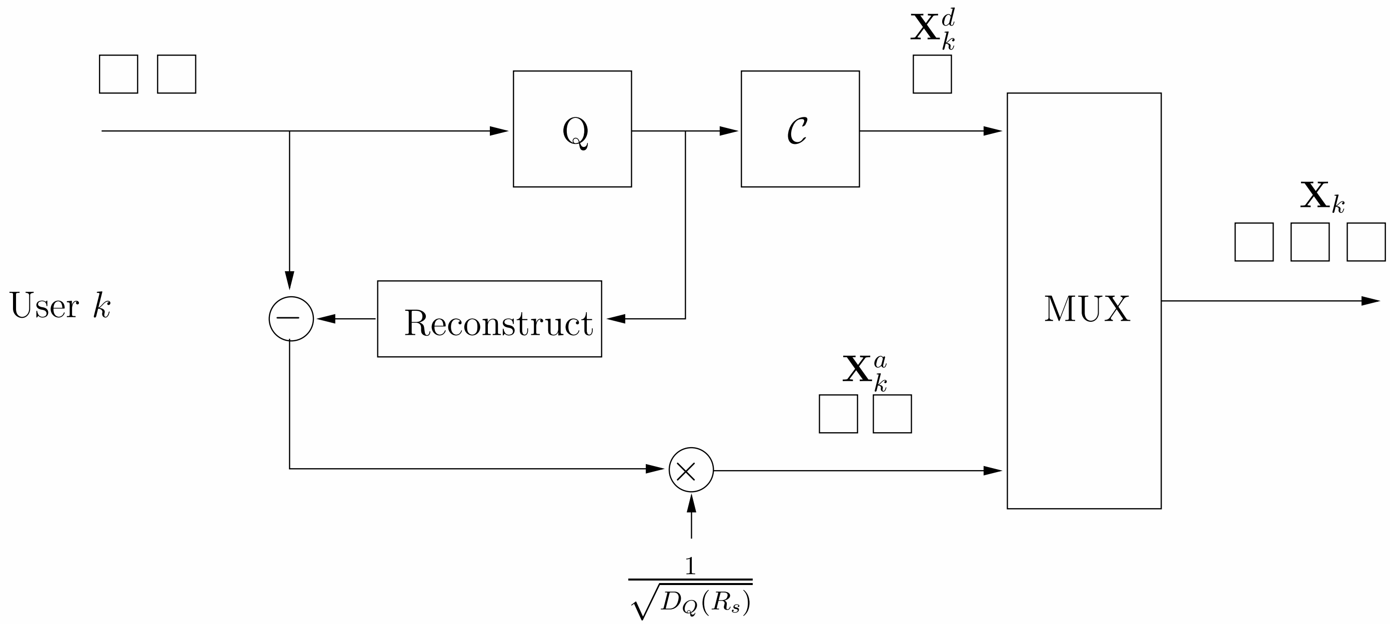
$$\delta_{sep}(b) \geq \frac{bK}{K+b}$$

- $K = 4$:



- Can we close the gap between the separated scheme and the upper bound?

Joint Source-Channel codes - Hybrid D/A scheme



- Consider $b > 1$ ("bandwidth expansion")
- Decoding at the BS:
  - ◇ First jointly decode "digital" data
  - ◇ If successful, obtain an MMSE estimate of quantization error and add to decoded data
- Distortion

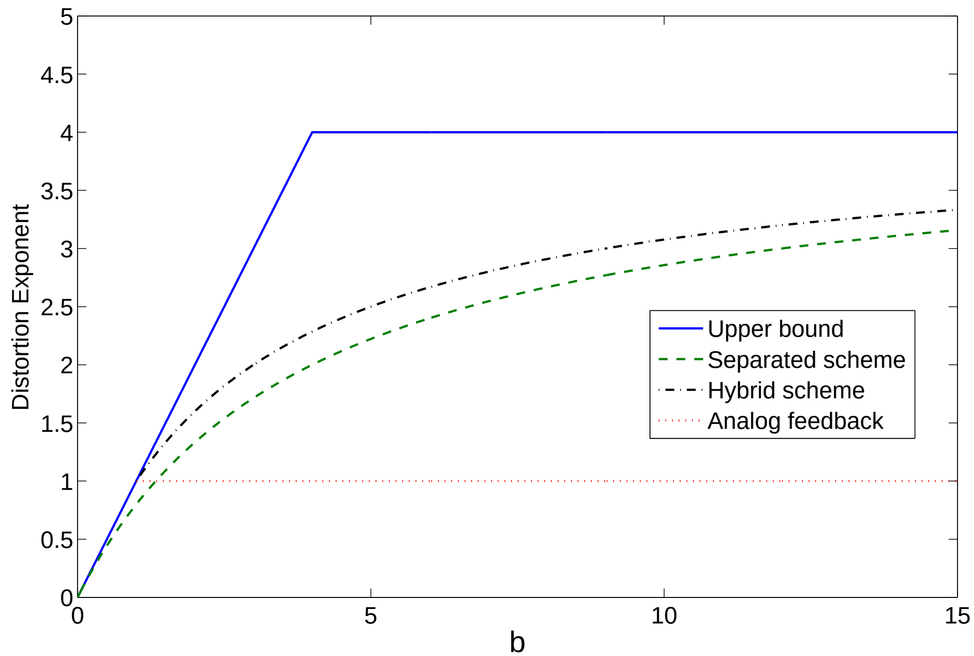
$$D_{hybrid}(\rho) = \int_{\mathcal{O}^c} \mathsf{mmse}(\mathbf{e}|\{\mathbf{H}_i\}) P(\{\mathbf{H}_i\}) d\{\mathbf{H}_i\} + \kappa P(\mathcal{O})$$

  - ◇ $\mathcal{O} \rightarrow$ MAC Outage event for joint detection of digital data

- Average over the channel, and choose rates to minimize $D_{hybrid}(\rho)$
- $N_t = 1$, $K = M$

$$\delta_{hybrid}(b) = 1 + \frac{(b-1)(K-1)}{K+b-1}.$$

- $K = M = 4$, $N_t = 1$:



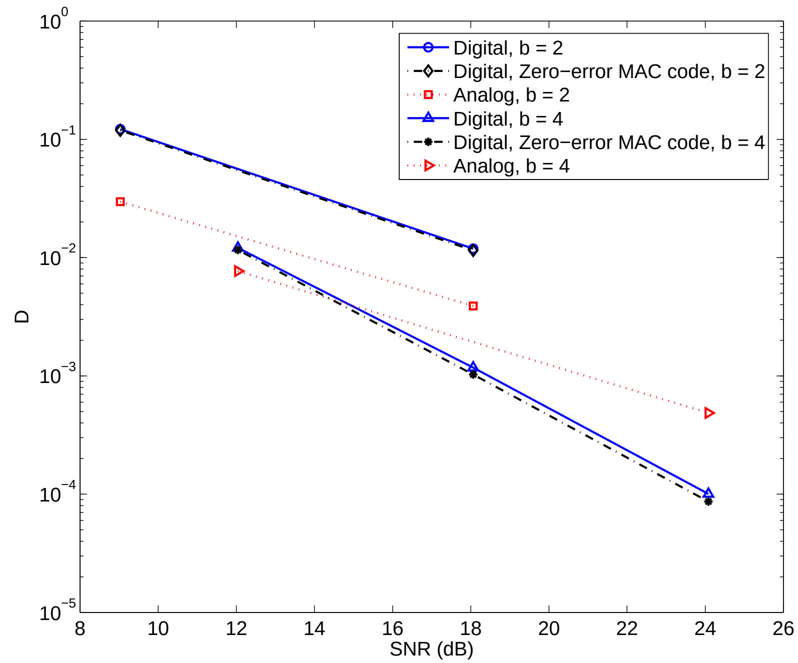
Simulation Results

- Compare performances of analog and digital feedback
- Digital feedback: separated source channel coding
  - ◇ Uniform scalar quantizer
  - ◇ Simple channel code: uncoded QAM!
  - ◇ Fix the multiplexing gain r_c that minimizes distortion, vary feedback rate with SNR:

$$R_c = r_c \log \rho$$

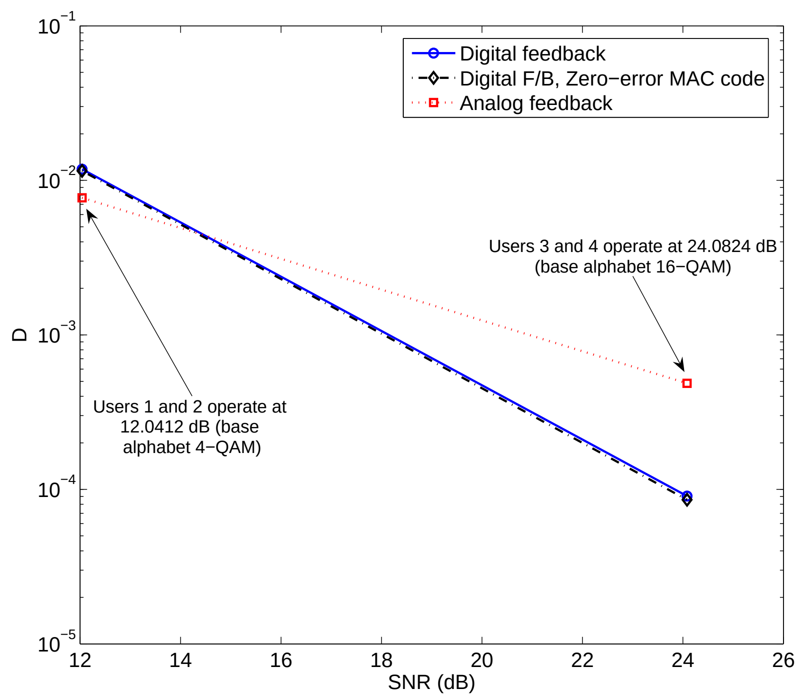
Equal SNRs for all users

- $N_t = 1$, $K = M = 4$
- Fix $T = 4$, vary $b = 2, 4$
- All users operate at the same SNR
- Analog feedback: $\delta(b) = 1 \; \forall \; b$
- Digital feedback: $\delta(2) = 1$, $\delta(4) = 2$
- Plot per-user distortion vs. SNR



Effect of SNR Imbalances

- Practical cellular system: SNRs for users close to BS and at cell edge differ by 10s of dBs
- Fix $K = M = 4$, $N_t = 1$, $b = 4$
  - ◇ Two high-SNR users operating at 24 dB (signal using 16-QAM)
  - ◇ Two low-SNR users operating at 12 dB (signal using 4-QAM)
- Distortions for the two user groups decay with the optimal exponent!



Conclusions

- Considered the problem of designing low complexity, low latency CSI feedback schemes
- Developed a framework in terms of minimizing distortion
- Presented separated source channel and joint source channel schemes that outperform analog feedback
- Validated the use of such schemes in practical scenarios using simple channel codes

Thank you!