Capacity Limits
of
Fibre-Optic Communication Systems

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Outline

- Introduction
- Information Theory
- The Optical Channel
- Impairments
- Nonlinearity Compensation
- Capacity of of the AWGN Channel
- Capacity of the Fiber Channel
- Conclusion

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Introduction

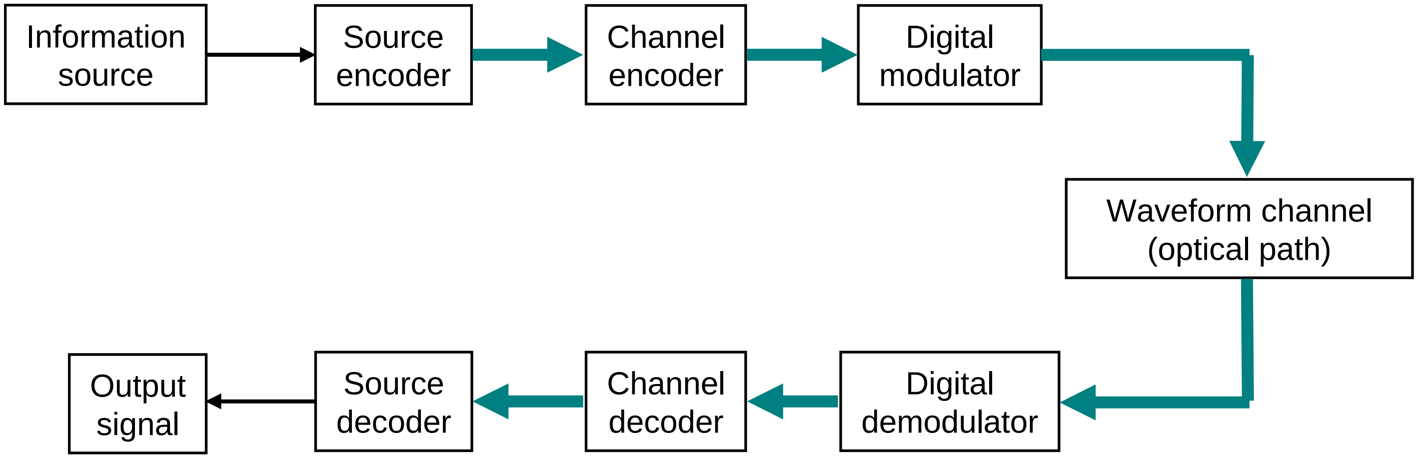
What are we trying to do?

Transmission of information over fibres is limited by

- **Noise** → Information theory
- **Fibre Kerr nonlinearity** → ???

Is there a fundamental limit to the transport of information over fibres due to Kerr fiber nonlinearity?

The Fiber Channel



Information source

Source encoder

Channel encoder

Digital modulator

Waveform channel (optical path)

Digital demodulator

Channel decoder

Source decoder

Output signal

→ “Fiber channel”

The waveform channel corresponds to nonlinear transmission over optical fibers

Array of Technologies to Increase Capacity?

Noise:

- Low-loss optical fibers
- Ideal low-noise distributed amplification
- Coding

Fiber nonlinearity:

- Large effective area fibers
- Advanced modulation formats
- Nyquist Signalling
- Reverse propagation at either transmitter and receiver
- Dispersion management
- Specially designed optical fibers

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- **Nyquist Signalling**
- **Reverse propagation at either transmitter and receiver**
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- Specially designed optical fibers

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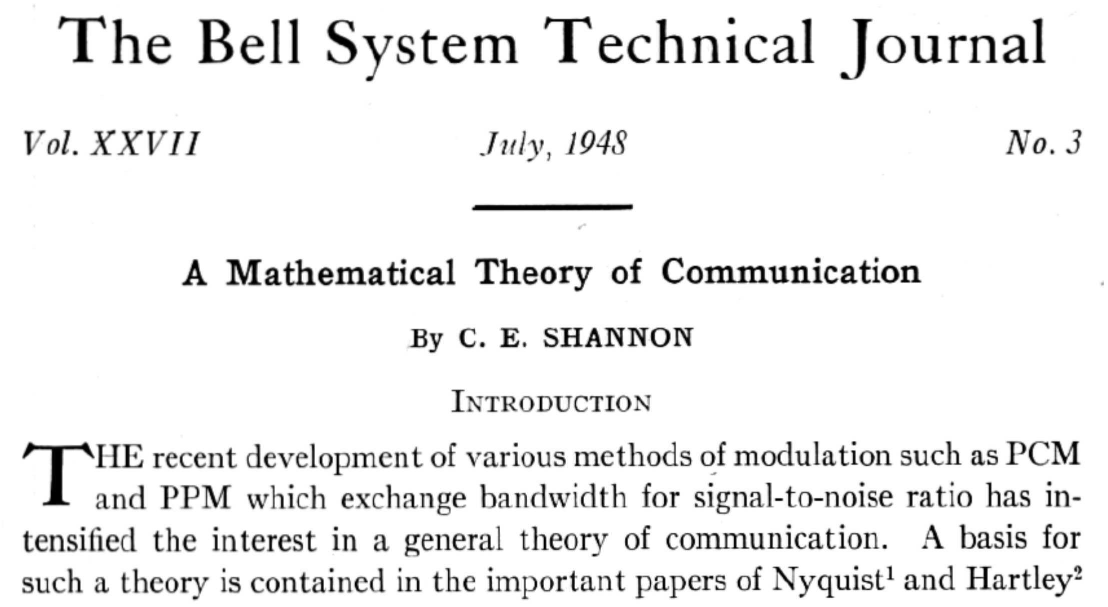
Information Theory

Information Theory: Impact of AWGN

Birth of information theory:

One paper (split in two) by Shannon in Bell System Technical Journal (1948)



The Bell System Technical Journal

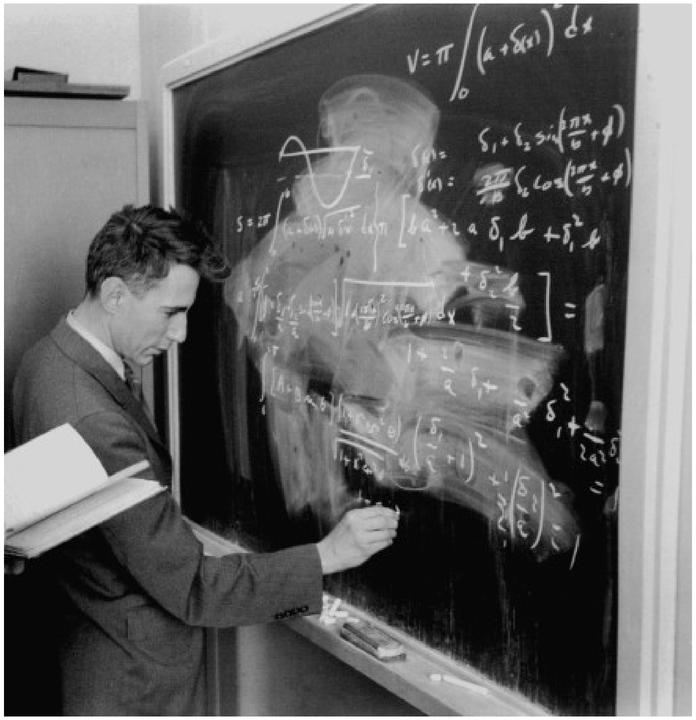
Vol. XXVII *July, 1948* *No. 3*

A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist[1] and Hartley[2]



Mathematical theory that calculates the maximum quantity of information that can be transmitted through an **additive white Gaussian noise (AWGN)** channel

Shannon's Formula of Channel Capacity (AWGN)

Shannon capacity:

$$C = B \log_2 (1 + SNR)$$

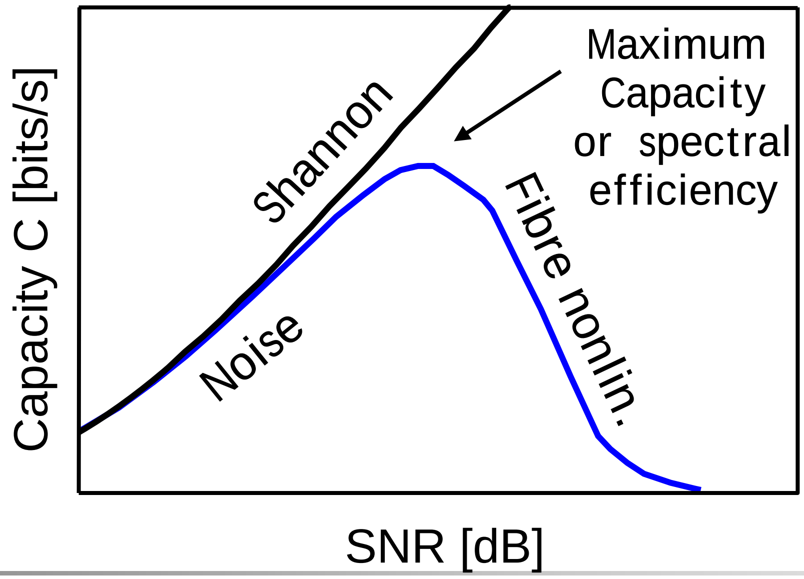
C: Channel capacity (bits/s)

B: WDM channel bandwidth (Hz)

SNR: Signal-to-noise ratio → Enery per symbol / power spectral density of the noise

Shannon formula implies using <u>optimum coding</u>!

Effect of fibre nonlinearity on 'fibre capacity'



Optimum Constellation for the AWGN Channel

Optimum constellation for the AWGN channel:

$$p_X(x) = \frac{1}{\pi P_s} e^{-(x_R^2 + x_I^2)/P_s}$$

p_X : Probability of having the symbol x

P_s : Signal average power

x_I : Imaginary part of symbol

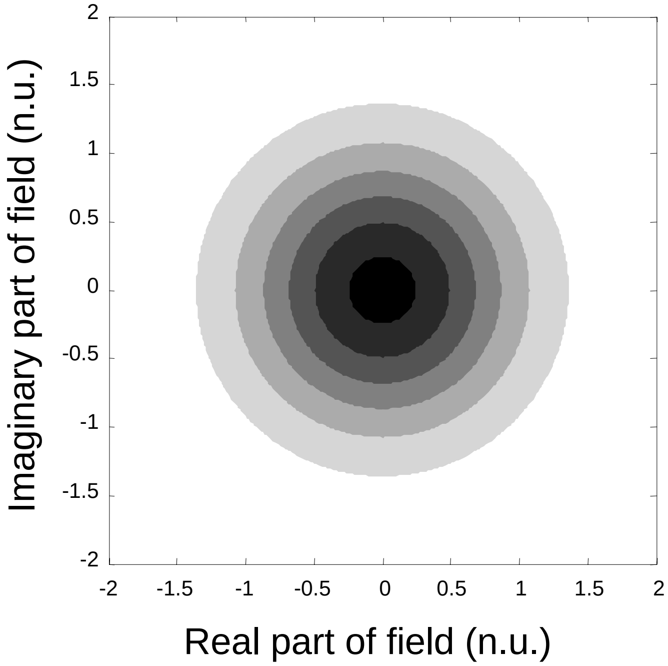
x_R : Real part of symbol

- The optimum constellation for the AWGN channel is a bidimensional Gaussian

Optimum Constellation for the AWGN Channel

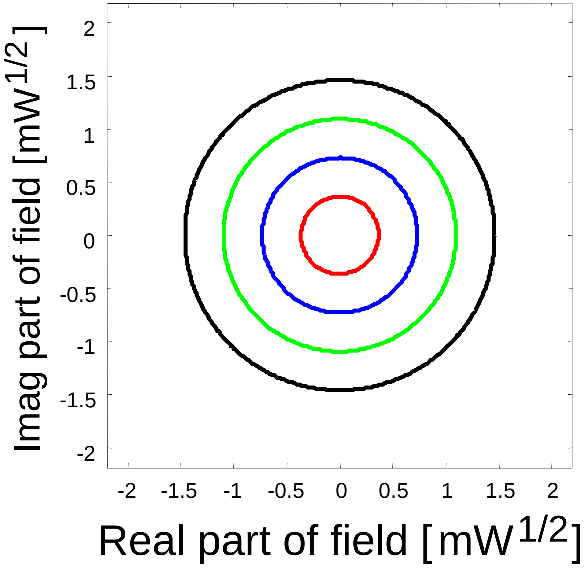
From bidimensional Gaussian to ring constellations:

Optimum constellation for AWGN



Ring constellation used for capacity study

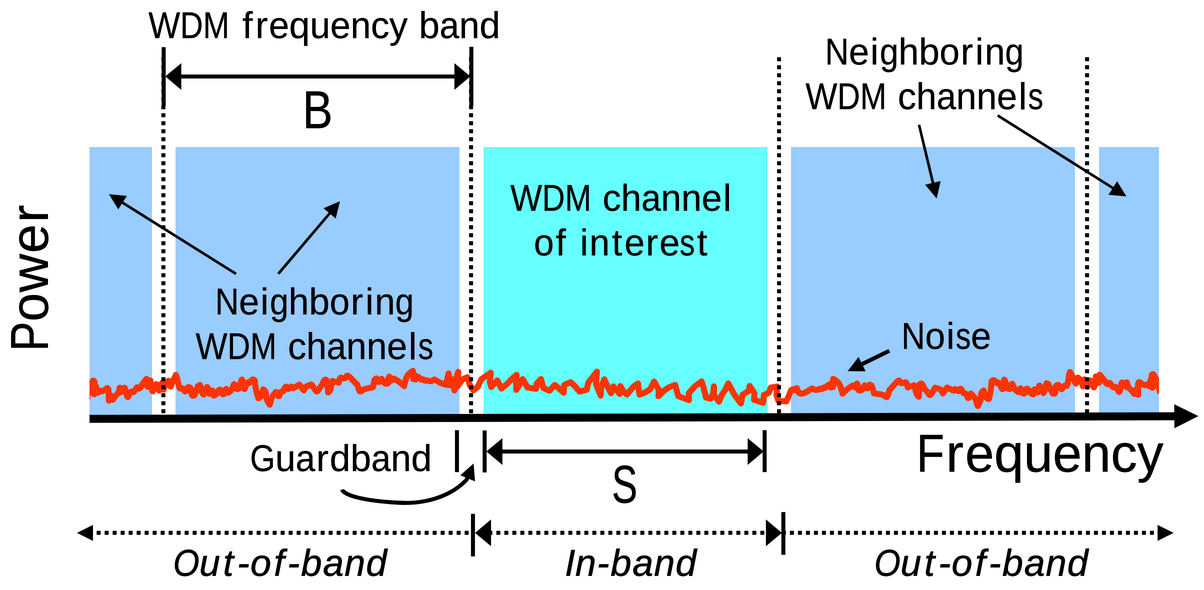
(Ex.: 4 rings with 0 dBm average power)



- Equal spacing between rings
- Equal frequency of occupation on each ring
- A ring constellation allows for simpler numerical evaluation of capacity

Optical Spectrum Allocation

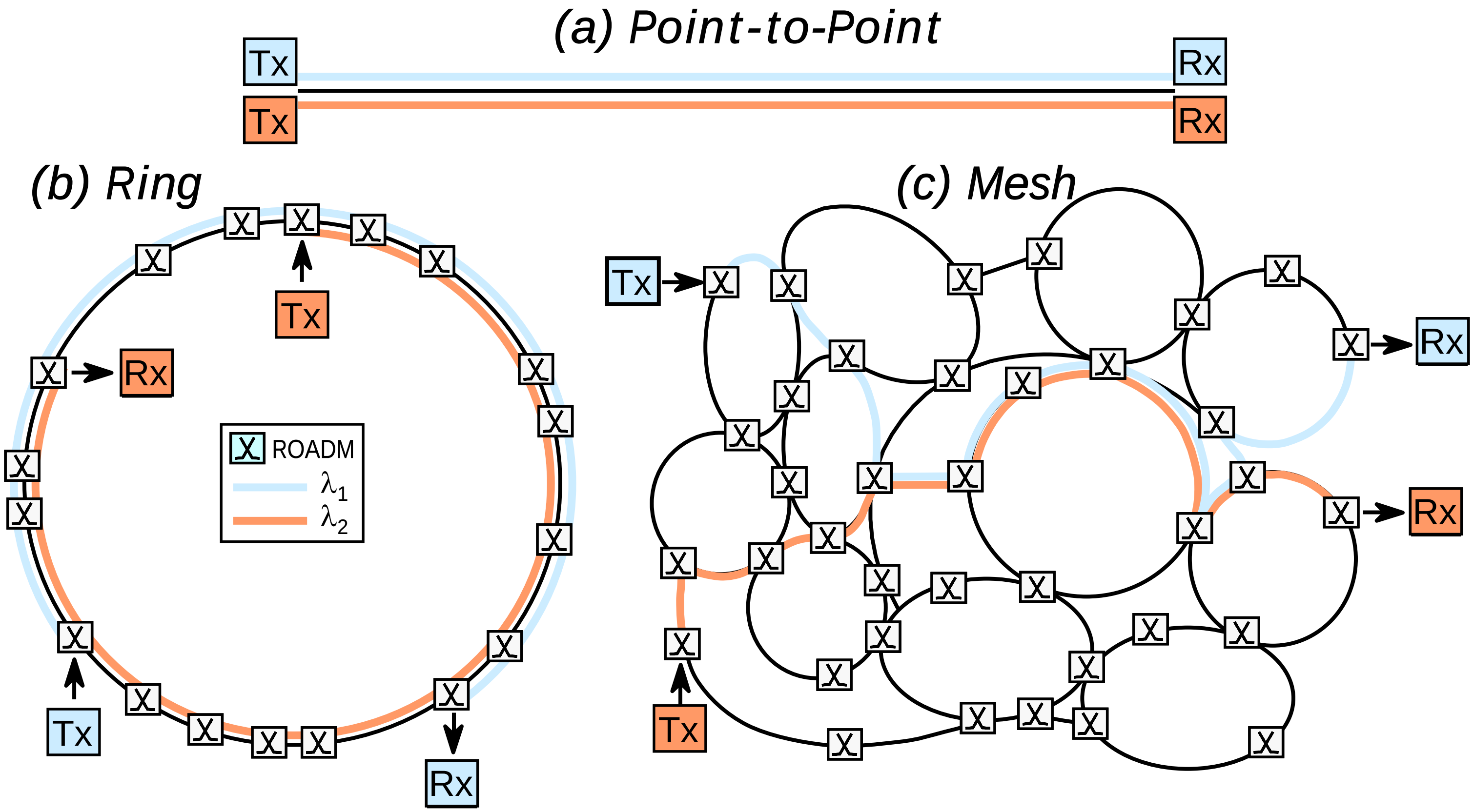
Wavelength-division multiplexing (WDM):



- We modulate at the Nyquist rate (signal bandwidth = symbol rate)
- Channels spacing is limited by signal bandwidth
- The ‘in-band’ frequencies signal and noise travel from the transmitter to the receiver
- The ‘out-of-band’ frequencies signal and noise are generally not available to the transmitter or the receiver

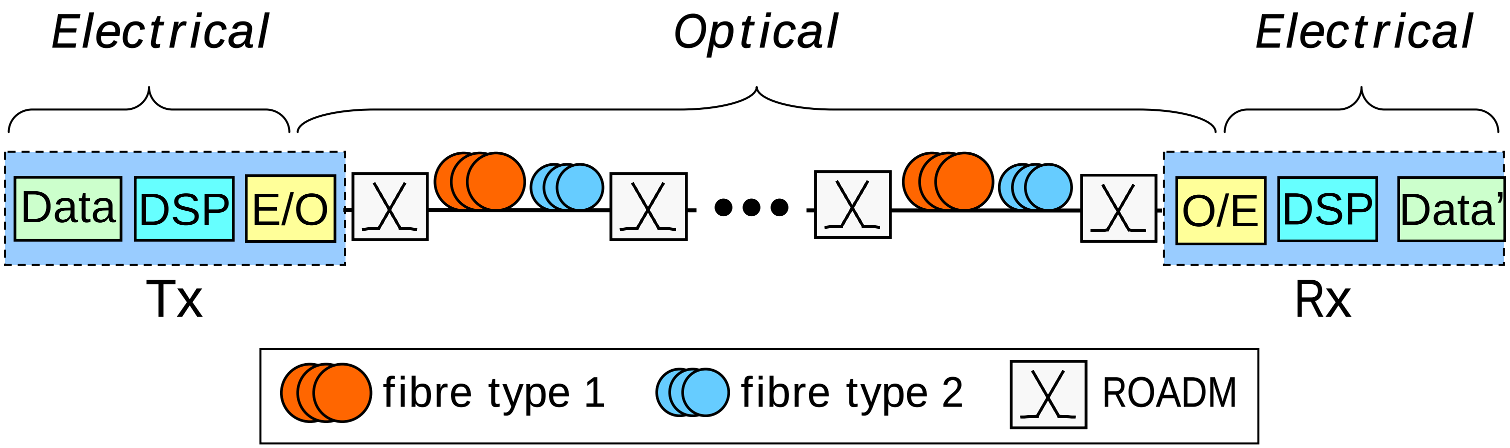
3 The Optical Channel

Optically-Routed Networks



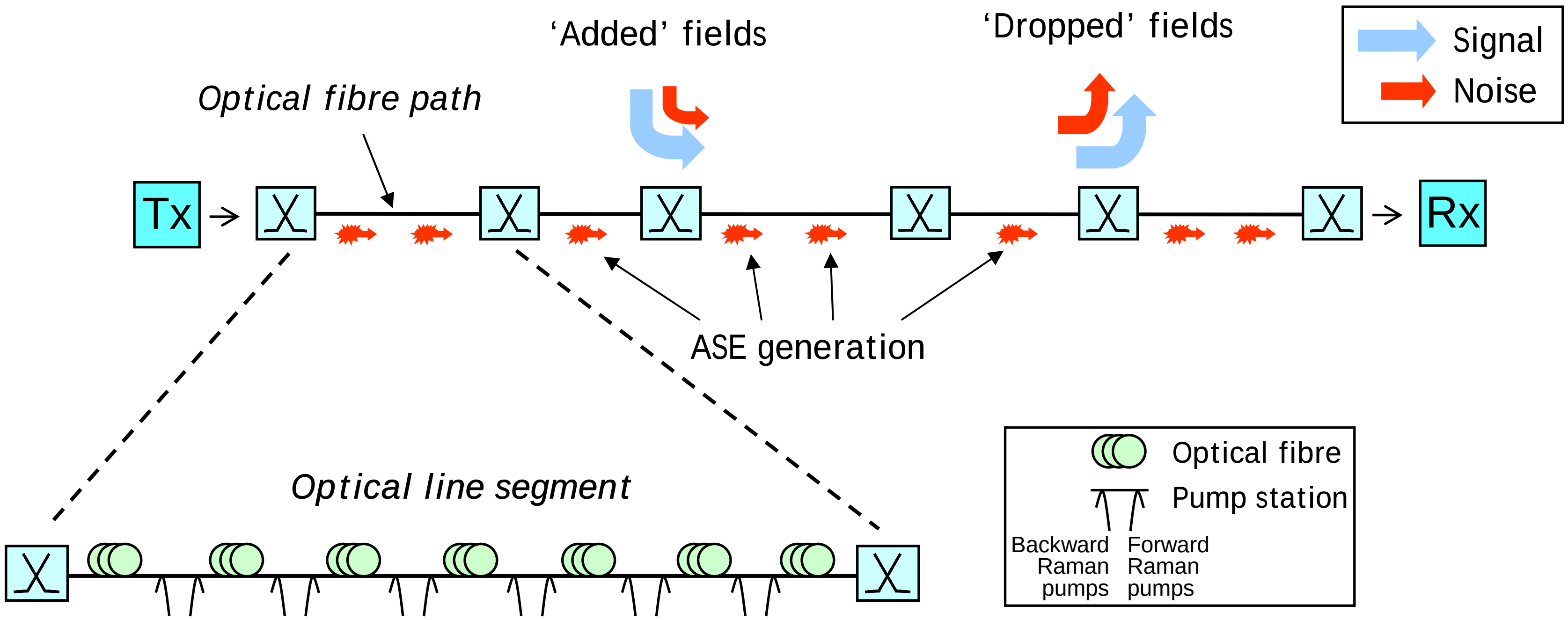
In optically-routed networks (rings/mesh), neighboring channels are not known but are transported over the same fibre!

The Optical Channel



- The optical path incorporates:
  - ➢ Distributed optical amplification
  - ➢ Optical filtering from ROADMs
  - ➢ Various fibre types distribution
- Arbitrary complex electronic processing is allowed at either ends (transmitter and receiver) of the optical path
- We do not consider the possible presence of optical regenerator

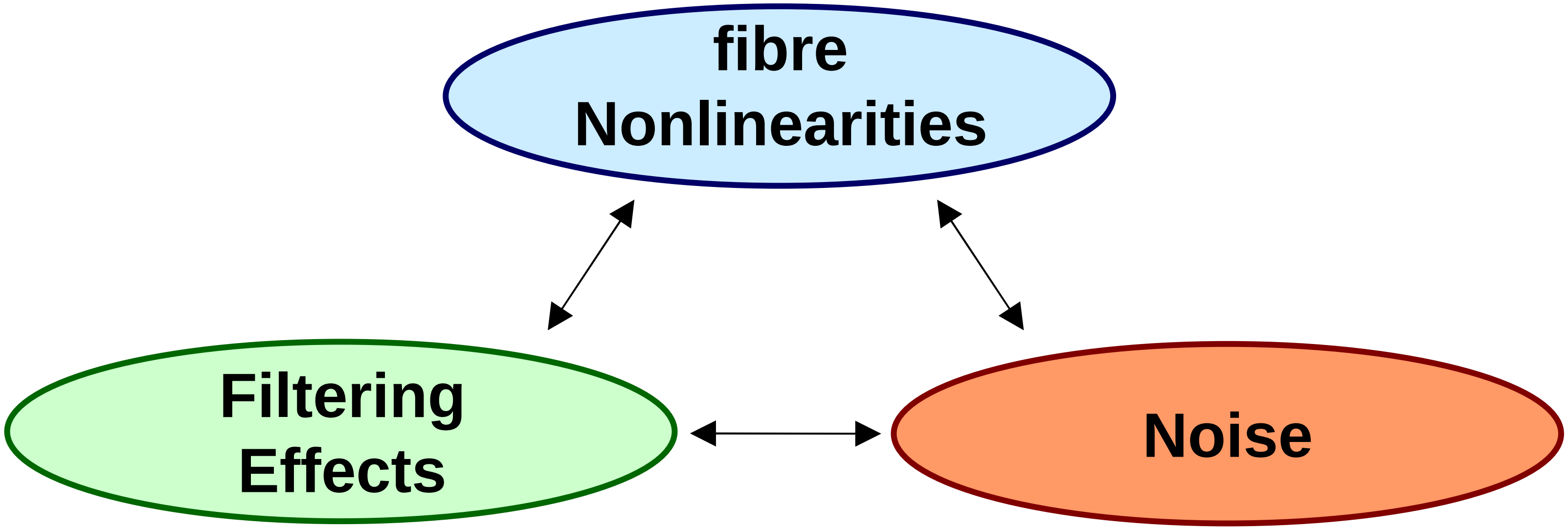
Optical Elements and Fields



The signal may be impaired by noise generation, optical filtering and fibre nonlinearity on its way from the transmitter (Tx) to the receiver (Rx)

4 Impairments

Physical Phenomena at Play



- Intra-channel nonlinearities
- Inter-channel nonlinearities

fibre Nonlinearities

Filtering Effects

- Chromatic Dispersion
- Optical Filtering

Noise

- Amplified Spontaneous Emission
- Double Rayleigh Scattering

Three phenomena are at play **simultaneously** during propagation

- Each physical effect influences the other
- Nonlinear transmission over fibres is not simply a transfer function!

Noise

Noise: Amplified Spontaneous Emission

- Amplified Spontaneous Emission is a **quantum noise**
- AWGN of Shannon is gaussian noise
- Can ASE be represented by AWGN?
- Answer by Jim Gordon in 1963 who says [1,2]:

"From this and from the Gaussian distribution of the output noise, it is clear that the amplification of the Gaussian noise input may be considered to have proceeded in a perfectly classical manner provided that we include the extra effective input photon to account for the response of the amplifier to the input zero-point fields. This result is valid for *arbitrarily small* input noise."

[1] J. P. Gordon, W. H. Louisell, and L. R. Walker, "Quantum fluctuations and noise in parametric processes II. Phys. Rev., Vol. 129, pp. 481–485 (1963).

[2] J. P. Gordon, L. R. Walker, and W. H. Louisell, "Quantum statistics of masers and attenuators," Phys. Rev., Vol. 130, pp. 806–812 (1963).

AWGN approximates well ASE even at low noise level!

Propagation

Propagation for Distributed Amplification

Generalized Nonlinear Schrodinger Equation (GNSE):

$$\frac{\partial E}{\partial z} + \frac{i}{2}\,\beta_2\,\frac{\partial^2 E}{\partial t^2} - i\,\gamma\,|E|^2 E = i\,\mathbf{n}(z,t)$$

White noise

E : Electrical field

β_2 : fibre dispersion

γ : Nonlinear coefficient

$$< \mathbf{n}(z,t)\,\mathbf{n}^*(z',t') > = \mathrm{n}_{\mathrm{sp}} K_T h\nu_s \alpha\,\delta(z-z',t-t')$$

n_{sp} : Spontaneous emission factor

K_T : Phonon occupancy factor

$h\nu_s$: Photon energy at signal wavelength

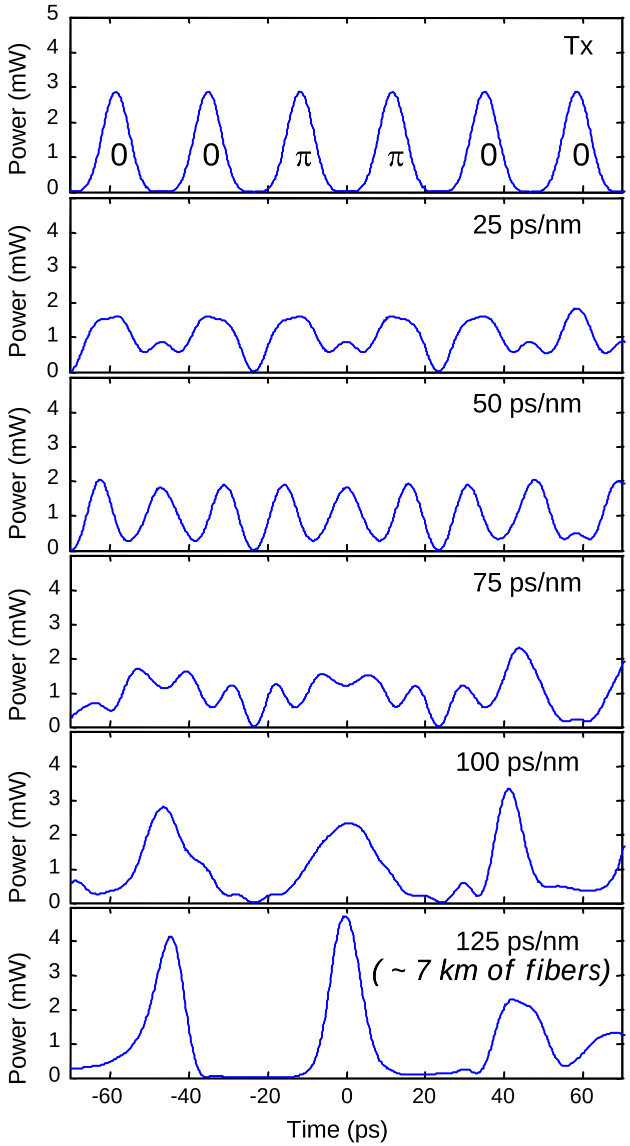
α : fibre loss coefficient

The field E contains **all WDM channels and all symbols**!

WDM: Wavelength-division multiplexing

Dispersion

Effect of Fiber Dispersion



Modulation format: 33% RZ-BPSK, 42.8 Gb/s

Fiber dispersion:

- Is a linear effect
- Causes optical pulses to overlap in time
- Creates pattern dependent intensity profile (i.e. memory effect)
- Can be perfectly compensated by:
  - ✓ *Electronic filters* (coherent detection)
  - ✓ *Special optical fibers* (dispersion-compensating fibers designed with negative dispersion)

Fiber dispersion by itself is not a problem!

Fiber Nonlinearity

Inter-Channel Nonlinearities

- The total field E is replaced by the sum of individual channel fields having distinct frequencies
- We separate the resulting equation into one equation per channel frequency and obtain:

Ex: 3 Channels: $E = E_1 e^{-i\omega_0 t} + E_2 e^{-i(\omega_0 + \Delta\omega)t} + E_3 e^{-i(\omega_0 + 2\Delta\omega)t}$

E_i = Channel *i* located at frequencies $\omega_0 + (i-1)\Delta\omega$

Equation of evolution of Channel 1 (E_1):

$$\overbrace{\frac{\partial E_1}{\partial z} + \frac{i}{2}\beta_2 \frac{\partial^2 E_1}{\partial T^2}}^{\text{Linear}} = \overbrace{i\gamma E_1 \left(|E_1|^2 + \underline{2|E_2|^2 + 2|E_3|^2} \right) + i\gamma\, E_2^2 E_3^*}^{\text{Nonlinear}}$$

SPM
Channel 1 modifies its own phase

XPM
Channel 2 and 3 modifies the phase of channel 1

FWM
Channel 2 and 3 exchanges energy with channel 1

SPM: Self-Phase Modulation
XPM: Cross-Phase Modulation
FWM: Four-Wave Mixing

Optical Solitons (Isolated Pulse)

Nonlinear Schrödinger Equation (NSE):

$$\frac{\partial E}{\partial z} = -\frac{i}{2}\beta_2 \frac{\partial^2 E}{\partial T^2} + i\gamma |E|^2 E$$

Normalization:

$u = E/(P_s)^{1/2}$,

$\xi = z/L_D$, and

$\tau = T / T_0$

$P_s = (\gamma\ L_D)^{-1}$: Soliton peak power

$\gamma = n_2\ \omega_0/(c\ A_{eff})$: Nonlinear coefficient

$L_D = T_0^2/|\beta_2|$: Dispersion length

n_2 : Fiber nonlinear coefficient

A_{eff} : Effective fiber core area

Normalized NSE:

$$\frac{\partial u}{\partial \xi} = -\frac{i}{2}\ sign(\beta_2)\ \frac{\partial^2 u}{\partial \tau^2} + i|u|^2 u$$

If $\beta_2 < 0$, Soliton Solution:

$$u(z,T) = u_0\ \mathrm{sech}(u_0 T)\ \exp[i\ u_0^2\ z\ /\ 2]$$

Isolated pulse solution

Soliton effect results in compensation of dispersion by nonlinearity !

Intra-Channel Nonlinearities

- Nonlinear interactions can be decomposed on a pulse by pulse basis
- Example: 3 Pulses from Channel 1

$$E_1 = E_{1,p1}(t+T_B) + E_{1,p2}(t) + E_{1,p3}(t-T_B)$$

$E_{1,pi}$ = Pulse from channel 1 centered at time T_i

Equation of evolution of Pulse 1 ($E_{1,p1}$):

$$\frac{\partial E_{1,p1}}{\partial z} + \overbrace{\frac{i}{2}\beta_2 \frac{\partial^2 E_{1,p1}}{\partial T^2}}^{\text{Linear}} = \overbrace{i\gamma E_{1,p1}\left(\underbrace{\left|E_{1,p1}\right|^2}_{\text{SPM}} + \underbrace{2\left|E_{1,p2}\right|^2 + 2\left|E_{1,p3}\right|^2}_{\text{IXPM}}\right) + \underbrace{i\gamma\, E_{1,p2}^{\;2} E_{1,p3}^{*}}_{\text{IFWM}}}^{\text{Nonlinear}}$$

SPM
Pulse 1 modifies
its own phase
(soliton effect)

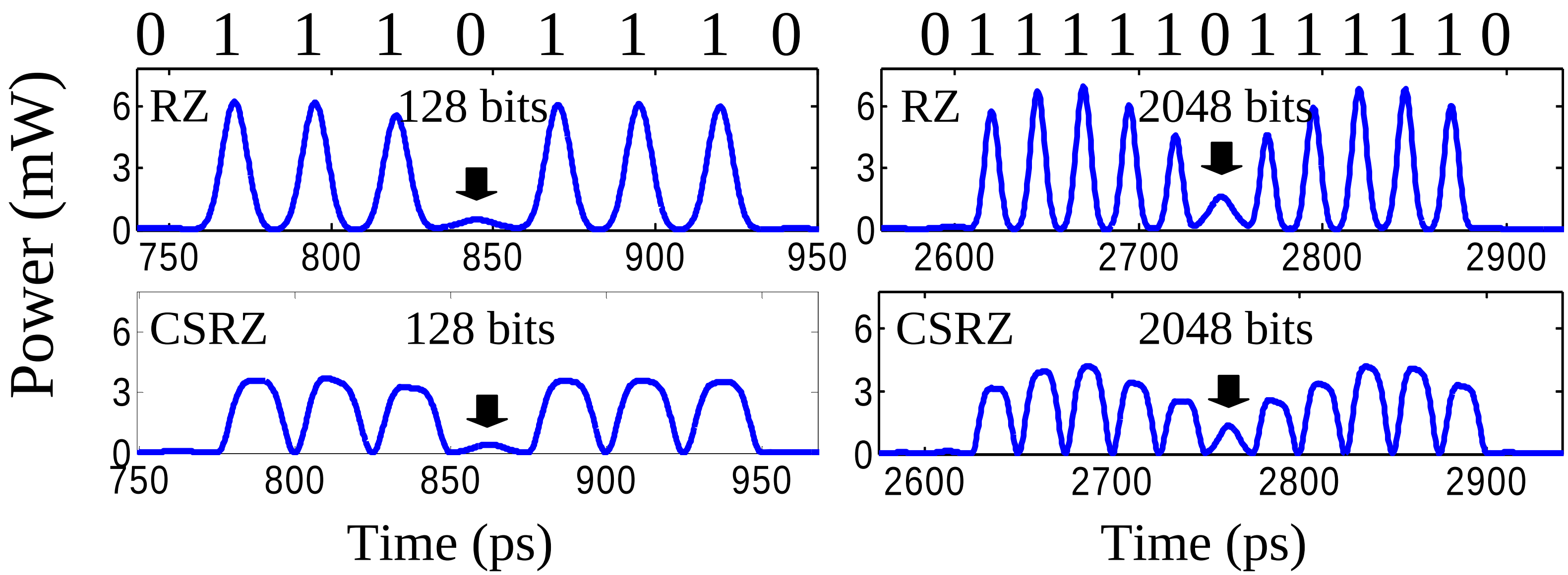
IXPM
Pulses 2 and 3
modifies
the phase of
pulse 1

IFWM
Pulses 2 and 3
exchanges
energy with
pulse 1

SPM: Isolated-pulse SPM
IXPM: Intra-Channel XPM
IFWM: Intra-Channel FWM

Example of Memory Effects from Intra-Channel Nonlinearities

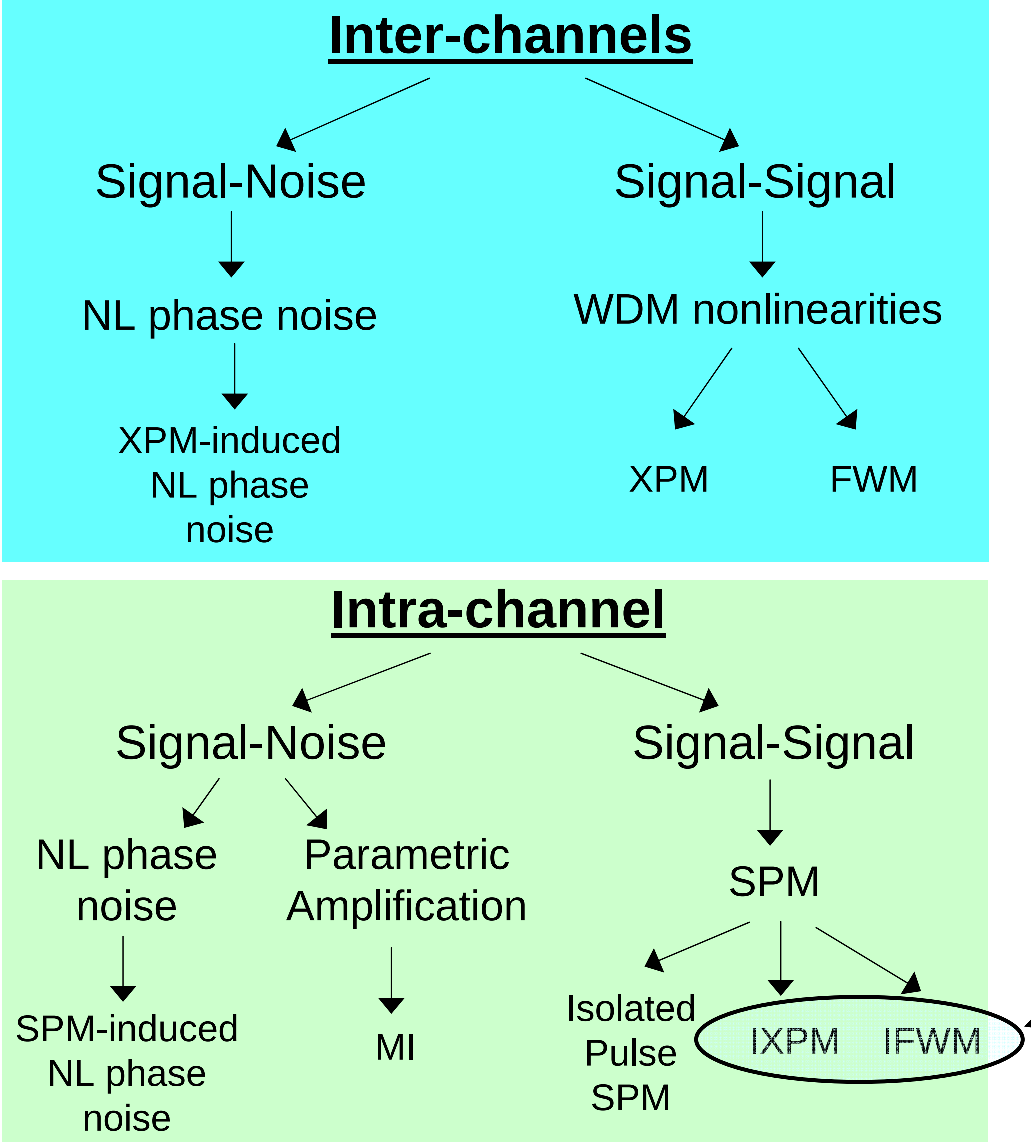
- Send binary data in a form of pulse
- After nonlinear transmission with get



- ‘Ghost’ or ‘shadow’ pulses were created by nonlinear transmission
- The more neighboring pulses, the large the ‘ghost’ pulses up to a certain number of neighbors
- Memory is caused by intra-channel fiber nonlinearities

RZ: Return-to-Zero
CSR: Carrier-suppressed RZ

Classification of fibre Nonlinearities



Inter-channels

Signal-Noise

NL phase noise

XPM-induced NL phase noise

Signal-Signal

WDM nonlinearities

XPM

FWM

Intra-channel

Signal-Noise

NL phase noise

SPM-induced NL phase noise

Parametric Amplification

MI

Signal-Signal

SPM

Isolated Pulse SPM

IXPM

IFWM

Nonlinear interactions with memory

List of Acronyms

- NL: nonlinear
- WDM: Wavelength-division multiplexing
- XPM: Cross-phase modulation
- SPM: Self-phase modulation
- MI: Modulation instability
- FWM: Four-wave mixing
- IXPM: Intra-channel XPM
- IFWM: Intra-channel FWM

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Nonlinearity Compensation

Reverse Propagation of Signal

Equation of propagation of signal:

$$\frac{\partial E}{\partial z} + \frac{i}{2}\beta_2 \frac{\partial^2 E}{\partial t^2} - i\gamma |E|^2 E = 0$$

Reverse propagation

$z \rightarrow - z$

is equivalent to:

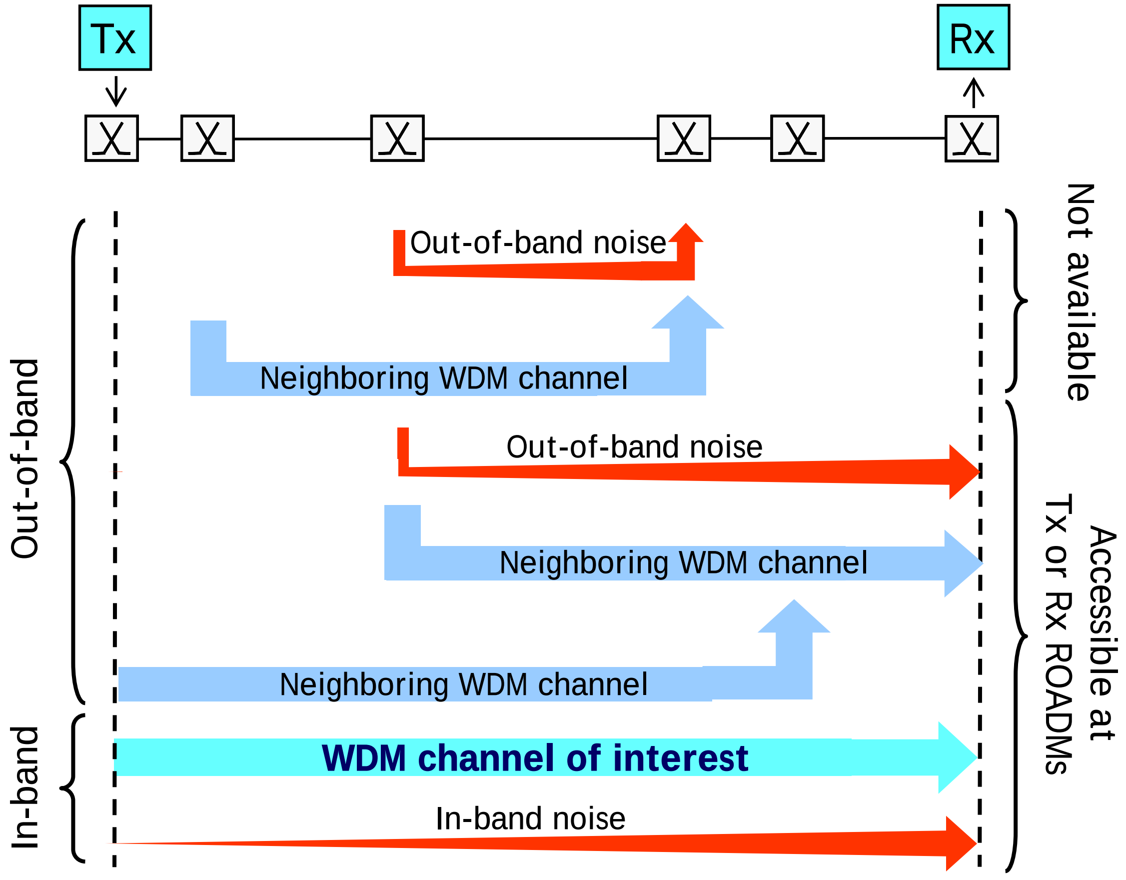
$\beta_2 \rightarrow - \beta_2$

$\gamma \rightarrow - \gamma$

Perfect backward propagation can be achieved if the evolution of all fields involved is known!

In optically-routed networks, the neighboring WDM fields are not known!

Nonlinear Transmission Compensation at Tx and Rx



- Propagating fields can be classified by:
  - *In-band* and *out-of-band* fields
  - *Available* and *non-available* fields

6

Capacity of the AWGN Channel

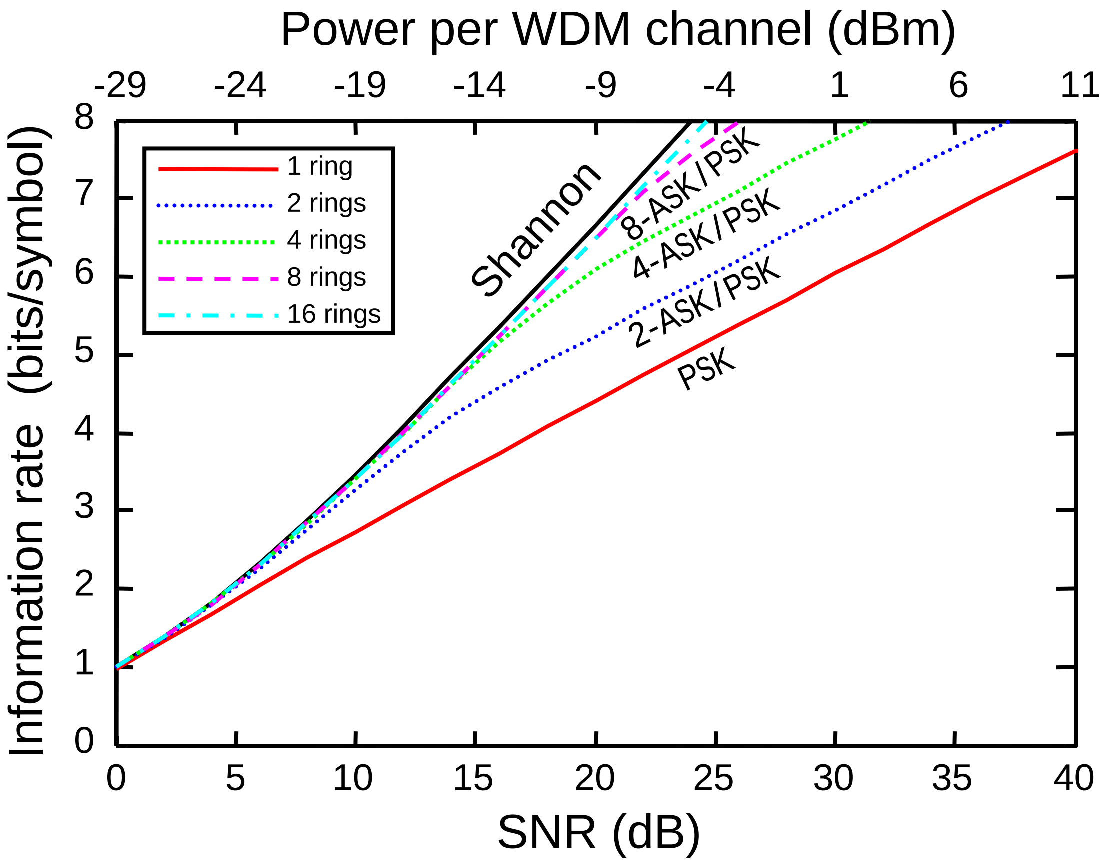
Capacity Calculations

- Memoryless channel capacity calculations

$$
\begin{aligned}
C/B &= \iint p_{Y,X}(y,x) \log_2 \frac{p_{Y,X}(y,x)}{p_Y(y) p_X(x)} \, dy \, dx \\
&= -\int p_Y(y) \log_2 p_Y(y) \, dy \\
&\quad + \iint p_{Y,X}(y,x) \log_2 p_{Y|X}(y|x) \, dy \, dx \\
&= H(Y) - H(Y|X) \ ,
\end{aligned}
$$

- From continuous memoryless to discrete memoryless channel (DMC) for simulations
- Memoryless motivated by the complete removal of intra-channel nonlinearities by reverse propagation

Information Rates for Ring Constellations



- At low SNR, less rings are necessary to achieve capacity
- At high SNR, only multiple rings can reach capacity

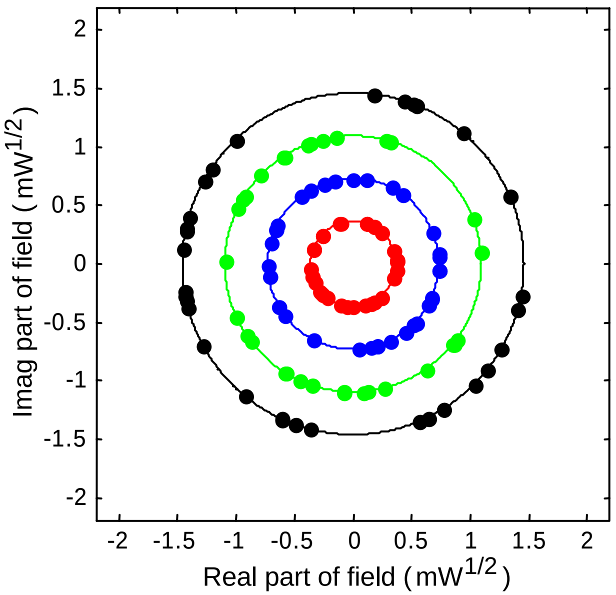
7

Capacity of the Fiber Channel

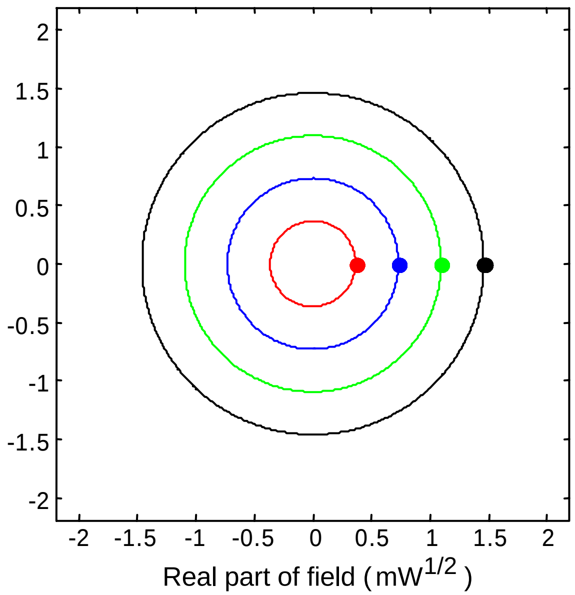
Backrotation of Constellation

Backrotation of constellation in the absence of nonlinearity and noise:

Original constellation

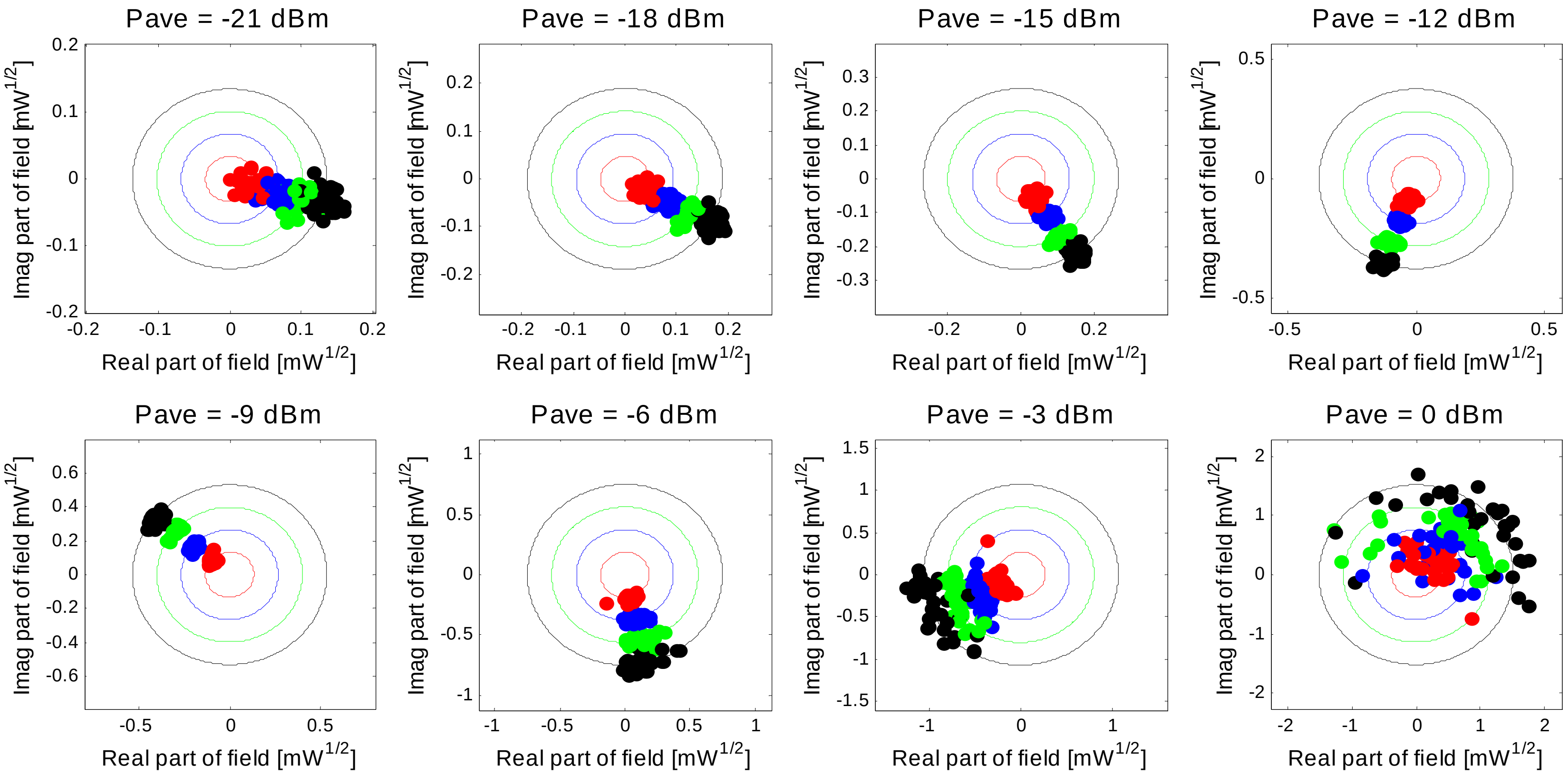


Backrotated constellation



- In the absence of nonlinearity and noise, all symbols belonging to a ring collapse to one point on the ring-hand side of the ring
- These points with become clouds in the presence of noise and uncompensated fibre nonlinearity

Evolution of Constellation with Signal Power

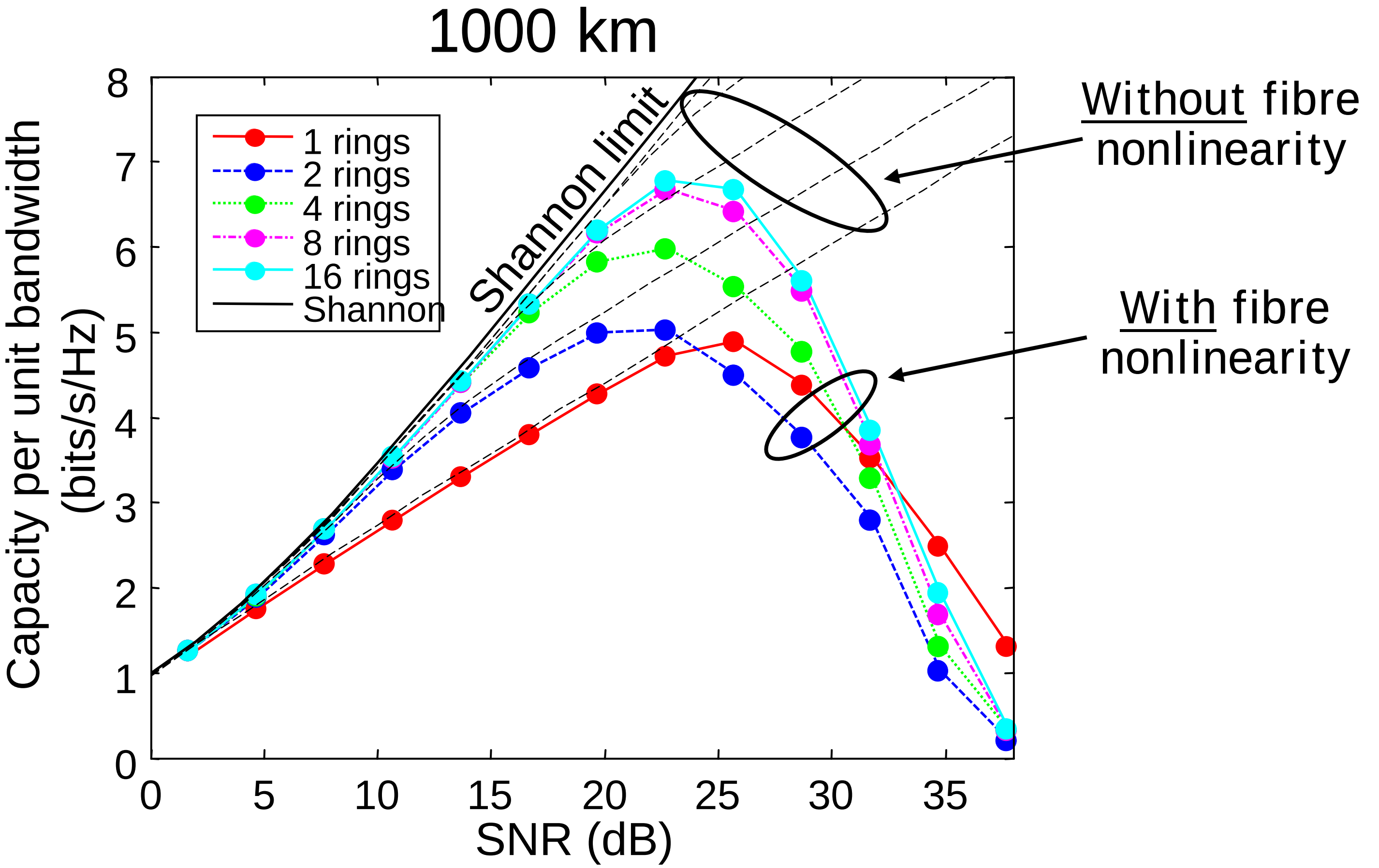


Symbol rate = 25 Gb/s, 50.0 GHz spacing

Evolution of the constellation with increasing power (at fixed noise level)

- At low powers, the clouds are large because of the low SNR
- At high powers, the clouds are large because of fibre nonlinearity

Capacity Estimates Results for Nonlinear Transmission



- Capacity can be increased by increasing the number of rings
- Increasing beyond 8 rings only marginally increases capacity
- This corresponds to an increase by approximatively one order of magnitude in spectral efficiency over record transmission experiments

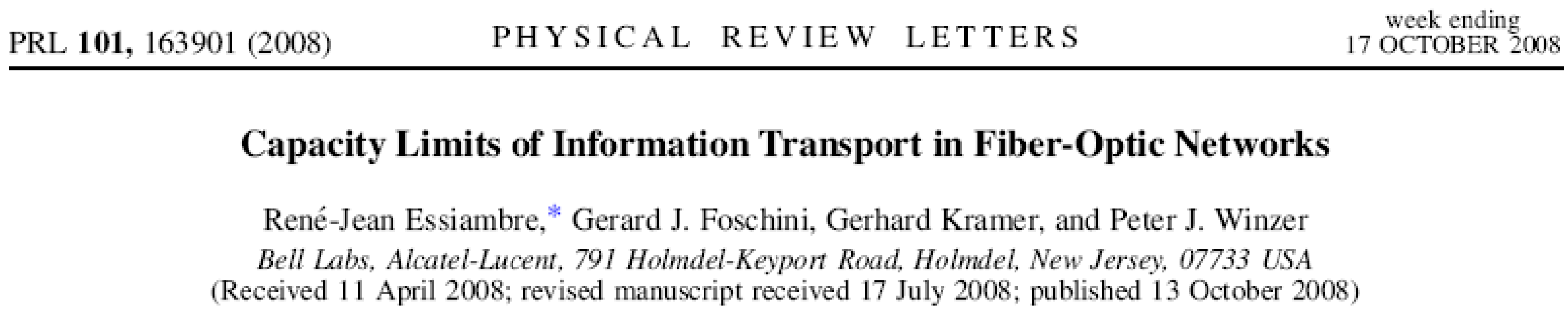
8

Conclusion

Conclusion

- Shannon’s information theory allowed to know the limit of detection for a signal impaired by noise
- Determining the maximum information that can be transmitted over a nonlinear fibre in optically-routed network can allow to set limits on optical network capacity
- Achieving capacity requires an array of advanced technologies
- So far, an increase of about one order of magnitude in spectral efficiency is predicted

More details can be found in:



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Other Important Topics we are working on:

- Blind equalization **for the optical channel**
- For polarization multiplexed signals, apply source separation **to the optical channel**
- Signal processing for implementing electronic **fiber nonlinearity** compensation
- Algorithms for **fiber nonlinearity mitigation**

Applying digital signal processing to increase the capacity of the optical channel is a very young field with a lot of opportunities to make original contributions www.alcatel-lucent.com