Rational Spectrum Utilization in OFDMA-Based Networks: a Game-Theoretic Approach

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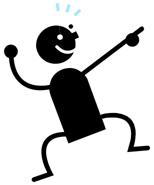
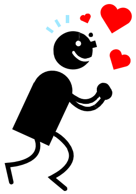


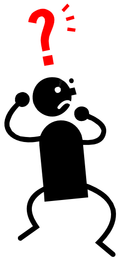
Outline

- Introduction
- Short review of the game-theoretic approach to OFDMA resource management
- The game for resources
  - players, utilities, strategies, payoffs
- The game for optimality (adaptive bit and power loading
  - players, utilities, strategies, payoffs
- Simulation results
- Conclusions



Motivation

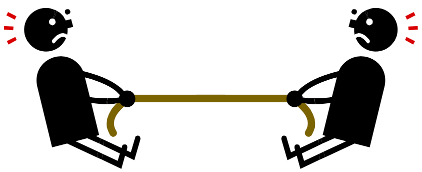
- The ambition to maximize the spectral efficiency and to rationalize the distribution of radio resources and the cost of their usage.
- Flexible radio node is expected to be less dependent on centralized network organization, and to be aware of its radio environment.
- Due to limited network resources dedicated to the control traffic, feed-back delays, sensing or estimation errors, the Channel State Information (CSI) may be erroneous, not valid or obsolete. The environmental awareness may be very limited in practice.
- The game theory has been considered as a tool to model decisive behavior of a node in a wireless network competitive environment.
- The radio environment can also be treated as a player, although its behavior is not rational and not reactive to the user's adopted strategies.



What is a game?

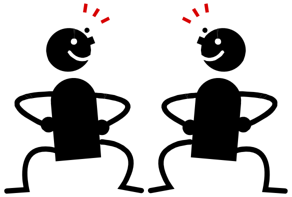
A strategic form of each game consists of three basic elements:

- the set of players
- the strategy (actions) space for each player,
- the payoff function, which measures the outcome of each player or a group of players



Non-cooperative games

	Static games	Dynamic games
Complete information games	• Complete information static game • Nash equilibrium • John Nash (1950, 1951)	• Complete information dynamic game • Subgame perfect Nash equil. • Reinhard Selten (1965)
Incomplete information games	• Incomplete information static game • Bayesian Nash equil. • John Harsanyi (1967-68)	• Incomplete information dynamic game • Perfect Bayesian Nash equil. • Reinhard Selten (1975)



Cooperative games

- Players can make binding commitments, form coalitions
- Analysis in cooperative game theory is centered around coalition formation and distribution of wealth gained through cooperation
- No equilibrium concept dominating the field (multiplicity of solutions due to the diversity of conflict situations)

Cournot oligopoly competition in the dynamic spectrum sharing

- The palyers: multiple secondary users (SUs)
- Strategies: the spectrum size Q_i requested by a user
- Payoffs: the profit $\pi_i(\mathbf{Q})$ (i.e. revenue minus cost) of the SU i depending on the spectrum demand $\mathbf{Q}$ from all SUs.
- The pricing function:

$$P(\mathbf{Q}) = x + y \cdot \left(\Sigma Q_j\right)^{\tau}$$
$$P(\mathbf{Q}) > w \cdot \Sigma Q_j$$

x, y, w, τ - non-negative constants, $\tau \geq 1$

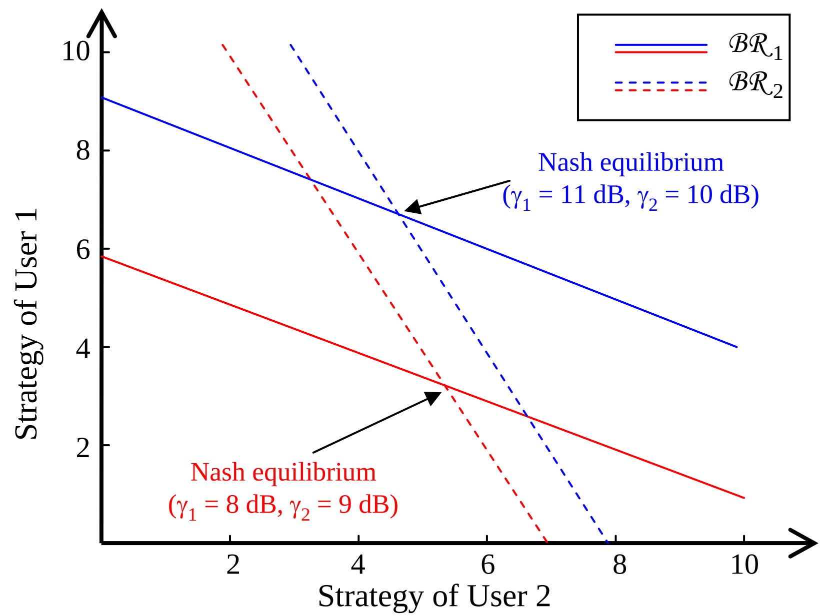
- The profit:

$$\pi_i(\mathbf{Q}) = r_i k_i Q_i - Q_i P(\mathbf{Q})$$

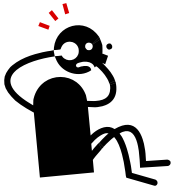
r_i - the revenue, k_i -spectral efficiency

- Best response function:

$$\mathcal{BR}_i(\mathbf{Q}_{-i}) = \arg\max_{Q_i} \pi_i\left[\mathbf{Q}_{-i} \cup \{Q_i\}\right]$$



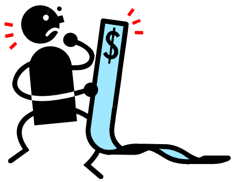
(BER = 10^{-4} , $x = 0$, $y = 1$, $w = 1$, $r_i = 10$, τ is adjusted)



Game-theoretic RRM for OFDMA

Game-theoretic scheduling for OFDMA has been considered in the literature in two major directions:

- Centralized SCs allocation (applies cooperative game theory, Nash-bargaining and arbitrary Pareto-optimal solutions):
  - Z. Han, Z. J. Ji, K. J. Ray Liu, „Fair Multiuser Channel Allocation for OFDMA Networks Using Nash Bargaining Solutions and Coalitions”, *IEEE Trans. Commun*., Vol. 53, No. 8, Aug. 2005, pp. 1366-1376
  - T. K. Chee, C.-C. Lim, L.J. Choi, “A Cooperative Game Theoretic Framework for Resource Allocation in OFDMA Systems”, IEEE Singapore International Conference on Communication Systems, ICCS 2006, Oct. 2006, pp. 1-5
  - H.W. Kuhn, “The Hungarian method for the assignment problem,” *Nav.Res. Logist.*, pp. 2:83–2:97, 1955
- Distributed decision making deploys non-cooperative games and results in Nash equilibrium as a game solution.
  - H. Kwon, B.G. Lee, “Distributed Resource Allocation through Noncooperative Game Approach in Multi-cell OFDMA Systems”, *IEEE ICC’06,* pp. 4345-4350, 2006



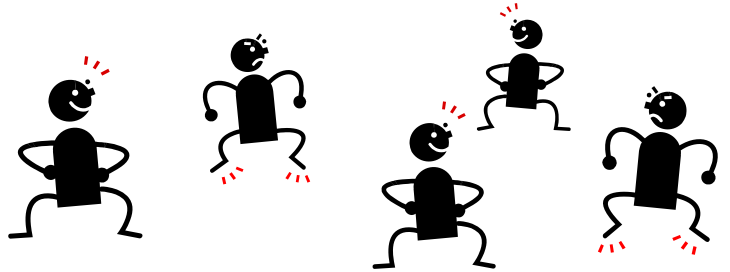
Resource pricing - related concepts

- Pricing of the bandwidth resources in the oligopoly model, particularly used in the Stakelberg model refers to bartering rather than taxation
- Taxation of resources – the tax (price) value t_x is periodically broadcast, so that each radio node adjusts its utility as a function of t_x. The aim of the central controller is to discover the value of t_x, which maximizes the global utility.

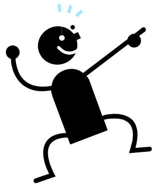
Related problem: The Tragedy of Commons

G. Hardin, “The Tragedy of the Commons”, Science, 162(1968), pp. 1243-1248.

The scenary



- Multiple mobile CR nodes appearing in the OFDMA-based network area
- These nodes are able to sense the radio environment, and detect available spectrum resources.
- The first goal of the CR node is to acquire radio resources and make the best use of them, i.e. maximize the spectrum utilization.
- An intelligent Control Unit (CU) of a CR node is an entity, with a key role in making decisions concerning which SCs it is going to utilize, and how it is going to utilize them.



The idea of the top-down approach to efficient and rational spectrum utilization

- First, allow a CR node to acquire resources, i.e. the SCs it is going to use, by applying a game model of common resource utilization
  - *the game for resources* (GfR) at the second OSI (top) layer,

and...

- Then, allow each node CU to play an independent game against the radio channel, assuming that the CSI is erroneous or obsolete
  - *the game for optimality* (GfO) at the physical (down) layer.

The game for resources

- The CR node senses the spectrum and takes decision on how many and which SCs she is going to occupy.
- The number of possible outcomes of the game equals:

$$\left[\sum_{i=0}^{I}\frac{N!}{i!(N-i)!}\right]^{K}$$

N - no. of available SCs,
K - no. of players,
I - the max. no. of SCs a player can take at a time

Analyzing the existence of the saddle points or the Nash equilibriums becomes very complicated!

- We simplify the game by letting the players take decisions subsequently as their demands occur, and by eliminating their strategic choices which are disadvantageous.
- To prevent a single player from occupying all resources, we limit number I, introduce some "social consciousness" mechanisms (such as network capacity factor) in the payoff function, and employ resource pricing.

GfR – strategies and payoffs

- The strategies of a CU are all possible numbers of SCs (form 0 to *I*) and each number *i* relates to *i* strongest SCs she detects. The strategies of the rest of the CR-nodes community are the numbers of SCs this community may occupy all together apart from the considered player (CU).

- The payoffs:

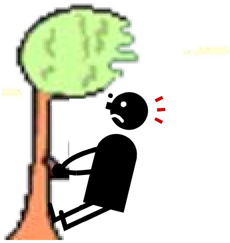
$$p_{i,j} = \left\{ \sum_{s \in \mathbf{S}_i} \log_2 [1 + \alpha \cdot P(f_s) CNR(f_s)] \right\} \cdot \left\{ N - \sum_{k=1}^{K} n_k \right\},$$

$$CNR(f_s) = \left| H(f_s) \right|^2 / G(f_s)$$

$$\tilde{p}_{i,j} = p_{i,j} - tax_i$$

$$tax_i = i \cdot r_i$$

where r_i is the tax rate, which can be fixed (linear tax) and not dependent on the chosen strategy, or it may depend on the number of chosen SCs (progressive tax).



The game for optimality: *the game against the nature*

- Players:
  - An intelligent Control Unit (CU) of an OFDM mobile node – rational player
  - radio mobile environment (radio channel) represented by the subcarrier gains – irrational player

- Strategies:
  - CU: power spectral densities (power levels) and constellation sizes at subcarriers,
  - the *s*-th subcarrier channel: *L* possible values of the Noise-to-Carrier gain Ratio (NCR):

$$CNR_l(f_s) = |H_l(f_s)|^2 / G(f_s)$$

GfO payoff function

- An example utility function:

results from the assumed BEP

$$R_{m,l}(f_s) = \Delta f \cdot \log_2\left(1 + \alpha P_m(f_s) CNR_l(f_s)\right) \text{ [bit/s]},$$

- More appropriate payoff for rational resource utilization provides flexiblity in determining preferences concerning the throughput and power consumption :

$$U_{m,l}(f_s) = x\varphi_1\{S_{m,l}(f_s)\} + (1-x)\varphi_2\{R_{m,l}(f_s)\}$$

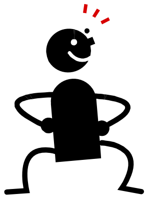
Power economizing component

Throughput maximizing component

$$S_{ml}(f_s) = P_l^{\text{opt}}(f_s) - P_m(f_s)$$

φ_1 , φ_2 – scaling (normalizing) functions of $S_{ml}(f_s)$ *and* $R_{ml}(f_s)$

x – weighting factor indicating substantiality of throughput maximization versus power savings.



The algorithm of a single GfO

1. Observation of the channel strategies
2. Definition of the CU own strategies

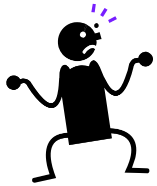
   CU strategies are in line with the WF principle for ”fake” CSI, i.e. for a given set of $CNR_l(f_s)$, the response power levels P_m (for $m = l$) equal:

$$P_m(f_s) = \begin{cases} W - \dfrac{1}{\alpha CNR_m(f_s)} & \text{for } CNR_m(f_s) > \dfrac{1}{\alpha W} \\ 0 & \text{otherwise,} \end{cases}$$

3. Calculation of the game matrix according to the utility function
4. Calculation of the expected values of payoffs B_i (based on the probabilities of the channel strategies)

$$B_m(f_s) = \sum_{l=1}^{L} U_{m,l}(f_s) \cdot \Pr\{CNR_l(f_s)\}$$

5. Selection of the CU strategy resulting in the highest expected payoff.



The problem of the erroneous CSI

Unknown are exact values of CNR, known is the range of these values

↓

Fixed number of the considered CNR values

↙ ↘

Unknown is the probability distribution of the CNR error

↓

Uniform CNR error probability distribution

Probability of the CNR error is known (shifted Rayleigh distribution)

↙ ↘

σ(SNR)

σ(-)

Exact values of CNR are known at the beginning of the game

↓

Varying (increasing) number of the considered CNR values

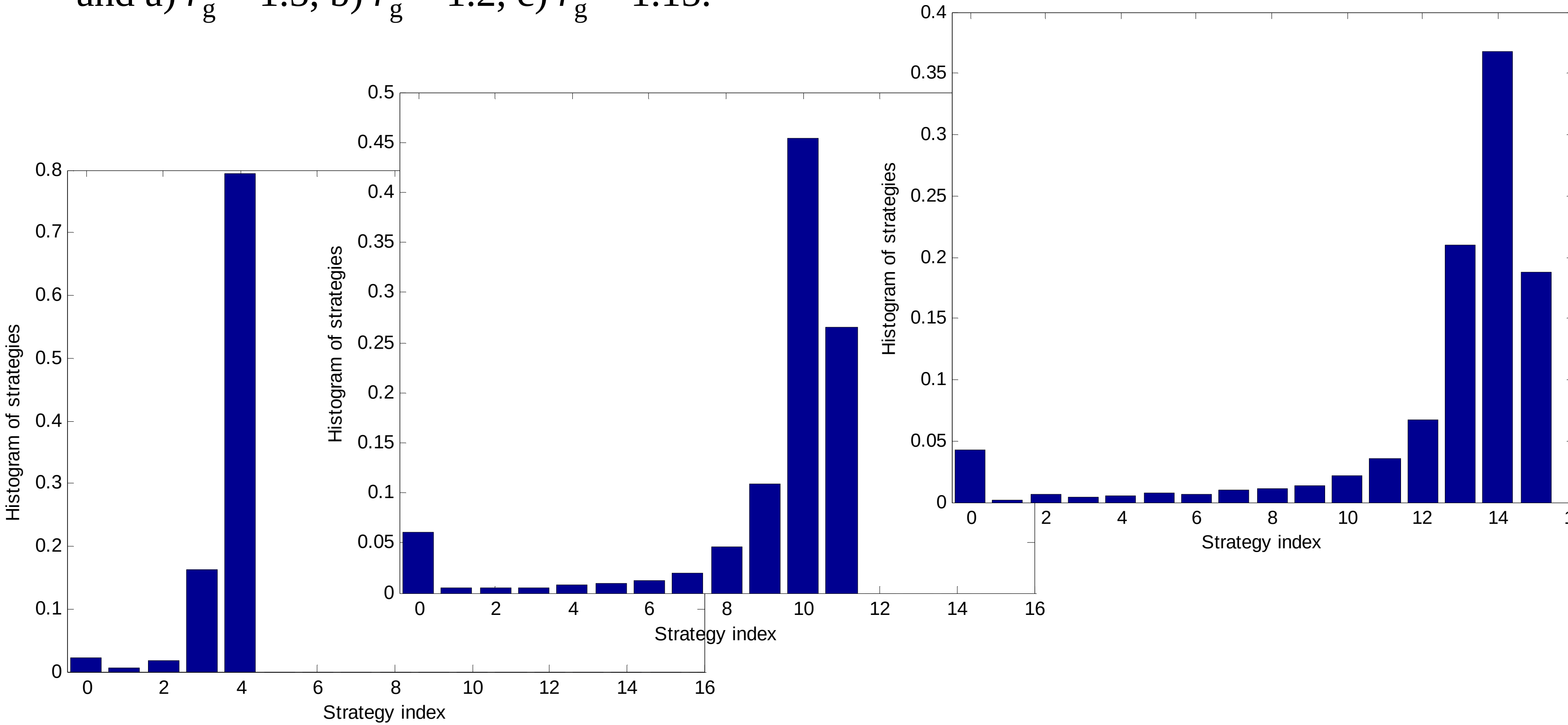
↓

Varying (increasing) CNR error standard deviation

Assumption can be made about fixed water level (within the coherence time) or its expected value can be calculated.

Simulation results (1)

The histograms of the chosen strategy index in the GfR for SNR = 30 dB, K = 8, and a) r_g = 1.5, b) r_g = 1.2, c) r_g = 1.15.



Simulation results (2)

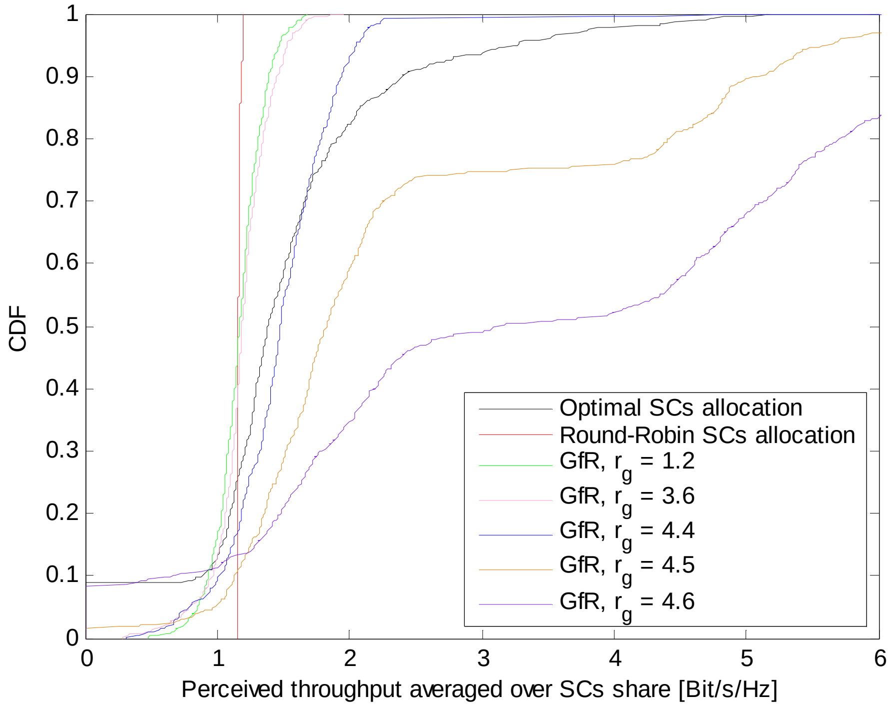
Performance metrics of the GfR and optimal SCs allocation

SNR = 30 dB		Game for resources			Optimal SCs allocation		
K	r_g	Nodes served [%]	SCs allocated [%]	Spectral efficiency [Bit/s/Hz]	Nodes served [%]	SCs allocated [%]	Spectral efficiency [Bit/s/Hz]
K = 4	1.2	100	100	1.16	91	100	1.42
	3.0	100	99.3	1.16			
	3.6	100	97.4	1.15			
	4.4	100	75.3	1.09			
	4.6	86.6	18.9	0.45			
	4.8	48.2	1.7	0.09			
K = 8	1.2	100	100	1.73	62	100	2.19
	3.0	100	99.9	1.77			
	3.6	100	99.5	1.75			
	4.4	100	85.9	1.67			
	4.6	96.3	22.3	0.68			
	4.8	49.3	2.3	0.14			
K = 16	1.2	100	100	2.43	40	100	3.11
	3.0	100	100	2.49			
	3.6	100	99.9	2.48			
	4.4	100	93.1	2.41			
	4.6	99.0	33.6	1.22			
	4.8	54.8	4.5	0.37			

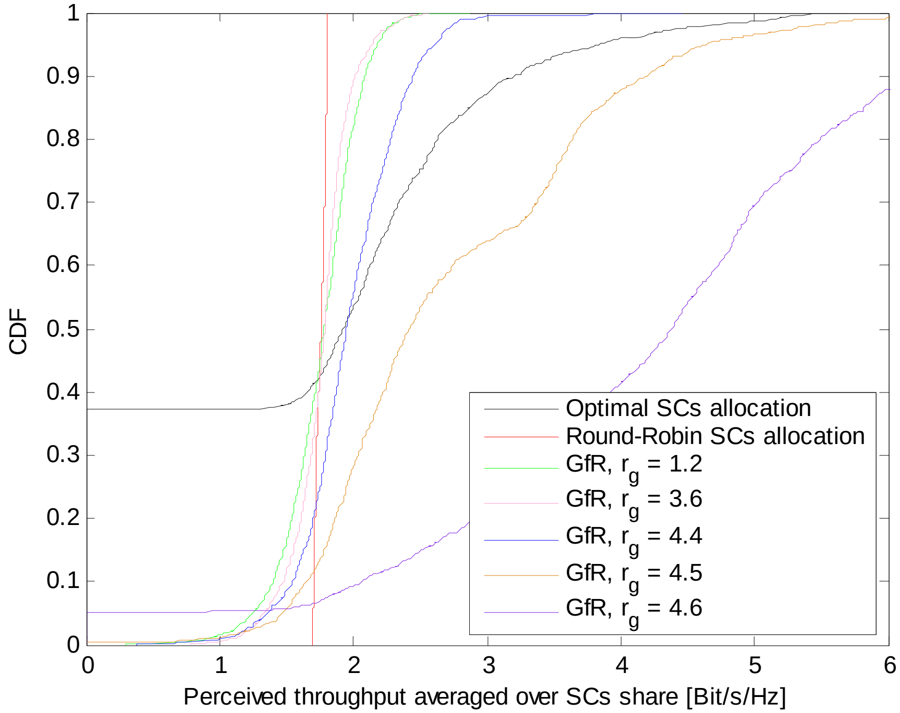
Simulation results (3)

CDF of the perceived throughput per frequency unit in the GfR for SNR = 30 dB and the number *K* of the competing CR-nodes:

K = 4

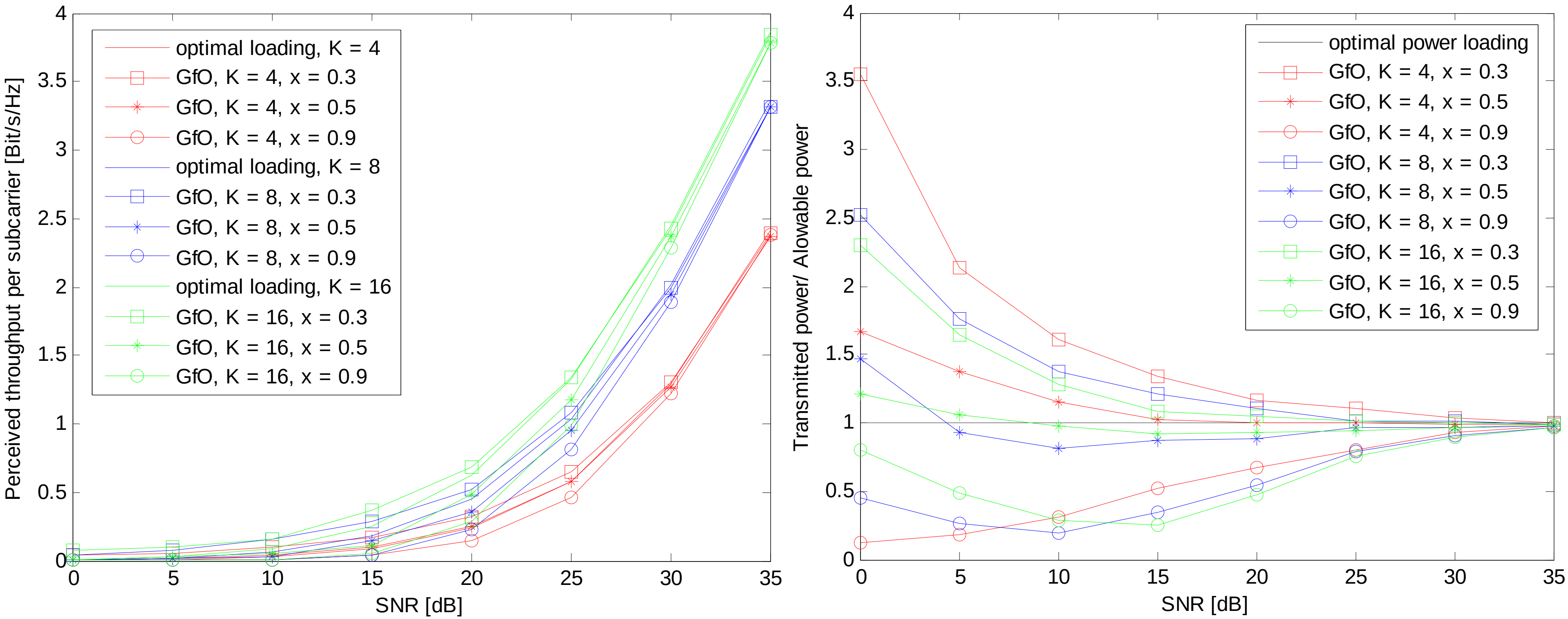


K = 8



Simulation results (4)

Achieved throughput averaged over the player's share of subcarriers (a), and the transmit power relative to the power limit (b); $P_e = 0.001$, $I = 16$, $N = 256$, $\sigma_\varepsilon = 1/\text{SNR}$.



Conclusions

- The top-down approach to the efficient dynamic spectrum access and rational spectrum utilization in the OFDMA-based CR-network scenery is to allow the nodes to play the game for resources and the game for optimality
- For the sake of efficiency, the payoff functions reflect the throughput achieved by a CR-node and the network potential to serve the CR-nodes community. For the sake of rationality, taxation of the SCs has been introduced in the GfR, as well as the power-economizing component in the payoff function in the GfO.
- In the GfR, we observe that there is some degree of freedom in establishing the tax rate value to approach either the maximum (but unfair) spectral efficiency or fairness in distributing available resources.
- In our simulation setup, $r_{\mathrm{g}} = 3$ resulted in the desirable compromise (spectral efficiency close to optimum and fairness close to the Round-Robin SCs allocation).

Conclusions, cont.

- In the GfO, the appropriate choice of the payoff contributions weighting factor *x*, reflecting CU preferences for maximizing the throughput versus economizing the power results in near-optimum solution as can be observed in the case of the ideal CSI available at the transmitter.
- Simulation experiments show that the GfO results (throughput and transmit power) approach the optimal bit-and-power loading algorithm performance even with the CSI of a limited reliability when the payoff-contributions weighting factor equals: $x = 0.5$.

Thank you.