Conflicts and Incentives in Wireless Cooperative Relaying: A Distributed Market Pricing Framework

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Cooperative relaying: a brief overview
  - basic concepts
  - difference from traditional approaches
  - sample of some recent work

The need for game theory and pricing in modelling of cooperative relaying systems
  - conflicts among cooperative relay nodes and flows

Our work: a distributed market pricing framework

Conclusion and future directions
Cooperative Relaying: the Basic Idea

- Transmission from sender to receiver unsuccessful
  - Overheard by relay node
  - Relay node transmits another signal
  - With the new signal, receiver can decode information

Figure: Cooperative relaying with one relay node

Figure: Cooperative relaying with multiple relay nodes
Differences from classic approaches

- Classic approaches are **layered**
  - decide about route in advance, other route remains unused

- Single-hop transmission (physical/MAC layer)
  - retransmission by sender only

- Routing via two hops (network layer)
  - direct link unused, even if direct transmission heard by receiver

- Cooperative communication
  - aka cooperative relaying / forwarding / retransmission / ARQ ...
  - Take advantage of wireless medium broadcast nature
  - Potential to combine benefits from both layers **opportunistically**
  - Especially useful for combating volatile nature of links
    - e.g. slow fading at packet time-scale
Physical-layer strategies

- Relaying by cooperative neighbor
  - amplify-and-forward
  - decode-and-forward
  - coded cooperation (cooperative FEC)
- Single-relay case widely studied in the literature
  - effective channel capacity / BER curves
  - tradeoffs and optimal schemes well-understood
Cooperation by multiple relays: existing approaches

- Choose one “best” relay
- Multiplex relay transmissions (frequency, code, ...)
  - needs multiple receiver circuits
- Cooperative diversity on physical layer
  - conceptually similar to MIMO
  - taking advantage of spatial diversity
  - need multiple receiver antennas or sophisticated “space-time codes”
  - extensive research in recent years
- Higher layers: variations on opportunistic routing
  - packet is broadcast, multiple ACKs returned, then next hop chosen
    - Selection Diversity Forwarding (SDF), Larsson 2001
    - Geographic Random Forwarding (GeRaF), Zorzi et al 2003
  - or implicit coordination by random waiting instead of ACKs
    - Extremely Opportunistic Routing (ExOR), Biswas and Morris 2005
Cooperative relaying

- cross-layer approach (joint PHY/MAC+routing)
- opportunistic (packet route not determined in advance, may be relayed by multiple nodes)
- defeats traditional routing in presence of short-term channel losses

Majority of existing literature: focus on optimization and/or performance bounds of single transmitter/receiver pair

- no consideration of conflict among multiple flows for same resource pool (cooperative relay nodes)
- no consideration of possibly selfish relay nodes
- related work from NUM theory / MANETs inapplicable
Most existing solutions focus on optimization perspective

Conflicts in cooperative relaying
- Helping one flow means cannot help others
- Increased interference among flows caused by cooperative relaying
- More complicated interactions among flows between different source/destination pairs in the presence of cooperative relaying
- Need distributed mechanisms for efficient allocation of cooperative relay nodes among the heterogeneous flows

Our work: investigate the conflicts and incentives in cooperative relaying using a distributed market pricing framework
Related work on applying game theory on distributed mechanism/protocol design in networks

- Apply pricing as a distributed control mechanism to orient the system towards a social optima
  - Network Utility Maximization (NUM) [1]
    - Not applicable in cooperative relaying
    - No clear distinction between “users” and “resources”
- Research on incentives in ad hoc networks: credit-based and reputation-based approach [2]
  - Not applicable in cooperative relaying, either
  - No pre-determined path, each packet may be relayed by multiple nodes
- Market-based frameworks, [3-5]
  - Our work can be viewed as the adaptation of market-based pricing methodology to the cooperative relaying context
Overview of Our Work

- A market pricing framework to study the conflicts and incentives in cooperative relaying

Main contributions
- First pricing framework for cooperative communication
- Game theoretic analysis on the proposed pricing framework
  - NE structure: existence, uniqueness, convergence, efficiency
  - Competition among source and relay nodes

Distinctive features of our work from previous research
- In our work: 2 types of players (the relay nodes and the flow sources), each of which is not only in competition with its peers but also with players of the other type
- The payment from a flow is shared among all relay nodes successfully participating in the relaying of that flow
  - A node’s utility depends not only on prices but also on the strategies of its peers
  - Competition scenario with more complex interactions among players
  - Non-concave utility, needs original study of the equilibrium properties
Network Model

- We consider a synchronized slotted wireless network consisting of
  - A set $\mathcal{F}$ of flows
  - $S_f$, $D_f$: source and destination of flow $f$
  - A set $\mathcal{R}$ of relay nodes
  - Simple memoryless channel model: either good or bad
  - $P_{sn}^f$ ($P_{nd}^f$): probability that the channel between $S_f$ and any relay node (any relay node and $D_f$) is good

- Each relay node can only relay one pkt at a time
- The transmission is successful if at least one relay node successfully relay the packet to its destination
Proposed Pricing Framework

- Flow sources pay the relay nodes in exchange for their cooperative relaying
- The payment $C^f$ (per successful pkt) is shared fairly among the relay nodes successfully relaying that pkt to its destination

**Figure:** Illustrating example on the pricing framework
Game Theoretic Model on the Pricing Framework

Leaders: Source nodes (flows)
Followers: Relay nodes
Strategy: source node $f$: payment $C^f$
          relay node $i$: $r_i = \{r_i^f\}$ where $r_i^f$: prob. that relay node $i$ helps flow $f$
Payoff: $U_f$ for source node $f$, $V_i$ for relay node $i$
Game rule: The leaders select their strategies
          by anticipating followers’ reaction
          The followers select their strategy based on knowledge of leaders’ strategy.

The proposed model can be applied in the following generic scenarios
- A set of “jobs” compete for the services of a pool of “workers”
- The jobs set their payment rates, workers are free to choose the job(s)
- The payment from each job is shared equally among all the workers that completed it successfully
Utility Functions

- Utility function of flow $f$: $U_f \triangleq u_f(P_{suc}^f) - C^f P_{suc}^f$
  - $P_{suc}^f$: prob. that a pkt of $f$ successfully arrives at dest.
  - $u_f(P_{suc}^f)$ characterizes the satisfaction level of $f$ at $P_{suc}^f$
  - $U_f \triangleq$ benefits - payment to relay nodes

- Utility function of relay $i$: $V_i \triangleq \sum_{f \in \mathcal{F}} C^f K^f r_i^f \sum_{l=0}^{R-1} \frac{P_f(l)}{l+1} - e^f r_i^f$
  - $(K^f \triangleq P_{sn}^f P_{nd}^f)$
  - $P_f(l) \triangleq \sum_{\mathcal{T} \subseteq \mathcal{R} \setminus \{i\}} \prod_{j_1 \in \mathcal{T}, |\mathcal{T}|=l} K^f r_{j_1}^f \prod_{j_2 \notin \mathcal{T}, j_2 \neq i} (1 - K^f r_{j_2}^f)$: prob. that there are $l$ additional nodes beside $i$ successfully relaying the pkt of $f$ to its dest.

- $\sum_{f \in \mathcal{F}} C^f K^f r_i^f \sum_{l=0}^{R-1} \frac{P_f(l)}{l+1}$: expected payment from flows

- $e^f$: transmission cost for relay nodes

- $V_i \triangleq$ share of received payment - cost of cooperation

- $P_{suc}^f = 1 - \prod_{i \in \mathcal{R}} (1 - K^f r_i^f)$
Solving the Stackelberg Cooperative Relaying Game

- Solve the followers’ game
  - For each relay node $i$: given the leaders’ strategies $C = \{C^f\}$ and the strategies of its peers $r_{-i} \triangleq \{r_j, j \neq i\}$, solve $r_i$ maximizing $V_i$:
    $$r_i^*(r_{-i}, C) = \arg\max V_i(r_i, r_{-i}, C)$$
  - Results: the followers’ equilibrium strategies $\{r_i^*(C)\}$
  - Difficulty: $V_i$ non-concave

- Solve the leaders’ game
  - For each flow $f$: with the knowledge of $r^*(C)$ and given $C^{-f}$, solve $C_{f}^*$ maximizing $U_f$: $C_{f}^* = \arg\max U_f(C^f, C^{-f}, r_i^*(\langle C^f, C^{-f}\rangle))$
  - Results: the leaders’ equilibrium strategies $\{C_{f}^*\}$

- Combine above equilibrium to derive system SNE
An Illustrating Example of the Followers’ Game

- A system of two flows, two relay nodes with perfectly reliable links: $C^1 = C^2 = 1$, $e^1 = e^2 = 0$, $u_f$ identical for two flows

**Diagram:**

- Symmetrical equilibrium: market competition
- Boundary equilibrium: market division

- Continuity property: the symmetrical equilibrium is continuous in $C$ while the boundary one is not

- This simple system indicates the followers’ game properties in general
The Followers’ Game: Equilibrium Analysis

**Theorem**

For any \( \{ C^f \} \), there exists a unique symmetrical equilibrium in the followers’ game.

**Theorem**

For any \( \{ C^f \} \), there exists at least a boundary equilibrium in the followers’ game.

- Besides, there is no other equilibrium
  - If a strictly interior equilibrium exists in the followers’ game, then it is symmetrical.
- The followers’ game: 1 symmetrical equilibrium + boundary equilibrium (may not be unique, may coincide with the symmetrical equilibrium)
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The Leaders’ Game: Equilibrium Analysis (1)

Theorem

If the followers always respond by playing in their (unique) symmetrical NE, then an equilibrium of the leaders’ game (i.e. an SNE of the overall system) exists and is unique.

Proof.

Derive the best response function \( B^f(C^{-f}) = \arg\max_{C^f} U_f(C^f, C^{-f}) \) and prove it continuous, bounded and concave in \( C \).
A boundary equilibrium of followers’ game may not lead to SNE for the system.

In the illustrating example (a system of 2 symmetrical flows with $e^f = 0$, $f = 1, 2$), the boundary equilibrium for followers’ game $\implies$ no further equilibrium for leaders’ game (no SNE of the Stackelberg game).

Equilibrium structure: substantially different from existing pricing methods.

**Theorem**

*If the followers always respond by playing in their (unique) symmetrical NE, any asynchronous myopic best-response update for the leader converges to the (unique) equilibrium of the leaders’ game.*
Numerical Studies: SNE Efficiency Analysis

- 2 flows and $R$ ($1 \leq R \leq 5$) relay nodes
- 2 kinds of utility functions
  - Power-law utility: $u_f(P_{suc}^f) = m_f(P_{suc}^f)^a$, $0 < a \leq 1$
  - Logarithmic utility: $u_f(P_{suc}^f) = m_f \log(1 + P_{suc}^f)$
- For each $R$, we run 100 random scenarios with $m_f \in [1, 100]$, $e^f \in [0, 10]$, $K^f \in [0, 1]$

Figure: Average ratio between $U_{opt}$ and $R = 4$

Figure: Histogram of $U_{SNE}/U_{opt}$ for
Numerical Studies: Large system scenario

- 100 relay nodes and 10 flows with identical parameters of $K^f = 0.6$, $u_f(P^f_{suc}) = P^f_{suc}$, and $e^f = e$ for all flows.
- Cooperation probabilities of all relay nodes with all flows are identical ($r_i^f = r, \forall i \in \mathcal{R}, f \in \mathcal{F}$), whether in the (unique) system equilibrium or in the optimal operating point.

**Figure:** Cooperation probability as a function of $e$.

**Figure:** Network utility at symmetric optimum and SNE.
Conclusion

A distributed market pricing framework to study conflicts and incentives in wireless cooperative relaying
- A game theoretical analysis on the resulting generic Stackelberg game
- Focus on structural properties on the equilibria and interactions among source and relay nodes

Perspective
- Some theoretical problems: analytical bounds for equilibrium efficiency, extensions to the general asymmetrical case
- Practical mechanism design: how to exchange pricing information among flows and relay nodes, how to enforce payments, etc.


Numerical Studies: An asymmetrical scenario

- Asymmetrical relay nodes: heterogeneous $K^f$ and $e^f$
  - 2 flows and 2 relay nodes, with $K_1^1 = K_2^1 = 0.8$, $K_1^2 = 0.5$, $K_2^2 = 0.4$, $e_1^1 = e_2^1 = 0$, $u_f(P^f_{suc}) = m_f P^f_{suc}$, where $m_1 = 1$ and $m_2 = 2$.
  - Symmetrical equilibrium $\rightarrow$ interior strategy profile $r = \{r^f_i\}$, $0 \leq r^f_i \leq 1$ satisfying $\frac{\partial V_i}{r^f_i} = 0$, $i, f = 1, 2$ (interior equilibrium)
  - Game dynamics: flows follow best-response update, under the assumption that the followers play an interior equilibrium
- Existence of interior and boundary equilibria in the followers’ game and the convergence to the SNE still hold in asymmetrical scenario

![Figure: Trajectory of prices: asymmetrical scenario](image-url)