Adaptive Passivity–based Control for Maximum Power Extraction of Stand–alone Windmill Systems

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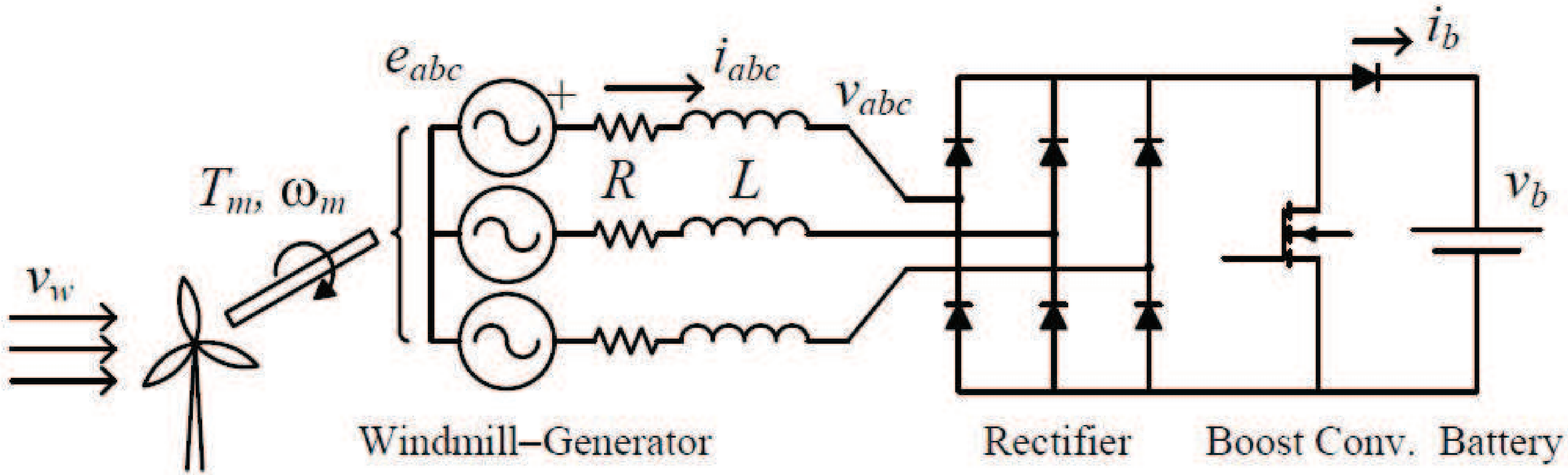
Layout

- Mathematical model of the small scale windmill system
- Wind Speed estimator and passivity–based controller
- Simulation results
- Conclusions and future work

Model of the System

Battery charging windmill system



PM Synchronous Generator and Wind Turbine

- In the rotor (dq) reference frame

$$\begin{aligned} L\dot{i}_d &= -Ri_d + Li_q\tfrac{P}{2}\omega_m - v_d, \\ L\dot{i}_q &= -Ri_q - Li_d\tfrac{P}{2}\omega_m + \phi\omega_e - v_q, \end{aligned}$$

  where i_d, i_q, v_d, v_q, are the currents and voltages in the dq reference frame, L and R are the stator winding's inductance and resistance, ω_m is the mechanical speed, ϕ is the permanent magnetic flux, and P is the number of pole pairs.

- The mechanical dynamics, with J the rotor inertia,

$$J\dot{\omega}_m = \underbrace{\frac{P_w}{\omega_m}}_{T_m} - \underbrace{\frac{3}{2}\frac{P}{2}\phi i_q}_{T_e}.$$

- The mechanical power at the windmill shaft

$$P_w = \frac{\rho A}{2} C_p(\lambda) v_w^3, \quad \lambda := \frac{r\omega_m}{v_w}.$$

  with v_w the unknown wind speed and $C_p(\lambda)$, the power coefficient.

Rectifier, dc/dc Converter and Full Model

- dq model

$$\begin{aligned} v_d &= \frac{i_d}{\sqrt{i_d^2+i_q^2}} M v_b D, \\ v_q &= \frac{i_q}{\sqrt{i_d^2+i_q^2}} M v_b D. \end{aligned} \qquad (1)$$

where is the v_b battery voltage, $M = \frac{\pi}{3\sqrt{3}}$ is the gain of the passive diode rectifier and D the duty ratio.

- Overall system

$$\begin{aligned} L\dot{x}_1 &= -Rx_1 + L_1 x_2 x_3 - C_1 x_1 u \\ L\dot{x}_2 &= -Rx_2 - L_1 x_1 x_3 + \phi_1 x_3 - C_1 x_2 u \qquad (FM) \\ J_1\dot{x}_3 &= -\phi_1 x_2 + \Phi(x_3, v_w), \end{aligned}$$

with $x :=$ col$(i_d, i_q, r\omega_m)$, the input $u := \frac{D}{\sqrt{x_1^2+x_2^2}}$, and the function

$$\Phi(x_3, v_w) := C_2 \frac{v_w^3}{x_3} C_p \left(\frac{x_3}{v_w} \right). \qquad (2)$$

Control Problem

- Operate the system at the point of maximum power extraction $\lambda_\star := \arg\max C_p(\lambda)$, which is typically known.
- If v_w is known the control task is the regulation of ω_m around the reference speed

$$\omega_m^* = \frac{\lambda_\star v_w}{r} =: \frac{1}{r} x_{3\star}.$$

- An on–line wind speed estimator is added to generate

$$\hat{\omega}_m^* = \frac{\lambda_\star \hat{v}_w}{r}.$$

- The problem is translated into asymptotic stabilization of the desired equilibrium $x_\star = (x_{1\star}^s, x_{2\star}, x_{3\star})$

Remark Nonlinearly parameterized nonlinear system, hence linear control and standard estimation are not applicable. Solved using passivity–based control (PBC) (Ortega, et al., Springer Book '98) and immersion and invariance (I&I) adaptation principles (Astolfi and Ortega, TAC'03, Springer Book '07).

Assumptions for Wind Speed Estimation

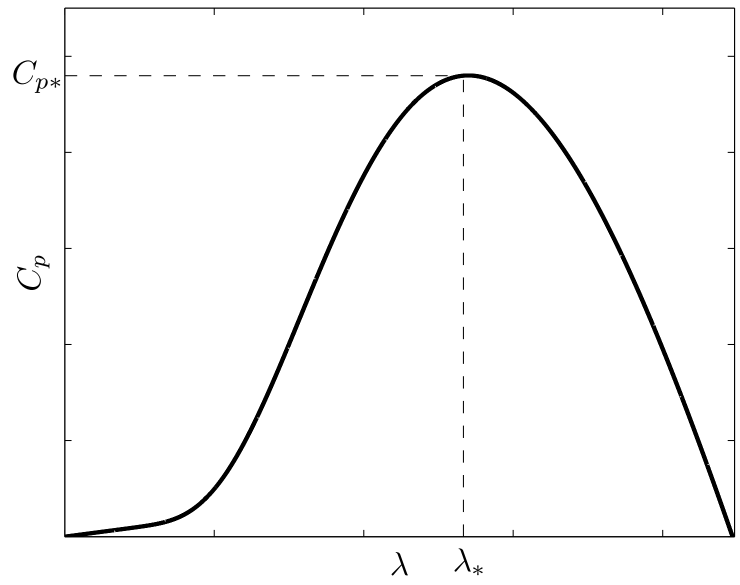
- Assumption 1 The power coefficient is a known, smooth, function

$$C_p : [0, \lambda_M] \to \mathbb{R}_+,$$

which verifies

$$C_p'(\lambda) \begin{cases} > 0 & \text{for} \quad \lambda \in [0, \lambda^\star) \\ = 0 & \text{for} \quad \lambda = \lambda^\star \\ < 0 & \text{for} \quad \lambda \in (\lambda^\star, \lambda_M], \end{cases}$$

where $\lambda^\star := \arg\max C_p(\lambda)$.



- Assumption 2 The wind speed v_w is an unknown positive constant.
- Assumption 3 The electrical torque T_e and the motor speed w_m are measurable.
- Assumption 4 For all $\lambda \in (0, \lambda^\star)$, the power coefficient verifies

$$\frac{3}{\lambda} C_p(\lambda) > C_p'(\lambda).$$

Some Remarks

- $C_p(\lambda)$ can be easily obtained from experimental data, and the algorithm implemented from a table look–up.
- Constant wind speed assumption only needed for the theory. An on–line estimator is able to track slowly–varying parameters, assumption justified by the time scale separation between the wind dynamics and the mechanical and electrical signals.
- On–line estimators average the noise—in contrast with differentiator–based or extended Kalman filter schemes currently used.
- Measuring w_m and T_e is standard practice in windmill systems.
- Theory applicable also if blade pitch β is included, i.e., $C_p(\lambda, \beta)$, or for more complete descriptions of the mechanical dynamics.
- Assumption 4 is satisfied in normal operating range (for Region 2), where the torque coefficient has negative slope. Indeed,

$$C_T(\lambda) := \frac{1}{\lambda} C_p(\lambda),$$

satisfies

$$C_T'(\lambda) \leq 0 \;\Rightarrow\; \text{Assumption 4.}$$

Main Estimation Result

Proposition (Ortega, *et al.*, IJACSP'12) Consider the system (FM), verifying Assumptions 1–4. The I&I estimator

$$\begin{aligned} \dot{\hat{v}}_w^I &= \gamma \left[T_e - \frac{\rho A}{2} \frac{(\hat{v}_w^I + \gamma \omega_m)^3}{\omega_m} C_p \left(\frac{r \omega_m}{\hat{v}_w^I + \gamma \omega_m} \right) \right] \\ \hat{v}_w &= \hat{v}_w^I + \gamma \omega_m , \end{aligned}$$

where $\gamma > 0$, is an adaptation gain, is asymptotically consistent, that is,

$$\lim_{t \to \infty} \hat{v}_w(t) = v_w .$$

Remarks

- Removing the term $\gamma \omega_m$ yields a copy of the mechanical equations.
- Comparative simulation study done by Vestas group (extended Kalman filters, approximate differentiation, neural networks): (Soltani, *et al.* IEEE–TCS (submitted)).

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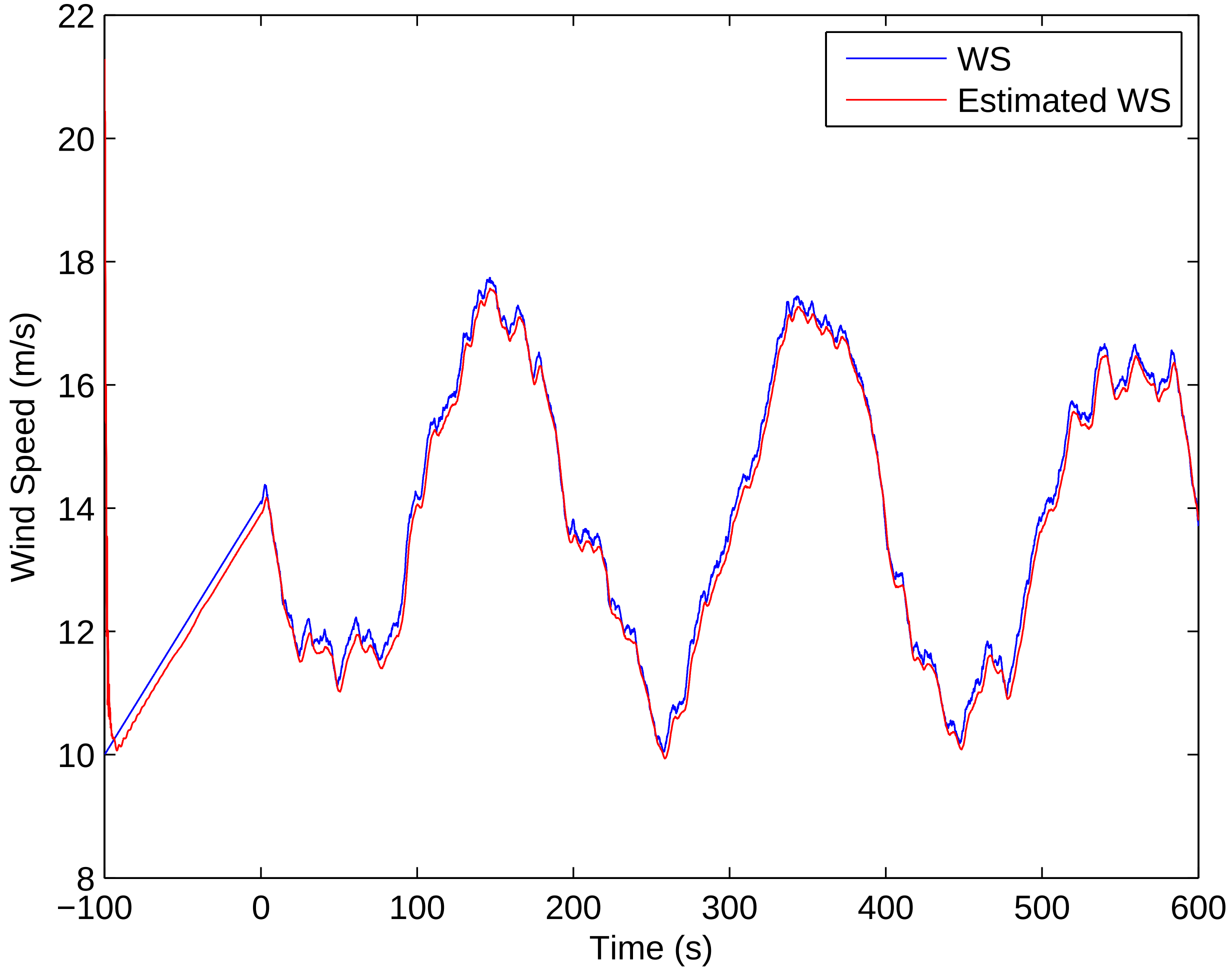
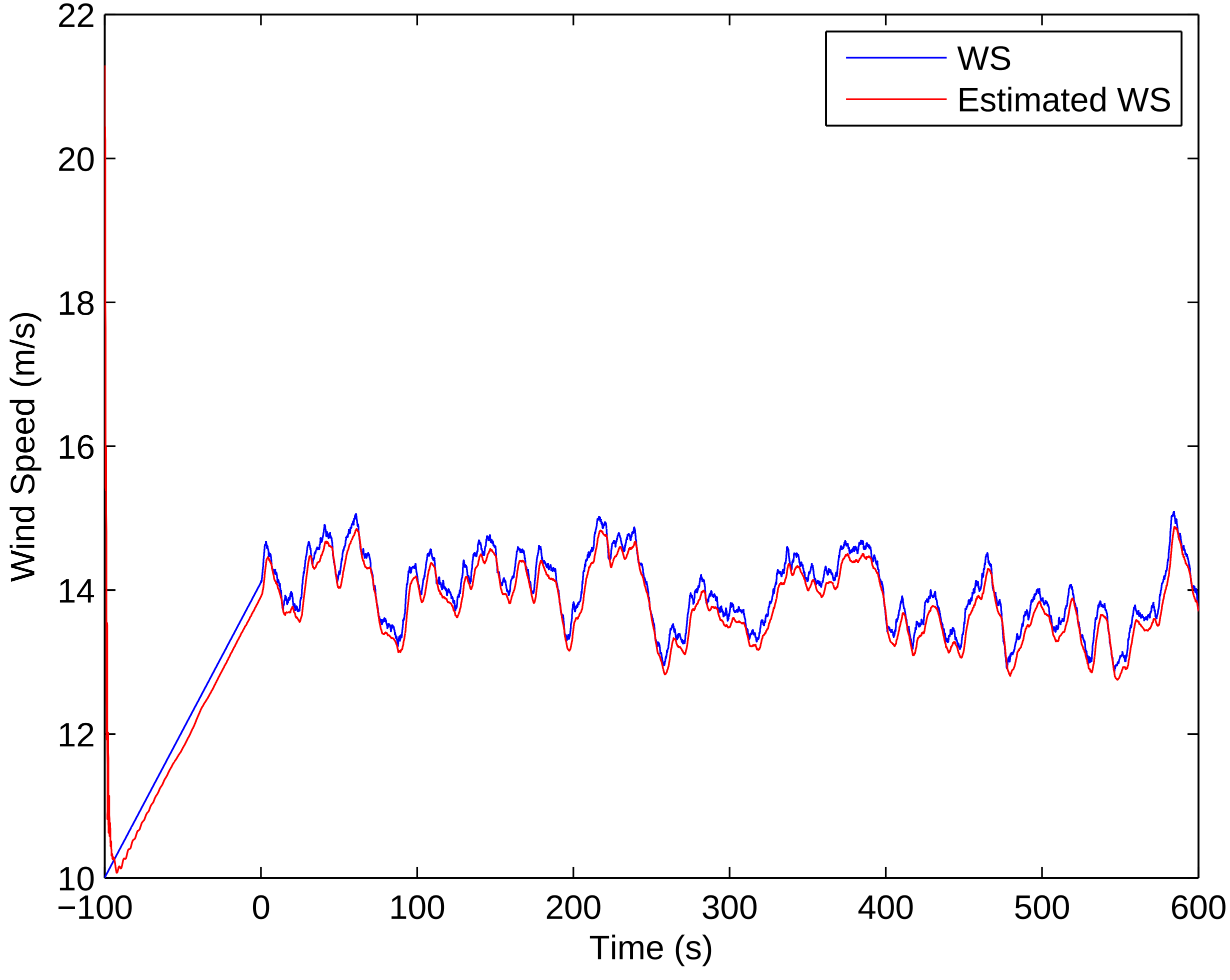
Simulation Results: Periodic Wind

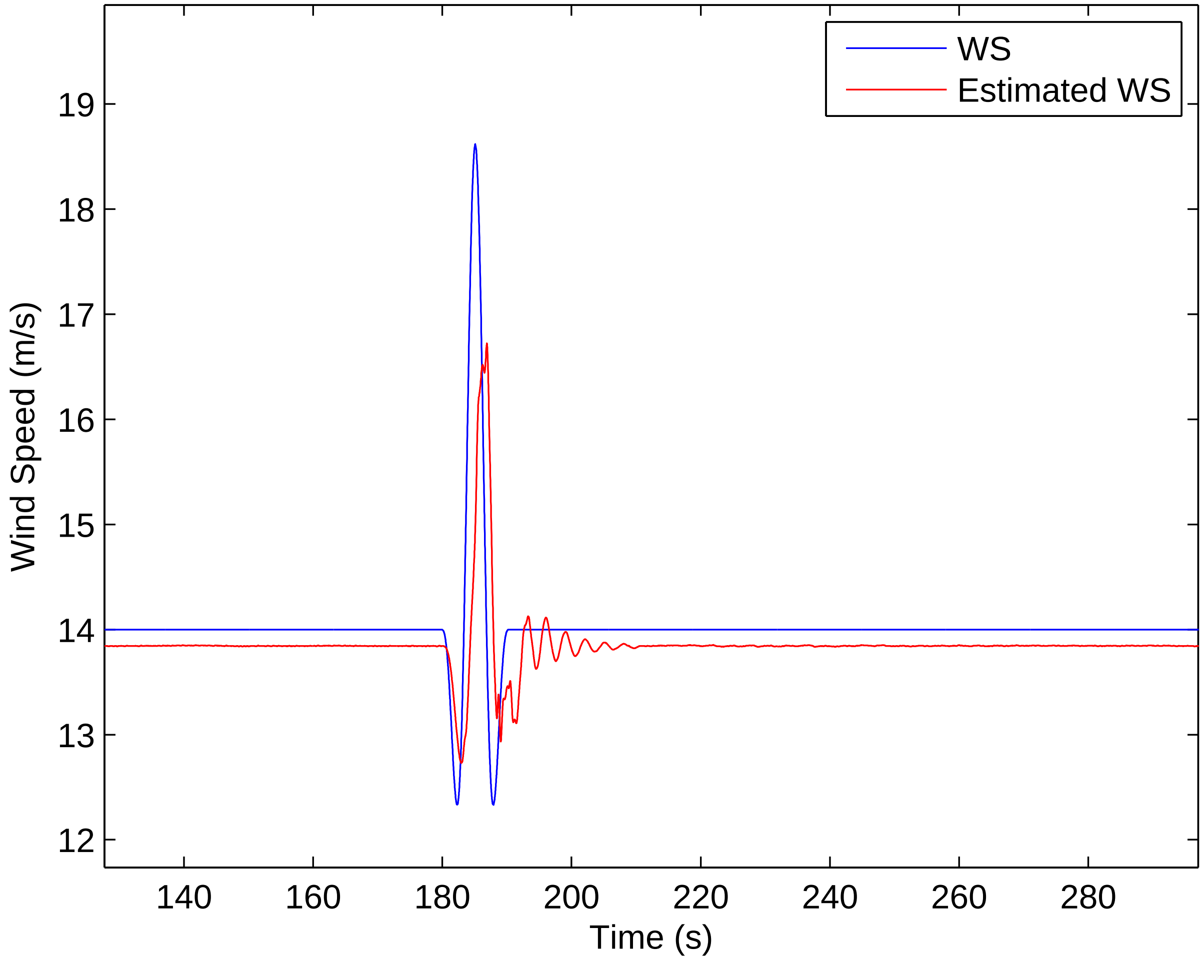
Done in Vestas professional software, with the full model, look–up table for $C_p(\lambda)$, and real wind data.



Simulation Results: Turbulent Wind



Simulation Results: Gust



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An Asymptotically Stable PBC with Known Wind Speed

Proposition Consider the system (FM), with known wind speed, in closed–loop with the PBC

$$\begin{aligned} L\dot{x}_{1d} &= -Rx_{1d} + \frac{L_1}{\phi_1}\Phi(x_{3\star}, v_w)x_3 - C_1 x_1 u \\ J_1\dot{x}_{3d} &= -\Phi(x_{3\star}, v_w) + \Phi(x_3, v_w) - R_{3a}(x_3 - x_{3d}) \\ u &= -\frac{1}{C_1 x_2}[\frac{R}{\phi_1}\Phi(x_{3\star}, v_w) + L_1 x_{1d} x_3 - \phi_1 x_{3d}], \end{aligned} \quad (3)$$

where $R_{3a} > 0$ is a damping injection. The equilibrium $x_\star$ is asymptotically stable.

Remarks

- The scheme is made adaptive replacing $\Phi(x_{3\star}, v_w)$ by $\Phi(\hat{x}_{3\star}, \hat{v}_w)$.
- An integral action can be added

$$J_1\dot{x}_{3d} = -\Phi(x_{3\star}, v_w) + \Phi(x_3, v_w) - R_{3a}(x_3 - x_{3d}) + \xi, \quad \dot{\xi} = -K_I e_3,$$

with $K_I > 0$ an integral gain.

Sketch of the Proof

(FM) written in Euler–Lagrange form (to reveal workless forces):

$$\mathcal{D}\dot{x} + [\mathcal{C}(x) + \mathcal{R}]x = G(x)u + b(x),$$

where

$$\mathcal{D} := \text{diag}\{L, L, J_1\} > 0, \mathcal{C}(x) = -\mathcal{C}^\top(x) := \begin{bmatrix} 0 & -L_1 x_3 & 0 \\ L_1 x_3 & 0 & -\phi_1 \\ 0 & \phi_1 & 0 \end{bmatrix},$$

$$\mathcal{R} := \text{diag}\{R, R, 0\} \geq 0, b(x) := \begin{bmatrix} 0 \\ 0 \\ \Phi(x_3, v_w) \end{bmatrix}, \; G(x) := \begin{bmatrix} -C_1 x_1 \\ -C_1 x_2 \\ 0 \end{bmatrix}.$$

The systems energy function $H(x) = \frac{1}{2}x^T \mathcal{D}x$, satisfies the power–balance equation

$$\dot{H} = \underbrace{-R|i_{dq}|^2}_{\text{dissipation}} - \underbrace{|v_{dq}||i_{dq}|}_{\text{elec. power}} + \underbrace{\frac{2}{3}P_w,}_{\text{mech. power}}$$

Energy Shaping and Damping Injection

Idea is to assign a new energy function

$$W(e) := \frac{1}{2} e^\top \mathcal{D} e,$$

where $e := x - x_d$ and $x_d :=$ col(x_{1d}, x_{2*}, x_{3d}).
The controller can be written as

$$\mathcal{D}\dot{x}_d + [\mathcal{C}(x) + \mathcal{R}]x_d = b(x) + G(x)u + u_{di}$$

where u_{di} is an additional damping injection signal: $u_{di} = -\mathcal{R}_a e$. This yields the error equation

$$\mathcal{D}\dot{e} + \mathcal{C}(x)e + (\mathcal{R} + \mathcal{R}_a)e = 0$$

Taking the derivative of W yields

$$\dot{W} = -e^\top (\mathcal{R} + \mathcal{R}_a)e \leq -2\frac{\min\{R, R_{3a}\}}{\max\{L, J_1\}} W$$

establishing that $e(t) \to 0$, exponentially fast. Proof completed with some signal chasing to prove that $x_d(t) \to x_\star$.

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Simulation Results

- Power coefficient given by the function

$$C_p(\lambda) = e^{-\frac{c_{p1}}{\lambda}} \left(\frac{c_{p2}}{\lambda} - c_{p3} \right) + c_{p4}\lambda,$$

where $c_{p1} = 21.0000$, $c_{p2} = 125.229$, $c_{p3} = 9.7803$, and $c_{p4} = 0.0068$—from Matlab package. This yields $\lambda_* = 8.1$ and $C_{p*} = 0.48$.

- Windmill/battery system parameters

Item	Value
Pole pairs	$P = 28$
Synchronous resistance	$R = 0.3676\ (\Omega)$
Synchronous reactance	$L = 3.55\ (mH)$
Flux	$\phi = 0.2867\ (Wb)$
Inertia	$J = 7.856\ (kg\ m^2)$
Blades radius	$r = 1.84\ (m)$
Battery voltage	$v_b = 48\ (V)$

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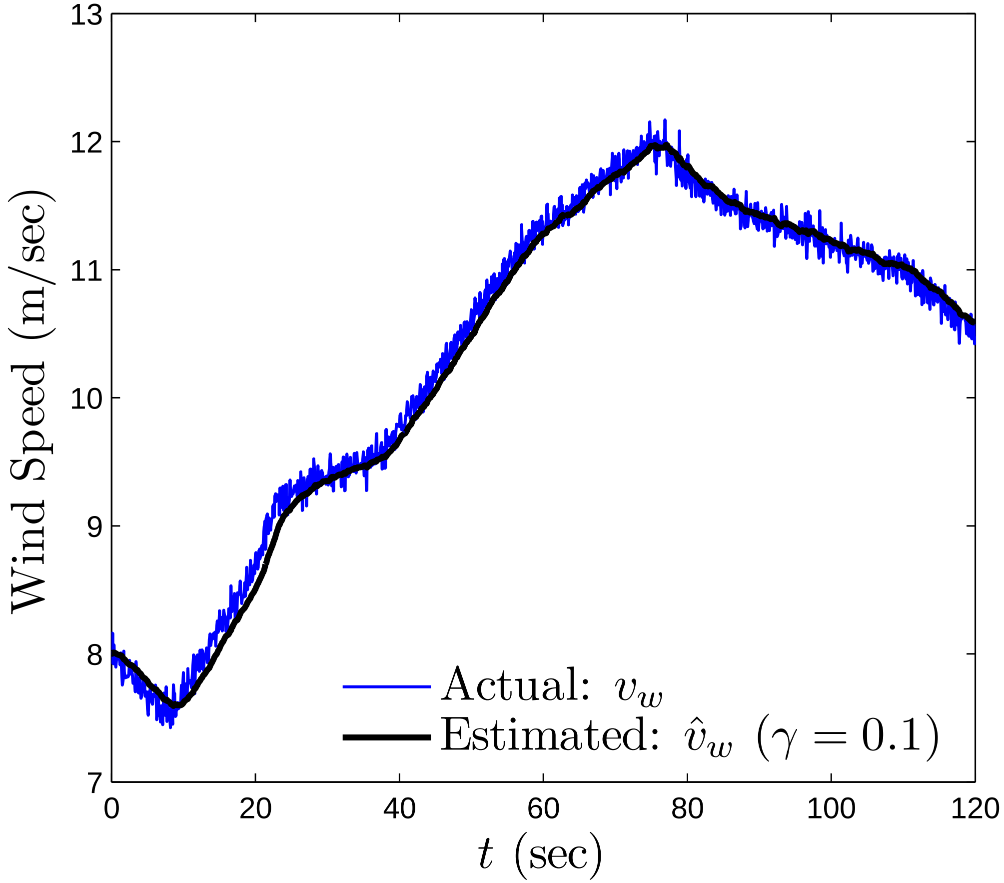
Wind Speed Estimator

- The estimator and controller parameters $\gamma = 0.1$, $R_{3a} = 0.01$ and $K_I = 1$.



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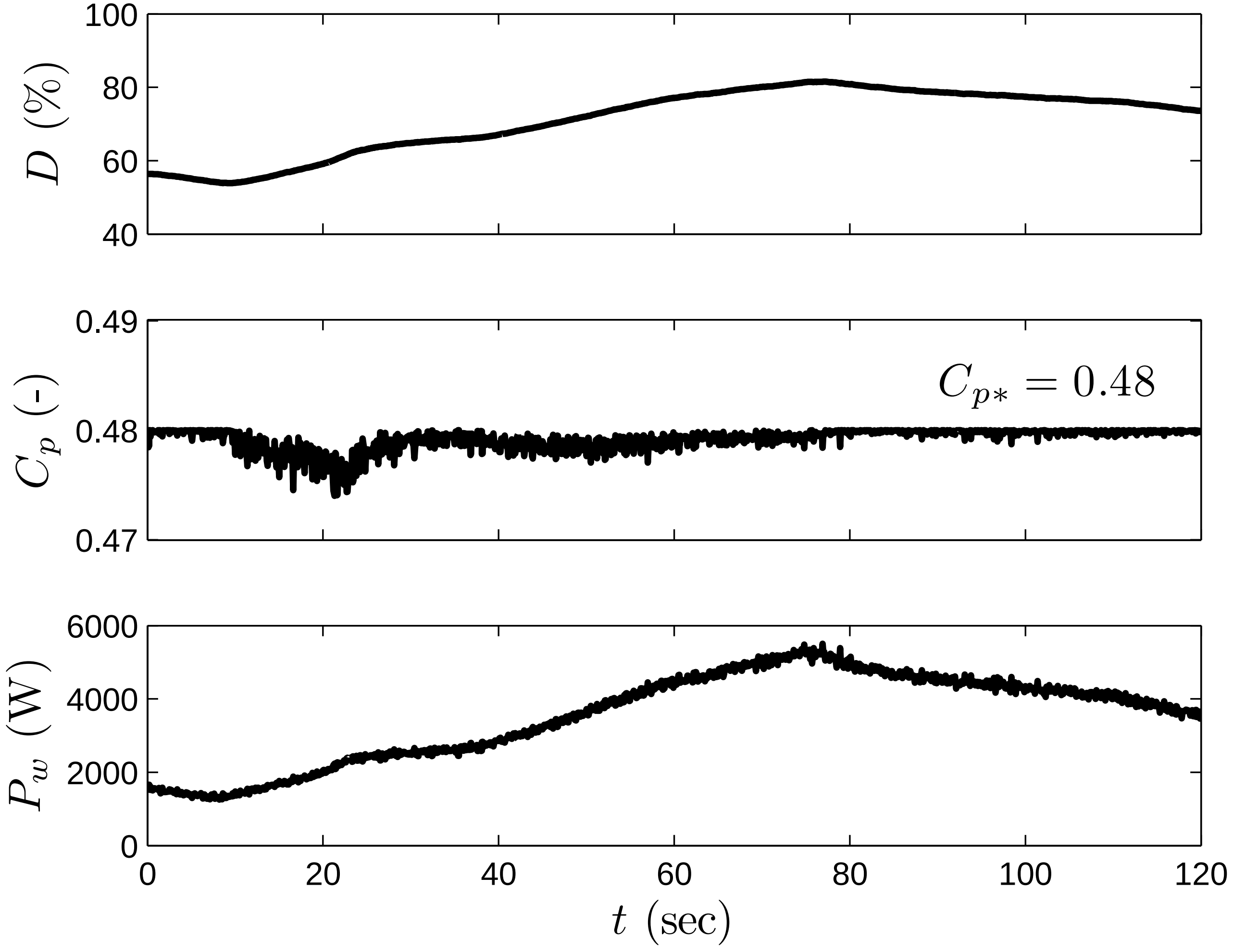
Duty Ratio, Power Coefficient and Power Capture



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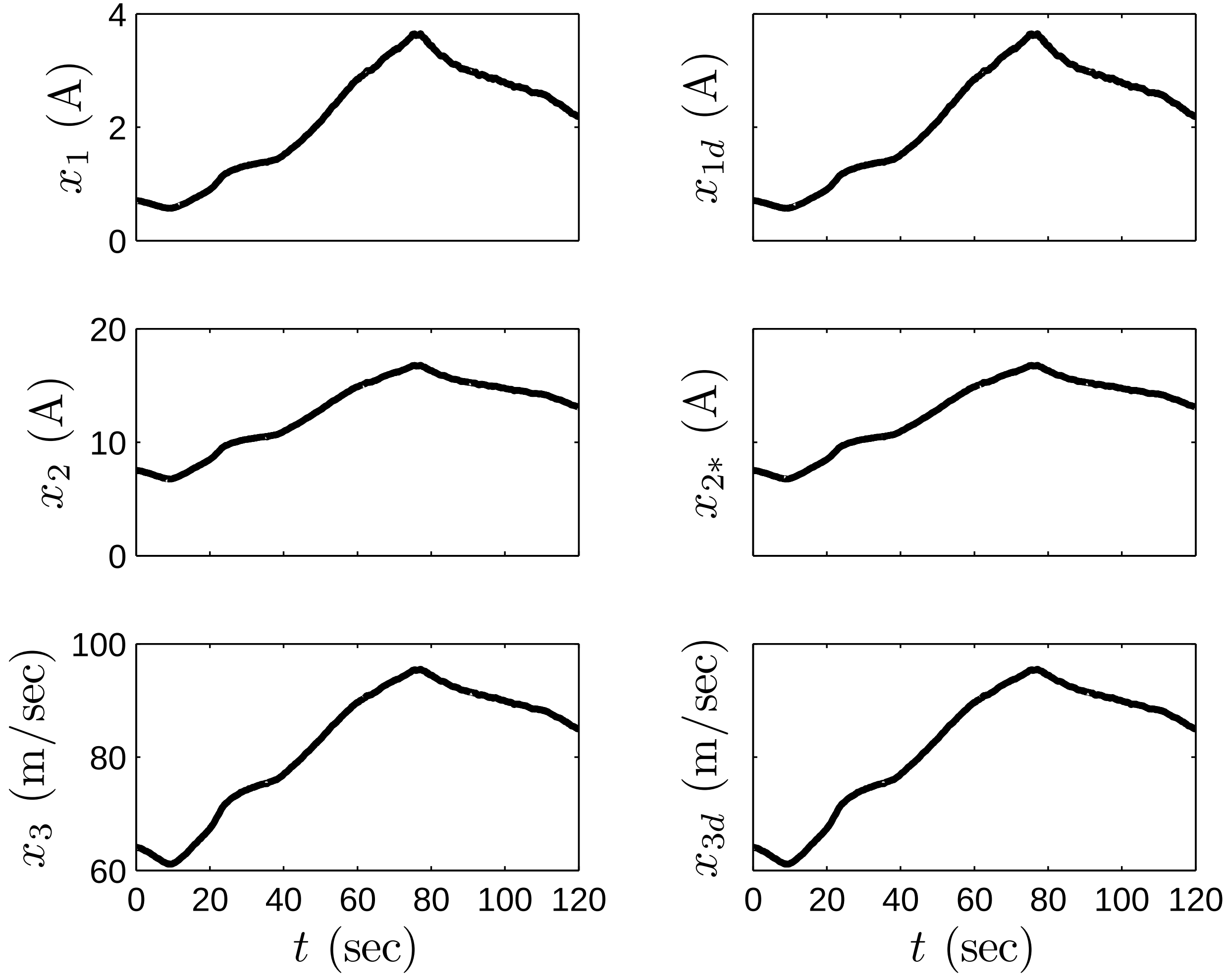
Evolution of all States



Future Work

- Analysis of the adaptive scheme:
  - Tough without persistency of excitation!
- Consider torsional modes:
  - Derive I&I estimator.
  - PBC.
  - Incorporate torsion minimization considerations.
- The stability result is only local, because stable invertibility of the system is needed.
  - New PBC's, for Hamiltonian models, obviate this assumption.
  - But a PDE needs to be solved.
- Comparison of adaptive PBC with ω_m^2–controller. Partial results for adaptive PI.
- Can Assumption 4 be relaxed?
  - It is not (globally) satisfied for several known turbines.
  - Analysis when it does not hold?