INRIA/Alcatel-Lucent Chair in Flexible Radio - SUPELEC

Optimal mobile association on hybrid networks: centralized and decentralized case

Alonso SILVA
alonso.silva@sophia.inria.fr
(joint work with E. Altman, M. Debbah, H. Tembine)

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Wireless Network

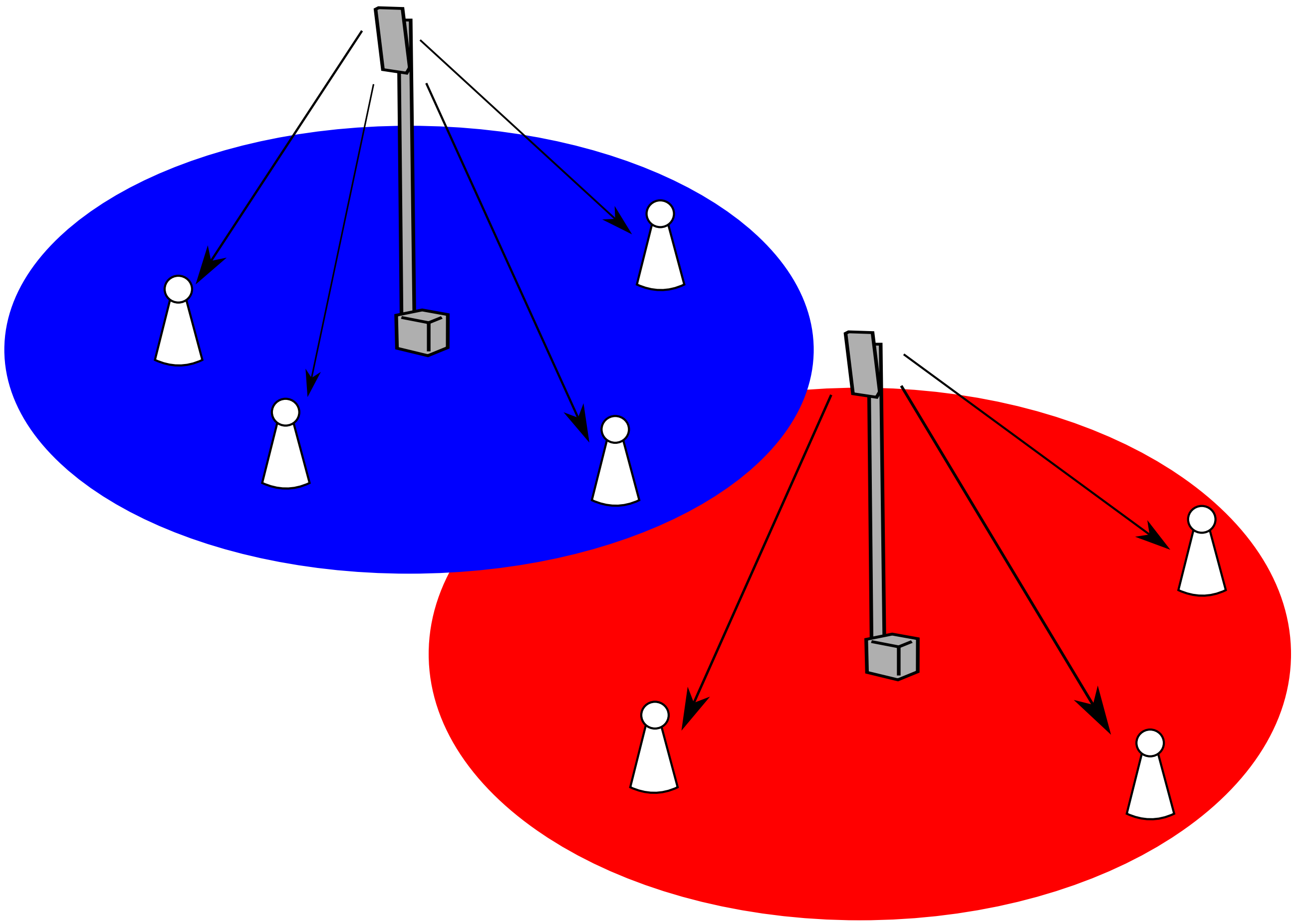
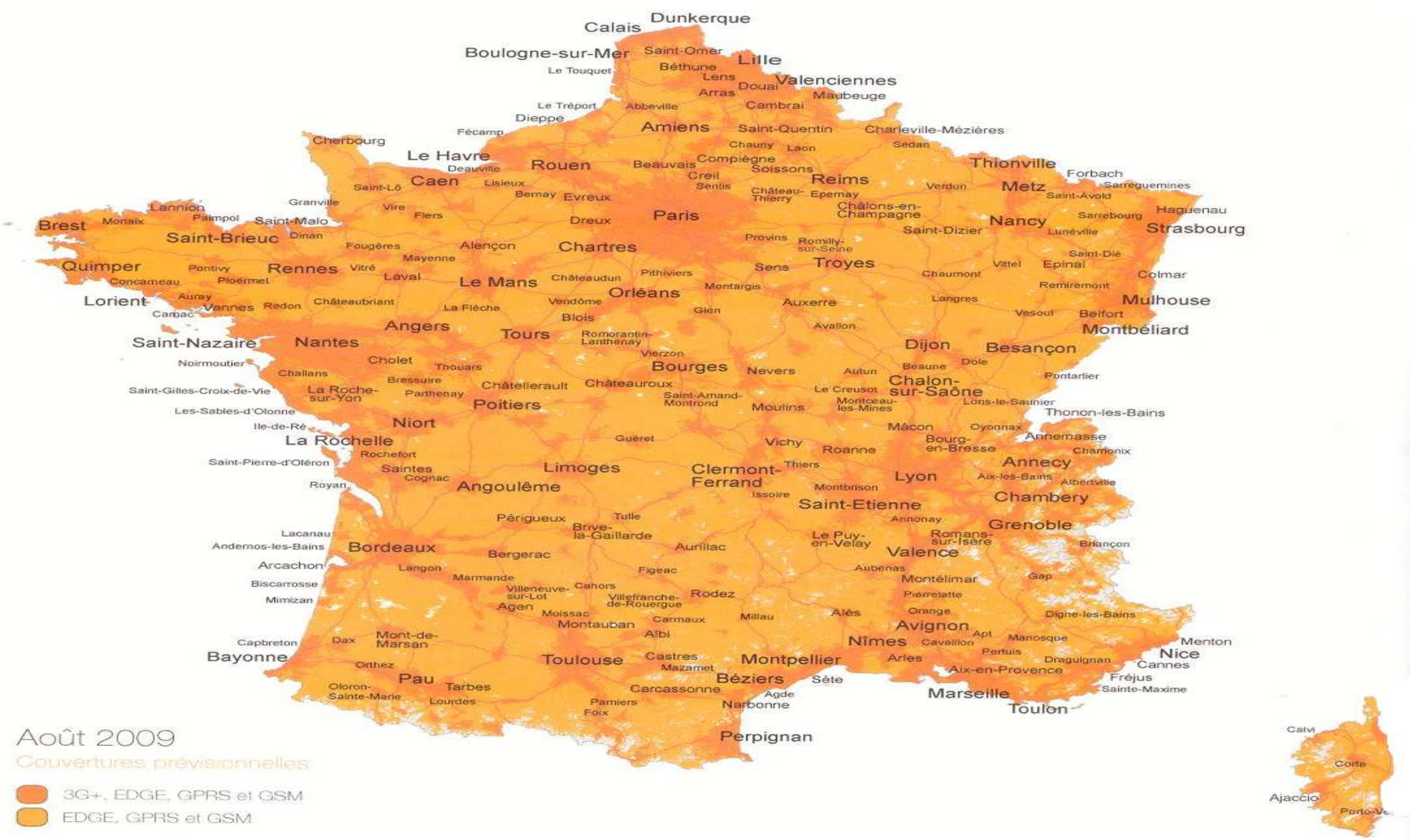


Figure 1: Mobile association to each base station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Le Tréport
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Le Havre
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Metz
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Flers
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Brest
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Paris
Provins
Châlons-en-Champagne
Saint-Dizier
Nancy
Sarrebourg
Haguenau
Lunéville
Strasbourg
Fougères
Alençon
Chartres
Mayenne
Quimper
Pontivy
Rennes
Vitré
Laval
Concarneau
Ploërmel
Le Mans
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Les-Sables-d'Olonne
Niort
Mâcon
Oyonnax
Ile-de-Ré
La Rochelle
Guéret
Vichy
Roanne
Bourg-en-Bresse
Annemasse
Rochefort
Chamonix
Saint-Pierre-d'Oléron
Saintes
Cognac
Limoges
Clermont-Ferrand
Thiers
Annecy
Royan
Angoulême
Lyon
Aix-les-Bains
Albertville
Montbrison
Issoire
Chambery
Saint-Etienne
Périgueux
Tulle
Annonay
Grenoble
Lacanau
Brive-la-Gaillarde
Le Puy-en-Velay
Romans-sur-Isère
Andernos-les-Bains
Bordeaux
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Briançon
Arcachon
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Marmande
Biscarrosse
Villeneuve-sur-Lot
Cahors
Villefranche-de-Rouergue
Rodez
Pierrelatte
Mimizan
Agen
Moissac
Orange
Digne-les-Bains
Carmaux
Millau
Alès
Avignon
Montauban
Albi
Apt
Manosque
Capbreton
Dax
Mont-de-Marsan
Nîmes
Cavaillon
Menton
Pertuis
Nice
Bayonne
Toulouse
Castres
Montpellier
Arles
Draguignan
Cannes
Orthez
Mazamet
Aix-en-Provence
Pau
Tarbes
Béziers
Sète
Fréjus
Oloron-Sainte-Marie
Carcassonne
Agde
Saint-Maxime
Lourdes
Pamiers
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Août 2009
Couvertures prévisionnelles
3G+, EDGE, GPRS et GSM
EDGE, GPRS et GSM

Definition of the problem

Consider:

- A grid area network D with large number of mobiles N, continuously distributed with distribution $\lambda(x, y)$ scaled so that

$$\iint_{\mathrm{D}} \lambda(x, y)\, dx\, dy = 1.$$

- K base stations $\mathrm{BS}_1, \mathrm{BS}_2, \ldots, \mathrm{BS}_K$ located at positions $(x_1, y_1), (x_2, y_2) \ldots, (x_K, y_K)$.

Our objective

Optimal mobile association in order to minimize the total power needed to maintain an average throughput of $\Theta > 0$ for each mobile of the network.

Examples of the distribution of users $\lambda(x, y)$:

- If the users are distributed uniformly in the network, then

$$\lambda(x, y) = \frac{1}{|D|}$$

where $|D|$ is the total area of the network.

- If the users are distributed according to different levels of population density, then

$$\lambda(x, y) = \begin{cases} \lambda_{\mathrm{HD}} \text{ if } (x, y) \text{ is at a High Density region,} \\ \lambda_{\mathrm{ND}} \text{ if } (x, y) \text{ is at a Normal Density region,} \\ \lambda_{\mathrm{LD}} \text{ if } (x, y) \text{ is at a Low Density region.} \end{cases}$$

where λ_{HD}, λ_{ND}, and λ_{HD} are defined similarly to the previous case.

- If the distribution of the users is radial with more mobile terminals in the center and less mobile terminals in the suburban areas then

$$\lambda(x, y) = \frac{R_D^2 - (x^2 + y^2)}{K_D},$$

where R_D is the radius of the network and K_D is a coefficient of normalization.

- If the distribution of users is a Poisson process with intensity ν, then

$$\lambda(x, y) = e^{-\nu \pi r^2}$$

where r is the polar coordinate representation of (x, y).

- Denote N_i the quantity of mobiles that are served by base station BS_i.
- Each cell C_i, to be determined, contains the mobiles served by base station BS_i

$$N_i(C_i) = N \iint_{C_i} \lambda(x, y)\, dx\, dy.$$

- We assume that the channel gain h_i at location (x_0, y_0) from base station BS_i is given by

$$h_i(x_0, y_0) = \left(R^2 + d_i^2(x_0, y_0)\right)^{-\xi/2}$$

where ξ is the path loss exponent and R is the high of the base station and $d_i(x_0, y_0)$ is the distance between base station BS_i and the mobile located at (x_0, y_0).

Downlink Case

The SNR (Signal to Noise Ratio) received at mobiles located at (x_0, y_0) is given by

$$\mathrm{SNR}_i(x_0, y_0) = \frac{P_i(x_0, y_0) h_i(x_0, y_0)}{\sigma^2},$$

where σ^2 is the noise power.

We assume that the instantaneous mobile throughput is given by the following expression, which is based on Shannon's capacity theorem:

$$\theta_i(x_0, y_0) = \log(1 + \mathrm{SNR}_i(x_0, y_0)).$$

Downlink Case: Rate of mobiles

We shall consider two different policies:

1. Round Robin Scheduling Policy: Centralized case.
2. Rate Fair Allocation Policy: Decentralized case.

Round Robin Scheduling Policy

We consider the policy that each base station BS_i devotes an equal fraction of time for transmission to each of its mobiles located within its cell C_i.

Then, as the number of mobiles located in cell C_i is $N_i(C_i)$, the average throughput of each mobile following the round robin scheduling policy will be given by:

$$\overline{\theta}_i(x_0, y_0) = \frac{1}{N_i} \log(1 + \mathrm{SNR}_i(x_0, y_0)).$$

Round Robin Scheduling Policy

The average throughput for each mobile has to be greater or equal than Θ, *i.e.*,

$$\overline{\theta}_i(x_0, y_0) = \frac{1}{N_i} \log(1 + \mathrm{SNR}_i(x_0, y_0)) \geq \Theta.$$

This equation is equivalent to

$$P_i(x_0, y_0) \geq \sigma^2 (2^{N_i \Theta} - 1)(R^2 + d_i^2(x_0, y_0))^{\xi/2}.$$

Round Robin Scheduling Policy

Then the problem, that we denote (RR), reads

$$(\mathrm{RR}) \quad \underset{C_i}{\mathrm{Min}} \sum_{i=1}^{K} \iint_{C_i} P_i(x,y)\lambda(x,y)\,dx\,dy.$$

where $\lambda(x,y)$ is the function of distribution of the users, such that,

$$P_i(x_0,y_0) \geq \sigma^2(2^{N_i\Theta}-1)(R^2+d_i^2(x_0,y_0))^{\xi/2}.$$

Rate Fair Allocation Policy

We consider the policy where each base station BS_i will maintain a constant power P_i sent to the mobile terminals within its cell.

$$P_i(x, y) = P_i \quad \text{for each } (x, y) \in C_i, \tag{1}$$

BUT, it will modify the fraction of time allowed to mobile terminals with different channel gains, in order that the average SNR of $\bar{\theta}(x, y)$ is satisfied.

Rate Fair Allocation Policy

Definition.- The Wardrop equilibrium is given by:

$$\text{If } \iint_{C_i} \lambda(x,y)\,dx\,dy > 0, \text{ then } r_i = \max_{1\le j\le K} r_j(C_j),$$

$$\text{and if } \iint_{C_i} \lambda(x,y)\,dx\,dy = 0, \text{ then } r_i \le \max_{1\le j\le K} r_j(C_j).$$

Rate Fair Allocation Policy

Similarly than for the Round Robin Scheduling Policy,

$$\text{(RF)} \quad \underset{C_i}{\mathrm{Min}} \sum_{i=1}^{K} \iint_{C_i} \sigma^2 (R^2 + d_i^2(x,y))^{\xi/2} \lambda(x,y)\, dx\, dy.$$

Uplink Case

We generalize Altman *et al.* INFOCOM 2009, to the two dimensional case

$$\mathrm{SINR}_i(x,y) = \frac{P_i(x,y)(R^2 + d_i(x,y)^2)^{-\xi/2}}{P_{\text{total}} + \sigma^2}\, dx,$$

where

$$P_{\text{total}} := \iint_D (R^2 + d_i(x,y)^2)^{-\xi/2} \lambda(x,y)\, dx\, dy.$$

As we want to guarantee an average SNR of $\Theta(x,y)$ to a mobile located at position (x,y) this condition is written as

$$\frac{P_i(x,y)(R^2 + d_i(x,y)^2)^{-\xi/2}}{P_{\text{total}} + \sigma^2}\, dx \geq \Theta(x,y).$$

Uplink Case

And then our problem reads

$$\text{(UL)} \quad \operatorname*{Min}_{C_i} \sum_{i=1}^{K} \iint_{C_i} P_i(x,y)\lambda(x,y)\Theta(x,y)\,dx\,dy$$

such that

$$P_i(x,y) \geq \Theta(x,y)(P_{\text{total}} + \sigma^2)(R^2 + d_i(x,y)^2)^{+\xi/2}.$$

Basics in Optimal Transportation

Mass transportation

The theory of mass transportation goes back to the original works by Monge [1781] and later by Kantorovich [1942].

The problem can be interpretated as the question:

"How do you best move given piles of sand to fill up given holes of the same total volume?"

Mass transportation

Example.- In $\mathbb{N}$

$$\mu = \frac{\delta_1 + \delta_2}{2} \quad \text{and} \quad \nu = \frac{\delta_2 + \delta_3}{2}.$$

Possible costs:

- $c_1(x, y) = |x - y|$,
- $c_2(x, y) = |x - y|^2$,
- $c_3(x, y) = \sqrt{|x - y|}$.

Possible Transports:

- We leave the mass in 2 and transport the mass in 1 into 3.
  The costs are 1, 2, and $1/\sqrt{2}$, respectively.
- We transport $T(x) = x + 1$. Then for all three cost functions the cost is one.

There are several transport applications!

Monge Problem

Monge's problem consists to search for the one that minimizes the cost, *i.e.*

$$(M) \qquad \inf_{T_{\#}\mu=\nu} \int_X c(x, T(x))\, d\mu(x).$$

Sometimes there are not transport applications.

Example.- There is no application sending $\mu = \delta_0$ into $\nu = \frac{\delta_0+\delta_1}{2}$.

We would have to split the mass, leave half of it in 0 and send half of it into 1.

Monge-Kantorovich Problem

We consider a more general strategy in which the mass in x will be distributed over the all space, following a distribution $d\lambda_x$.

Then $d\pi(x,y) = d\lambda_x(y)d\mu(x)$ is the transported mass from x to y.

The cost of transporting the mass in x will be

$$\int_Y c(x,y)\, d\lambda_x(y).$$

and the total transport cost will be

$$\int_{X\times Y} c(x,y)\, d\lambda_x(y) d\mu(x) = \int_{X\times Y} c(x,y)\, d\pi(x,y).$$

Monge-Kantorovich Problem

Kantorovich's problem consists to search the optimal transport plan

$$(K) \qquad \inf_{\pi \in \Pi(\mu,\nu)} \int_{X \times Y} c(x,y)\, d\pi(x,y)$$

Theorem.- (K) has a solution.

MK-Sum Case

Consider the problem $(\mathrm{P1})$

$$\underset{C_i}{\mathrm{Min}} \sum_{i=1}^{K} \iint_{C_i} \left[F(d_i(x,y)) + s_i \left(\iint_{C_i} \lambda(\omega, z)\, d\omega\, dz \right) \right] \lambda(x,y)\, dx\;\, dy,$$

where C_i is the cell partition of D. Suppose that s_i are continuously differentiable, non-decreasing, and convex functions. The problem $(\mathrm{P1})$ admits a solution that verifies

$$(\mathrm{S1}) \left\{ \begin{array}{ll} C_i = & \{x : F(d_i(x,y)) + s_i(N_i) + N_i \cdot s_i'(N_i) \leq \\ & \leq F(d_j(x,y)) + s_j(N_j) + N_j \cdot s_j'(N_j)\} \\ N_i = & \iint_{C_i} \lambda(\omega, z)\, d\omega\, dz. \end{array} \right.$$

MK-Multiplication Case

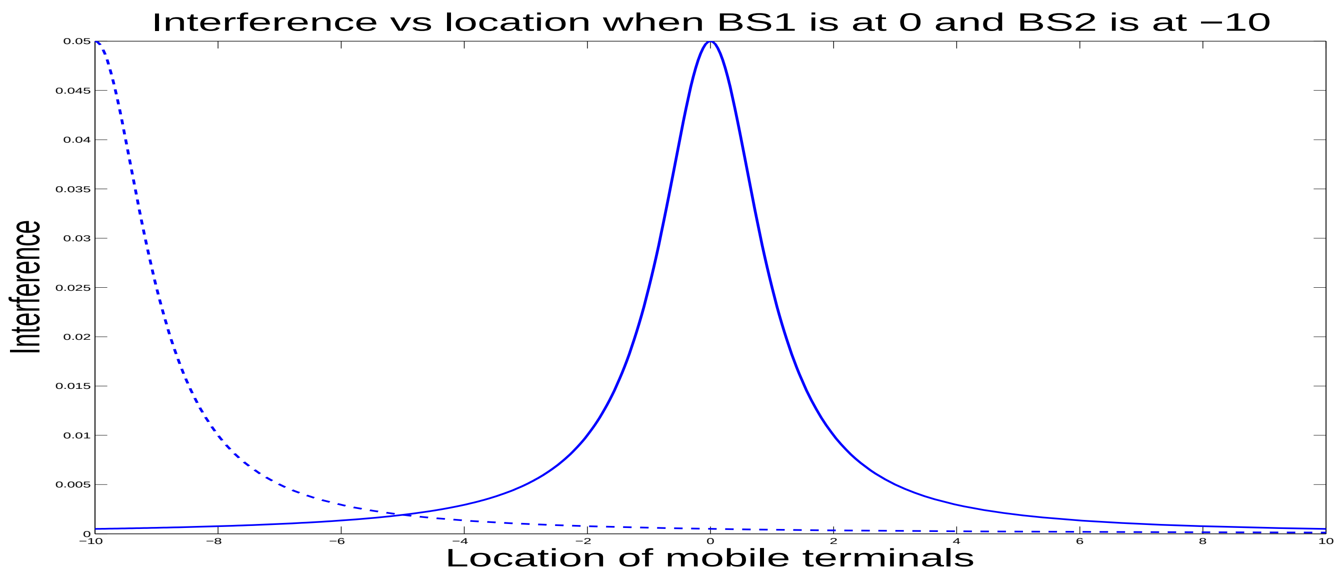
Consider the problem $(\mathrm{P2})$

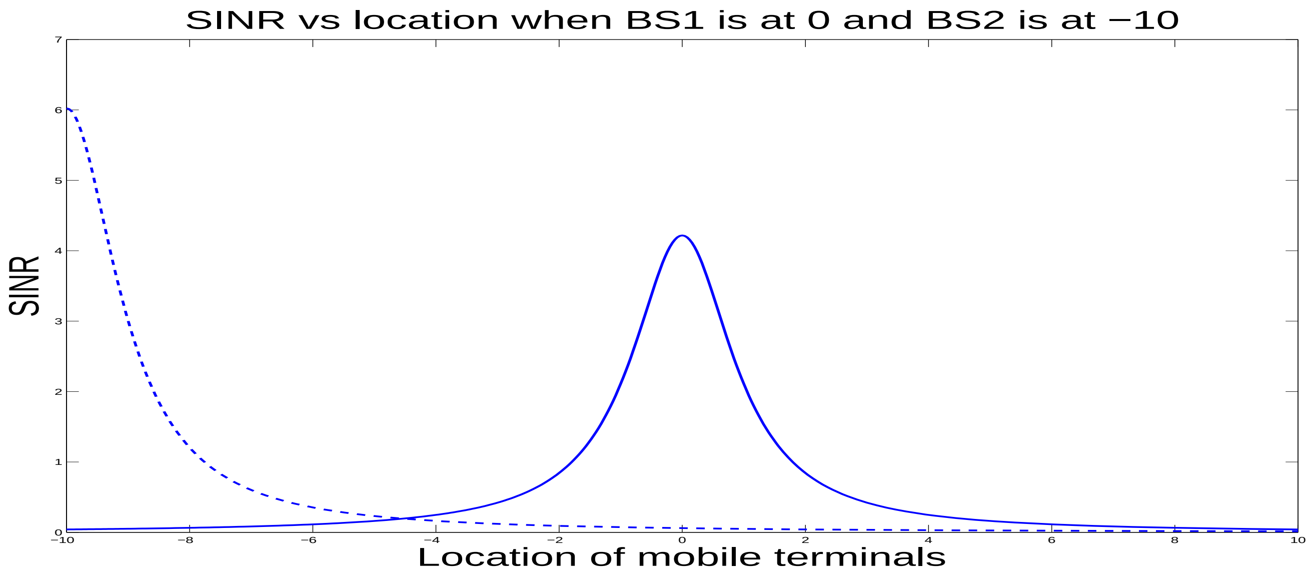
$$\operatorname*{Min}_{C_i} \sum_{i=1}^{K} \iint_{C_i} \left[F(d_i(x,y)) \cdot m_i \left(\iint_{C_i} \lambda(\omega, z)\, d\omega\, dz \right) \right] \lambda(x,y)\, dx\, dy$$

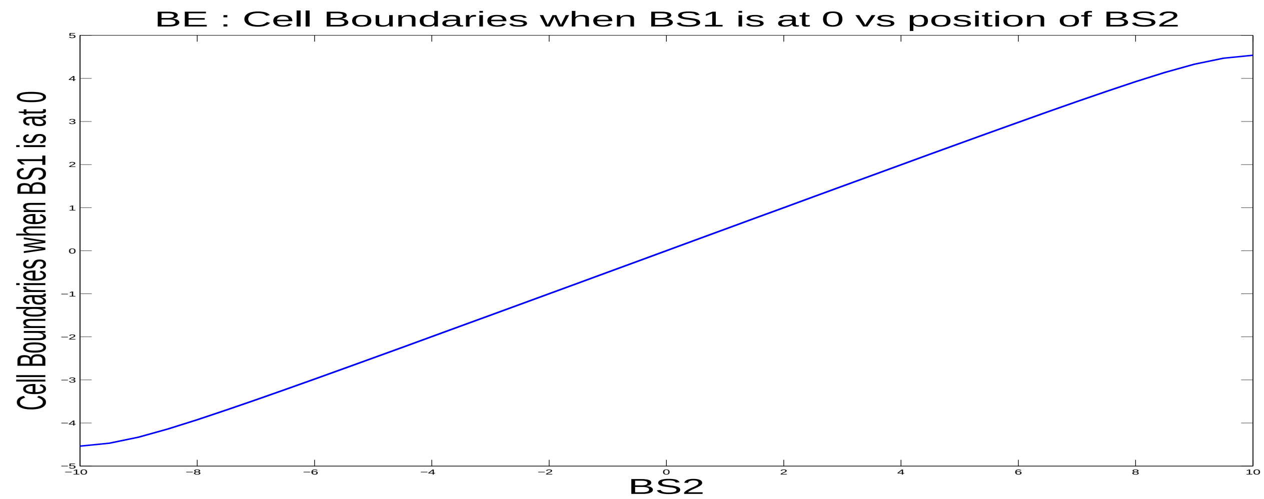
where C_i is the cell partition of D. Suppose that m_i are derivable. The problem $(\mathrm{P2})$ admits a solution that verifies

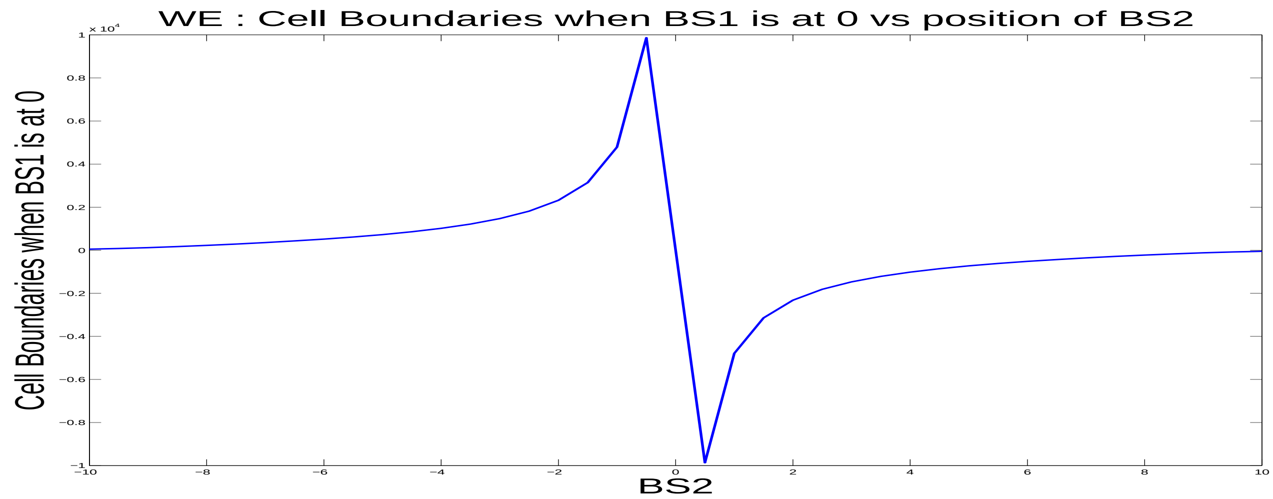
$$(\mathrm{S2}) \begin{cases} C_i = & \{x \;:\; m_i(N_i) F(d_i(x,y))\, \lambda(x,y) + U_i(x,y) \leq \\ & \leq m_j(N_j) F(d_j(x,y))\, \lambda(x,y) + U_j(x,y)\} \\ U_i = & m_i'(N_i) \iint_{C_i} F(d_i(x,y)) \lambda(x,y)\, dx\, dy \\ N_i = & \iint_{C_i} \lambda(\omega, z)\, d\omega\, dz. \end{cases}$$

Application

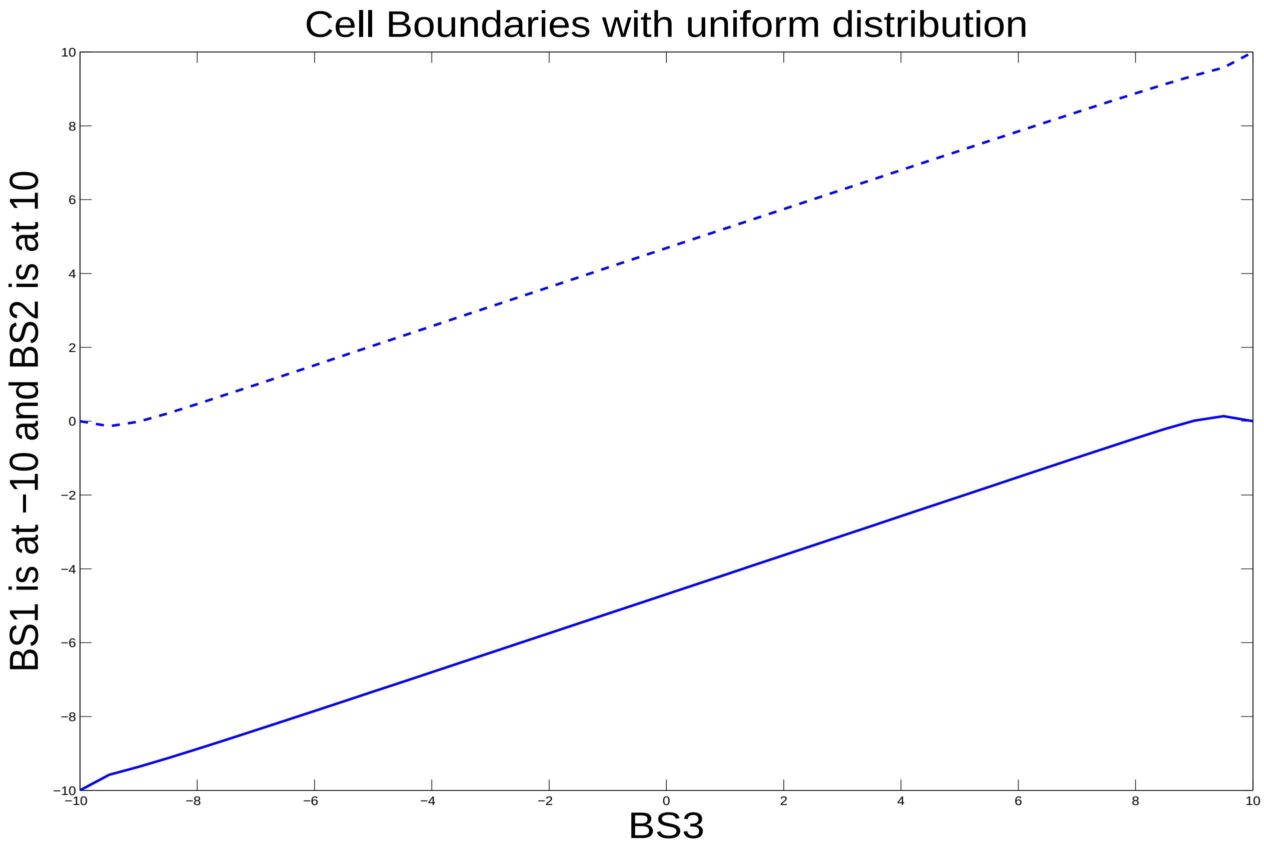

Interference vs location when BS1 is at 0 and BS2 is at −10
Interference
Location of mobile terminals
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
−10
−8
−6
−4
−2
0
2
4
6
8
10


SINR vs location when BS1 is at 0 and BS2 is at −10
SINR
Location of mobile terminals
7
6
5
4
3
2
1
0
−10
−8
−6
−4
−2
0
2
4
6
8
10

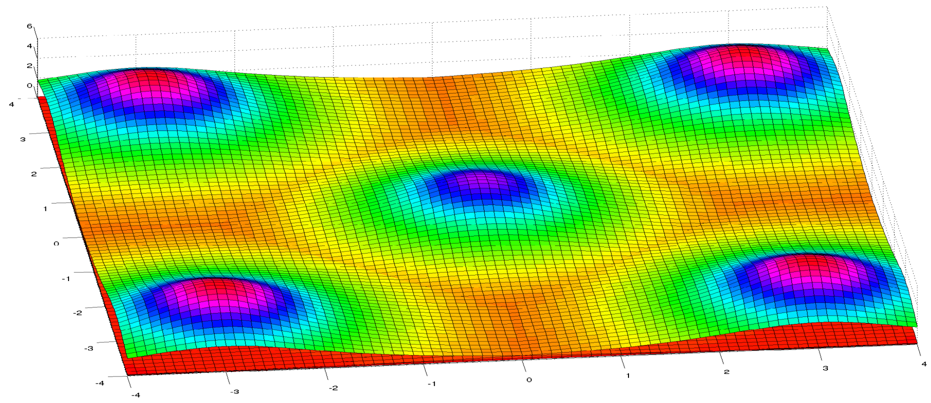

BE : Cell Boundaries when BS1 is at 0 vs position of BS2
Cell Boundaries when BS1 is at 0
BS2


WE : Cell Boundaries when BS1 is at 0 vs position of BS2
Cell Boundaries when BS1 is at 0
BS2

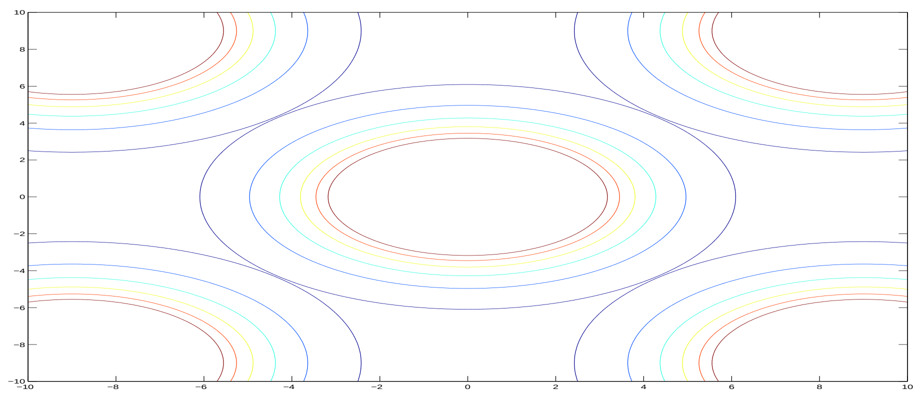
Decentralized Case



Decentralized Case

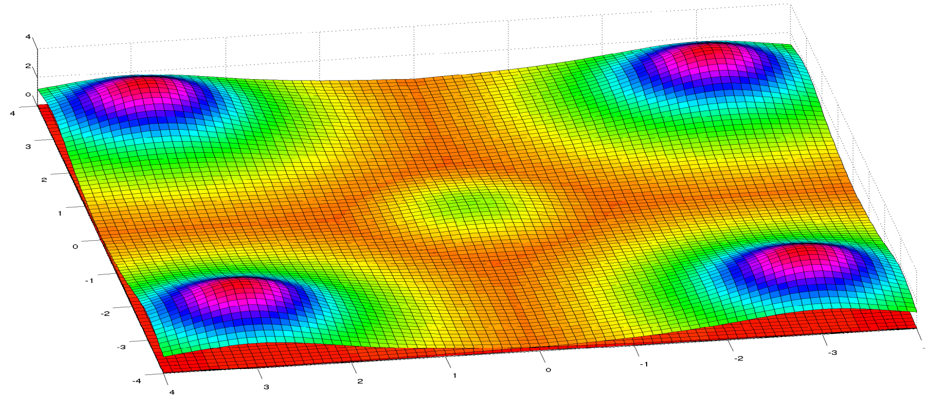


Cell boundaries with uniform distribution of users.

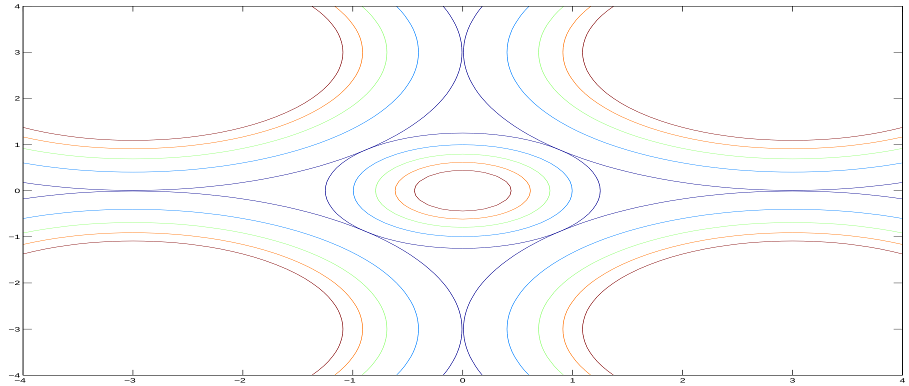


Cell contours

Decentralized Case



Non-Homogeneous case: Cell boundaries.



Non-Homogeneous case: Cell contours.

Example: Round Robin Scheduling Policy

Consider a network of $N = 2500$ mobile terminals distributed according to $\lambda(x)$ in $[0, L]$

- $\mathrm{BS}_1 = 0$ and $\mathrm{BS}_2 = L$

Then the system of equations is reduced to find x such that:

$$(2^{N_1\theta} - 1)(1 + x^2)\lambda(x) + 2^{N_1\theta}\theta \log 2 \left[x + \frac{x^3}{3}\right]$$
$$= (2^{N_2\theta} - 1)(1 + (1 - x)^2)\lambda(x) +$$
$$2^{N_2\theta}\theta \log 2 \left[\frac{4}{3} - 2x + x^2 - \frac{x^3}{3}\right]$$

This is a fixed point equation on x.

Example: Round Robin Scheduling Policy

- When mobile terminals are distributed uniformly, the optimal solution is given by $[0, 1/2)$ and $[1/2, 1]$,
- Then it is the case of Voronoi cells and the number of mobile terminals connected to each base station is equal and given by $N_1 = N_2 = 1250$.

Example: Round Robin Scheduling Policy

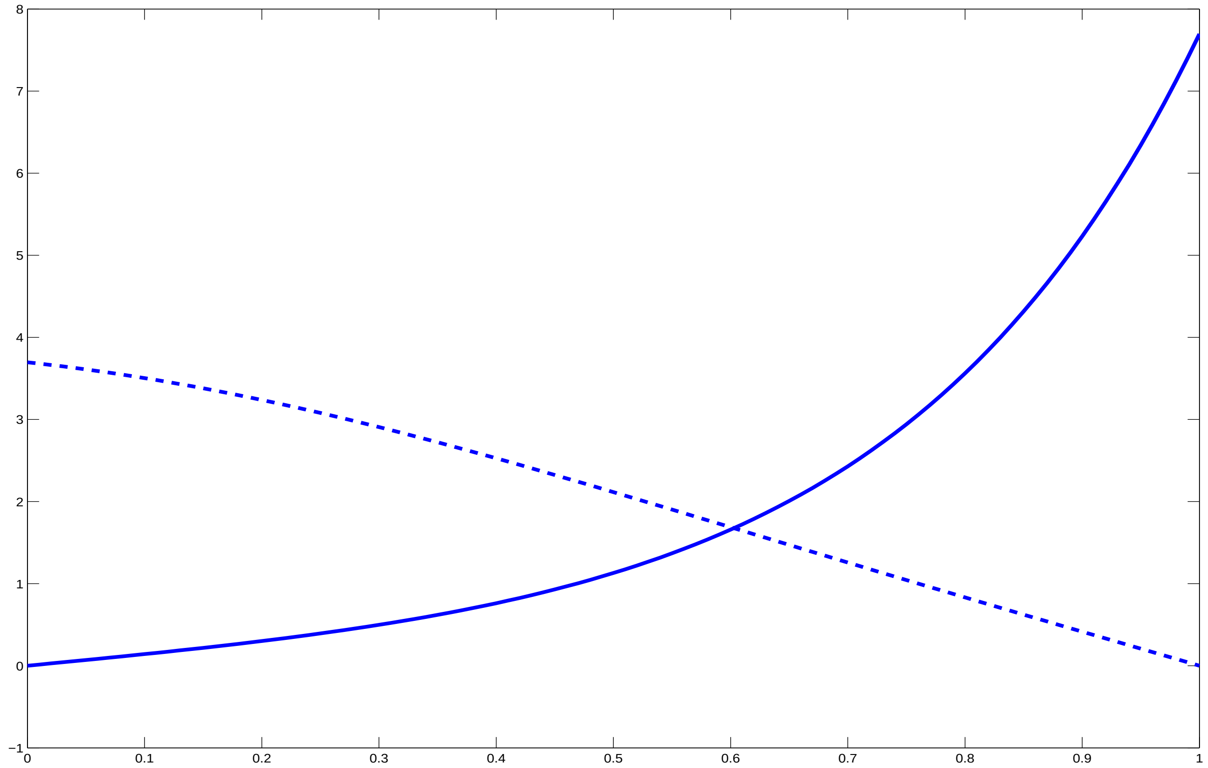


Figure 2: Thresholds determining the cell boundaries (vertical axis) when $\lambda(x) = 2x$

Example: Round Robin Scheduling Policy

- When mobile terminals are distributed uniformly, the optimal solution is given by $[0, 1/2)$ and $[1/2, 1]$,
- Then it is the case of Voronoi cells and the number of mobile terminals connected to each base station is equal and given by $N_1 = N_2 = 1250$.

However when the distribution of mobile terminals is increasingly more concentrated at location 1,

- If $\lambda(x) = 2x$, the optimal solution is given by $[0, q)$ and $[q, 1]$ with $q = 0.6027$
- The quantity of mobile terminals connecting to BS_1 is equal to $N_1 = 908$ and the quantity of mobile terminals connecting to BS_2 is equal to $N_2 = 1592$

Conclusions and Further Work

- We proposed a new approach using optimal transport theory for the mobile association problem for the uplink and downlink case.
- We obtained the optimal mobile association under different policies from the service provider and from the user point of view.