Performance Analysis of Relay Networks with Channel code in Low SNR Regime

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Introduction

➢ We analyze the performance of the three-node relay network with channel coding in the quasi-static block Rayleigh fading plus Gaussian noise channels. Demodulate-and-Forward relaying.

➢ In order to achieve high spectrum efficiency, the relay can either forward the whole or a part of the estimated codeword to the destination namely *partial relaying*.

➢ We compute an upper bound of the Bit Error Rate (BER) of the proposed scheme. From the upper bound, we derive a so-called *instantaneous diversity* order in low and medium SNR region which is essential to practical systems.

➢ The instantaneous diversity depends on both the amount of information forwarded by the relay and the minimum distance of the channel code.

➢ The proposed scheme can achieve full diversity gain in a given SNR region of interest while obtaining 32% spectrum efficiency improvement compared to classical relay network under appropriate conditions.

System model

➢ System model is depicted in Fig.1. All the channels are orthogonal, quasi-static block Rayleigh fading plus Gaussian noise.

➢ Convolutive code and Binary Phase Shift Keying (BPSK) modulation are used.

➢ The relay uses Maximum Likelihood (ML) detector and DMF relaying protocol.

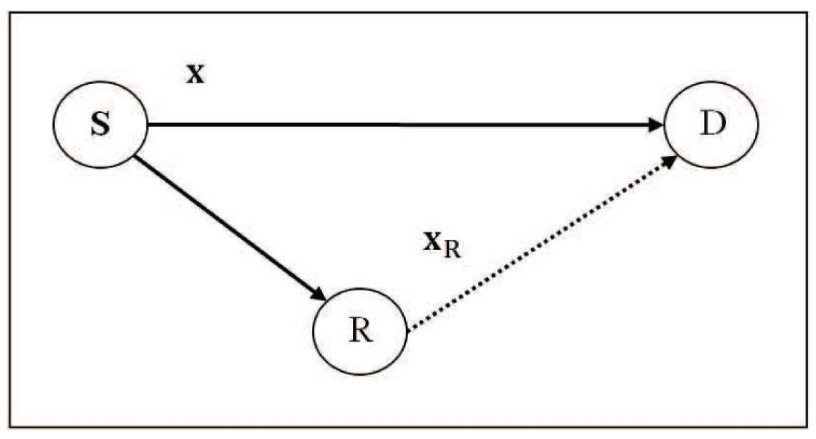

Figure 1: Three-node relay network

➢ At the end of first frame, the relay: i) estimate $M = \delta N$, $0 < \delta \le 1$ data symbols to form an estimated codeword and ii) forwards the estimated codeword to the destination.

➢ The destination applies C-MRC detector to detect coded bits:

$$\mathbb{M}(x_k) = \begin{cases} \left| y_{SD,k} - \sqrt{P_{SD}} h_{SD} x_k \right|^2, & \text{if} \quad k \notin \Theta, \\ \left| y_{SD,k} - \sqrt{P_{SD}} h_{SD} x_k \right|^2 + \lambda \left| y_{RD,k} - \sqrt{P_{RD}} h_{RD} x_k \right|^2, & \text{if} \quad k \in \Theta \end{cases}$$

where Θ denotes the indexes set of relayed symbols, $\lambda = \min(P_{SD} |h_{SD}|^2, P_{RD} |h_{RD}|^2) / P_{RD} |h_{RD}|^2$

➢ The C-MRC detector [1] computes LLR of coded bits and send them to channel decoder (BCJR algorithm) [3].

BER Analysis

➢ The received signal at the destination is output of 2-block channels: the first block consists of *M* symbols that see two channel S-D and R-D, the second block consists of *N-M* symbols that see one channel S-D. The BER of the proposed system is given as follows:

$$BER \le \sum_{d=d_H}^{\infty} w(d) \mathrm{P}_u(d)$$

where d_H is the minimum distance of the channel code, *w(d)* is the distance spectrum of the code and $\mathrm{P}_u(d)$ is unconditioned error probability (UEP) of receiving a codeword with weight *d* when all-zero codeword was transmitted.

➢ Define $\mathbf{W} = (d_1, d_2)$ with $d = d_1 + d_2$ as the weight pattern that d_1 weights locatesin the first block and d_2 weights locate in the second block, we have

$$\mathrm{P}_u = \sum_{\mathbf{W}} \mathrm{P}_u(d \mid \mathbf{W}) \times p(\mathbf{W}), \qquad \text{with} \quad p(\mathbf{W}) = \frac{C_{d_1}^{M} \times C_{d_2}^{N-M}}{C_d^N}$$

➢ The UEP $\mathrm{P}_u(d|\mathbf{W})$ is computed as the integration over fading channels of the conditioned error probability (CEP) $\mathrm{P}_c(d|\mathbf{W})$:

$$\mathrm{P}_u(d \mid \mathbf{W}) = \mathbb{E}_{\gamma_{SD}, \gamma_{SR}, \gamma_{RD}} \left\{ \mathrm{P}_c(d \mid \mathbf{W}) \right\}, \qquad \text{With}$$

$$\mathrm{P}_c(d \mid \mathbf{W}) = (1 - Q(\sqrt{\gamma_{SR}})) \Pr\left\{ \mathbb{M}(\mathbf{x}) - \mathbb{M}(\tilde{\mathbf{x}}) > 0 \mid \mathbf{x}_R \right\} + Q(\sqrt{\gamma_{SR}}) \Pr\left\{ \mathbb{M}(\mathbf{x}) - \mathbb{M}(\tilde{\mathbf{x}}) > 0 \mid \hat{\mathbf{x}}_R \right\}.$$

➢ Taking into account all possible values of pattern $\mathbf{W}$ and following the same integral method as in [2], we obtain:

$$\mathrm{P}_u(d) \le \frac{1}{4d\bar{\gamma}_{SD}} p(D_1) + \left(\frac{\alpha}{d\bar{\gamma}_{SR}\bar{\gamma}_{SD}} + \frac{\beta}{d\bar{\gamma}_{RD}\bar{\gamma}_{SD}} \right) \sum_{\mathbf{W} \ne D_1} \frac{p(\mathbf{W})}{d_1}$$

with $\alpha = 3/16$, $\beta = (45+\mathrm{sqrt}(5))/160$ and $D_1 = (0, d)$.
The probability of the pattern D_1 is given as:

$$p(D_1) \simeq (1-\delta)^d.$$

If *d* is large (strong code) or δ is large, the impact of diversity one factor is small, the proposed system can achieves full diversity order of 2.

Diversity Analysis

▪ Classical definition of diversity order is defined as negative exponent of the average BER in log-log scale in the infinity SNR region.

▪ In this paper, we define diversity order at certain SNR, which is called as *Instantaneous Diversity Order,* as follows:

$$\mathrm{Div}(\gamma) \triangleq -\lim_{\Delta \to 1} \frac{\log[BER(\Delta\gamma)] - \log[BER(\gamma)]}{\log[\Delta\gamma] - \log[\gamma]}$$

▪ The key idea: the instantaneous diversity allows to analyze diversity order at any SNR region.

▪ The instantaneous diversity of partial relaying is given by

$$\mathrm{Div}(\gamma) = 1 + \frac{B}{B + A\gamma},$$

$$\text{where} \quad B = \left(\frac{3}{4g_{RD}} + \frac{45+\sqrt{5}}{40 g_{SR}} \right) \sum_{\mathbf{W} \ne D_1} \frac{p(\mathbf{W})}{d_1},$$

$$A = (1-\delta)^d, \; g_{RD} = \frac{\bar{\gamma}_{RD}}{\bar{\gamma}_{SD}}, \; g_{SR} = \frac{\bar{\gamma}_{SR}}{\bar{\gamma}_{SD}}$$

If the relay forwards the whole estimated codeword (δ = 1, then *A* = 0), the system achieves diversity 2 for all SNR

If the relay keeps silent, the system achieve diversity 1.

Numerical Results

Simulation setups:

▪ BPSK modulation, data message length of 200 bits, the relay locates at the middle of the source and the destination.

▪ Three convolutive codes of rate 1/3 are consivered: CC [3 2 3] with d_H =5, CC [5 7 5] with d_H = 7 and CC [123 135 157] with d_H = 15. The first five weights *d* are used to compute the bound.

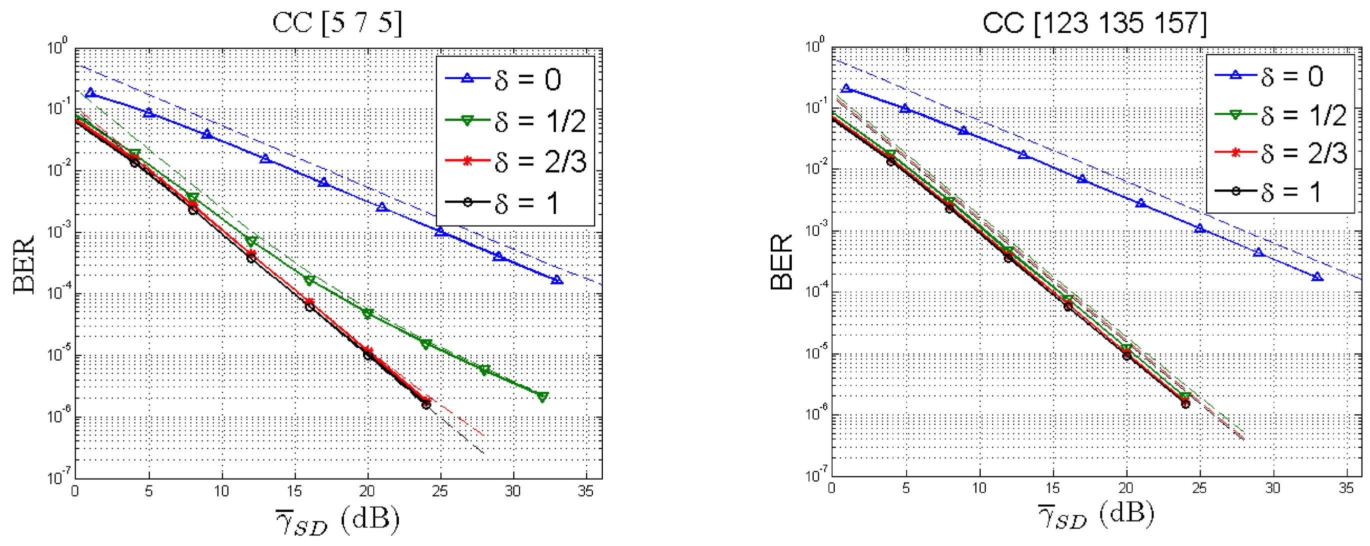

Figure 2: Effect of δ on performance of partial relaying for different channel codes. Solid marked curves: simulation results, dotted curves: the upper bound.

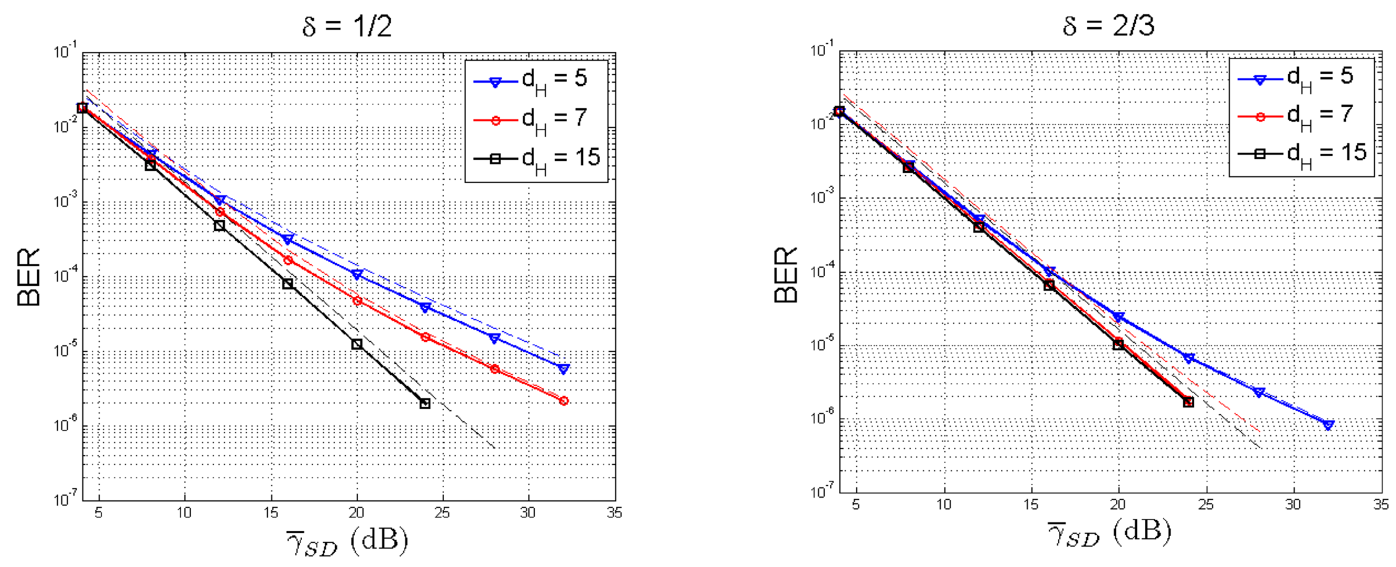

Figure 3: Effect of minimum distance d_H on performance of the proposed network for different δ. Solid marked curves: simulation results, dotted curves: the upper bound.

Performance:

▪ The upper bound is quite tight compared with the simulations, providing a theoretical tool to evaluate the system diversity order.

▪ The proposed system achieves full diversity in operating SNR (BER = 10^{-6}) while save 32% spectrum efficiency (δ = ½) with the strong code [123 135 157], and save 20% spectrum efficiency (δ = 2/3) with the medium code [5 7 5].

▪ Potential to apply in multiple-source networks where on relay can aid more sources thanks to partial relaying.

References

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