Achievability of Efficient Satisfaction Equilibria in Self-Configuring Networks [2]

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Game in Satisfaction Form [3]

A game in satisfaction form is fully described by the following triplet

$$\hat{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right). \quad (1)$$

- $\mathcal{K}$ represents the set of players,
- $\mathcal{A}_k$ is the strategy set of player $k \in \mathcal{K}$,
- the correspondence f_k determines the set of actions of player k that allows its satisfaction given the actions played by all the other players.

Efficient Satisfaction Equilibrium

An action profile $\mathbf{a}^*$ is an ESE for the game $\hat{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right)$, with cost functions $\{c_k\}_{k\in\mathcal{K}}$, if

$$\forall k \in \mathcal{K}, \quad a_k^* \in f_k\left(\mathbf{a}_{-k}^*\right), \quad (2)$$

and

$$\forall k \in \mathcal{K},\ \forall a_k \in f_k(\mathbf{a}_{-k}^*),\ c_k(a_k) \geq c_k(a_k^*). \quad (3)$$

Best response dynamics

$$BR_k(\mathbf{a}_{-k}) = \arg\min_{a_k \in f_k(\mathbf{a}_{-k})} c_k(a_k). \quad (4)$$

At step $n+1$ of the algorithm, only one player updates its action:

$$a_k^{(n+1)} = BR_k(a_1^{(n+1)}, \dots, a_{k-1}^{(n+1)}, a_{k+1}^{(n)}, \dots, a_K^{(n)}). \quad (5)$$

Theorem (Convergence of the BRD [1, 5])

Assume that for all $k \in \mathcal{K}$, $f_k(\cdot)$ is nonempty and compact for all the values of their arguments, $f_k(\cdot)$ has either the ascending or the descending property and $f_k(\cdot)$ is continuous. Then the following holds:

- *(i) An ESE exists.*
- *(ii) If the dynamics start with the action profile associated with the highest or lowest effort in $c_k(\cdot)$, for all $k \in \mathcal{K}$, the BRD converge monotonically to an ESE.*
- *(iii) If the dynamics start from an SE, the trajectory of the best response converges to an ESE. It monotonically evolves in all components.*
- *(iv) In a two-player game, the BRD converge to an ESE from any starting point.*

Power control game [4, 6]

- K transmitter/receiver pairs denoted by index $k \in \mathcal{K}$.
- Transmitter k uses power level $p_k \in \mathcal{A}_k$, with $\mathcal{A}_k$ defined as a compact sublattice.
- For every couple $(i,j) \in \mathcal{K}^2$, we denote by g_{ij} the channel gain coefficient between transmitter i and receiver j.
- The considered metric for each pair k is the Shannon rate given by

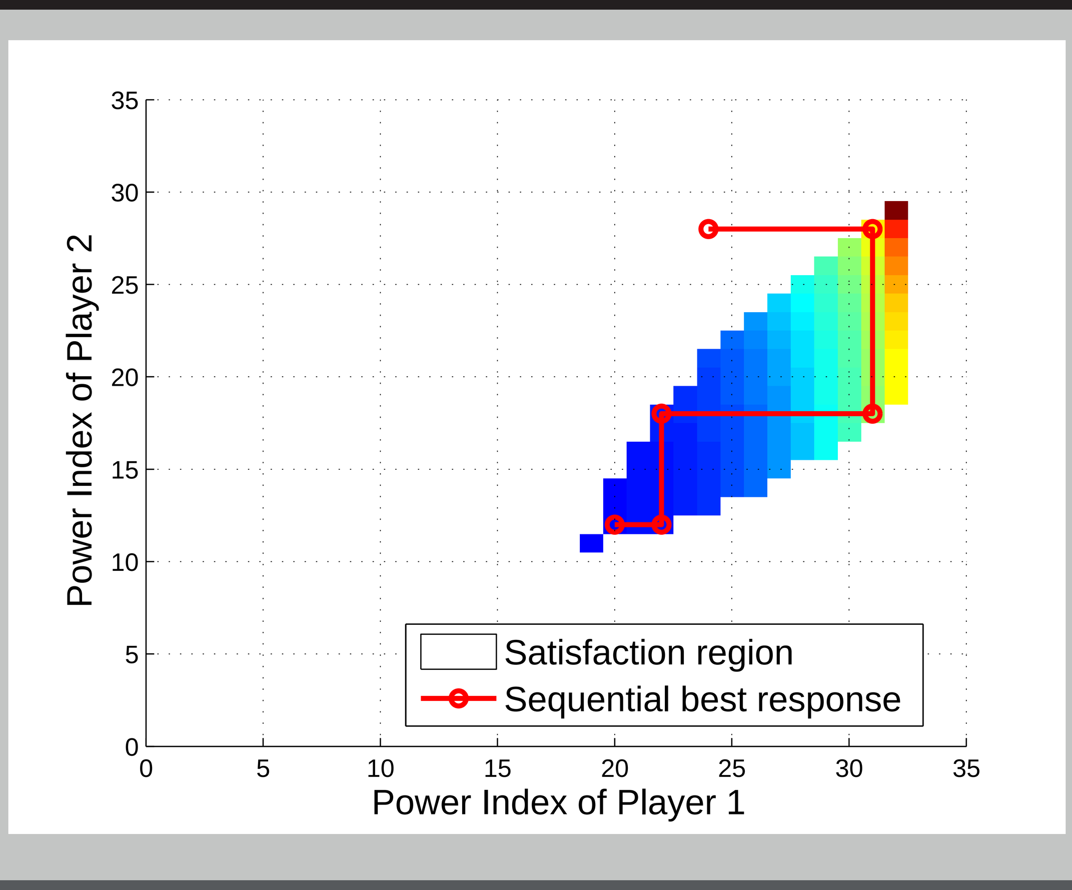
$$u_k(p_k, \mathbf{p}_{-k}) = \log_2\left(1 + \frac{p_k g_{kk}}{\sigma_k^2 + \Sigma_{j\neq k} p_j g_{jk}}\right) \quad \text{[bps/Hz]}, \quad (6)$$

where σ_k^2 is the noise variance at receiver k.

The QoS requirement for each pair k is to have a channel capacity $u_k(p_k, \mathbf{p}_{-k})$ higher than a given threshold Γ_k bps/Hz. The satisfaction correspondence of link k is then

$$f_k(\mathbf{p}_{-k}) = \{p_k \in \mathcal{A}_k \mid u_k(p_k, \mathbf{p}_{-k}) \geq \Gamma_k\}$$
$$= \left\{p_k \in \mathcal{A}_k \mid p_k \geq (2^{\Gamma_k} - 1)\frac{\sigma_k^2 + \Sigma_{j\neq k} p_j g_{jk}}{g_{kk}}\right\}. \quad (7)$$

BRD in the 2-player uplink power control game



BRD in the 3-player uplink power control game

Let us consider $K = 3$ pairs of transmitters/receivers. For all $k \in \mathcal{K}$, transmitter k uses power level $a_k \in \{p^{\min}, p^{\max}\}$. Let consider channel gains such that

$$\begin{aligned} f_1(p^{\min}, p^{\min}) &= f_3(p^{\min}, p^{\min}) = \{p^{\min}, p^{\max}\}, \\ f_1(p^{\min}, p^{\max}) &= f_3(p^{\min}, p^{\max}) = \{p^{\min}, p^{\max}\}, \\ f_1(p^{\max}, p^{\min}) &= f_3(p^{\max}, p^{\min}) = \{p^{\max}\}, \\ f_1(p^{\max}, p^{\max}) &= f_3(p^{\max}, p^{\max}) = \{p^{\max}\}, \end{aligned} \quad (8)$$

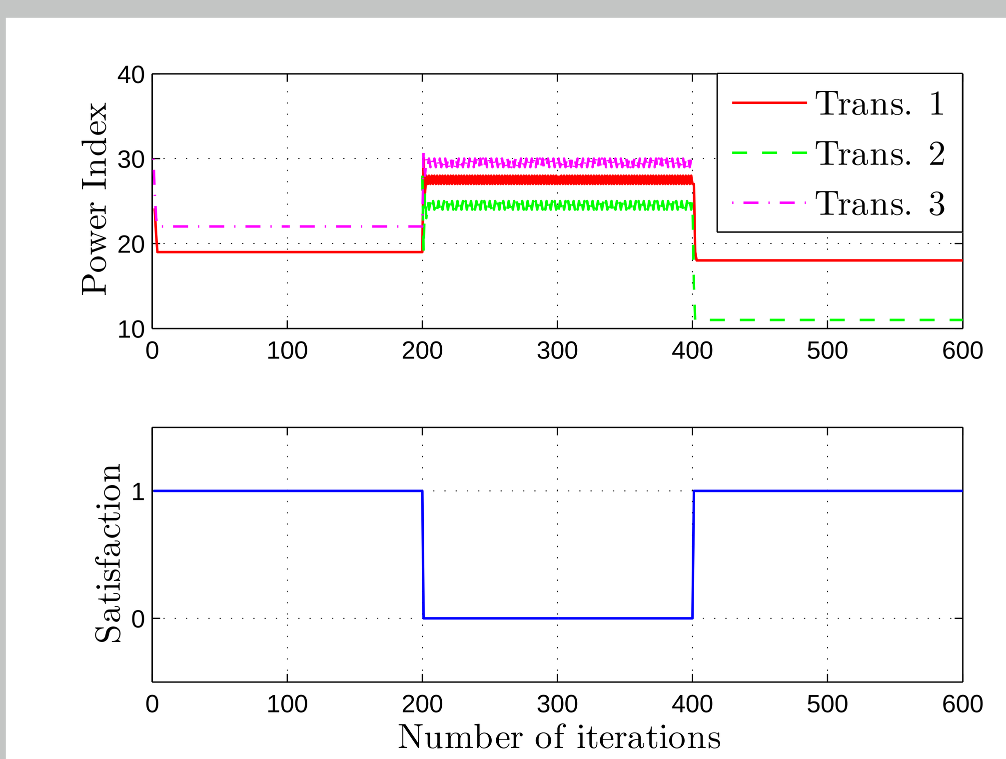
and

$$\begin{aligned} f_2(p^{\min}, p^{\min}) &= \{p^{\min}, p^{\max}\}, \\ f_2(p^{\min}, p^{\max}) &= \{p^{\max}\}, \\ f_2(p^{\max}, p^{\min}) &= \{p^{\min}, p^{\max}\}, \\ f_2(p^{\max}, p^{\max}) &= \{p^{\max}\}. \end{aligned} \quad (9)$$

For each pair k, the cost of the power level is defined by the identity cost function $c_k(a_k) = a_k$. This game has two ESEs:

- $(p^{\min}, p^{\min}, p^{\min})$ where all the players transmit at their lowest power level. No player has interest in deviating from its action since any other action has a higher cost (even though the player would remain satisfied).
- $(p^{\max}, p^{\max}, p^{\max})$ where all the players have to transmit at maximum power to be satisfied. If one deviates from its action, it will not be satisfied anymore.
- Depending on the initial action profile of the BRD, the BRD may not converge to an ESE. It is the case if the BRD starts at $\mathbf{p}^{(0)} = (p^{\max}, p^{\min}, p^{\max})$.

BRD in the 3-player uplink power control game



Robust blind response dynamics

Define the robust blind response (RBR) by $RBR_k: \mathcal{A} \to \mathcal{A}_k$, such that :

$$(a_k, \mathbf{a}_{-k}) \to \begin{cases} a'_k, & \text{if } a'_k \in f_k(\mathbf{a}_{-k}) \text{ and } c_k(a'_k) \leq c_k(a_k), \\ a'_k, & \text{if } a'_k \in f_k(\mathbf{a}_{-k}) \text{ and } a_k \notin f_k(\mathbf{a}_{-k}), \\ a_k, & \text{otherwise}, \end{cases} \quad (10)$$

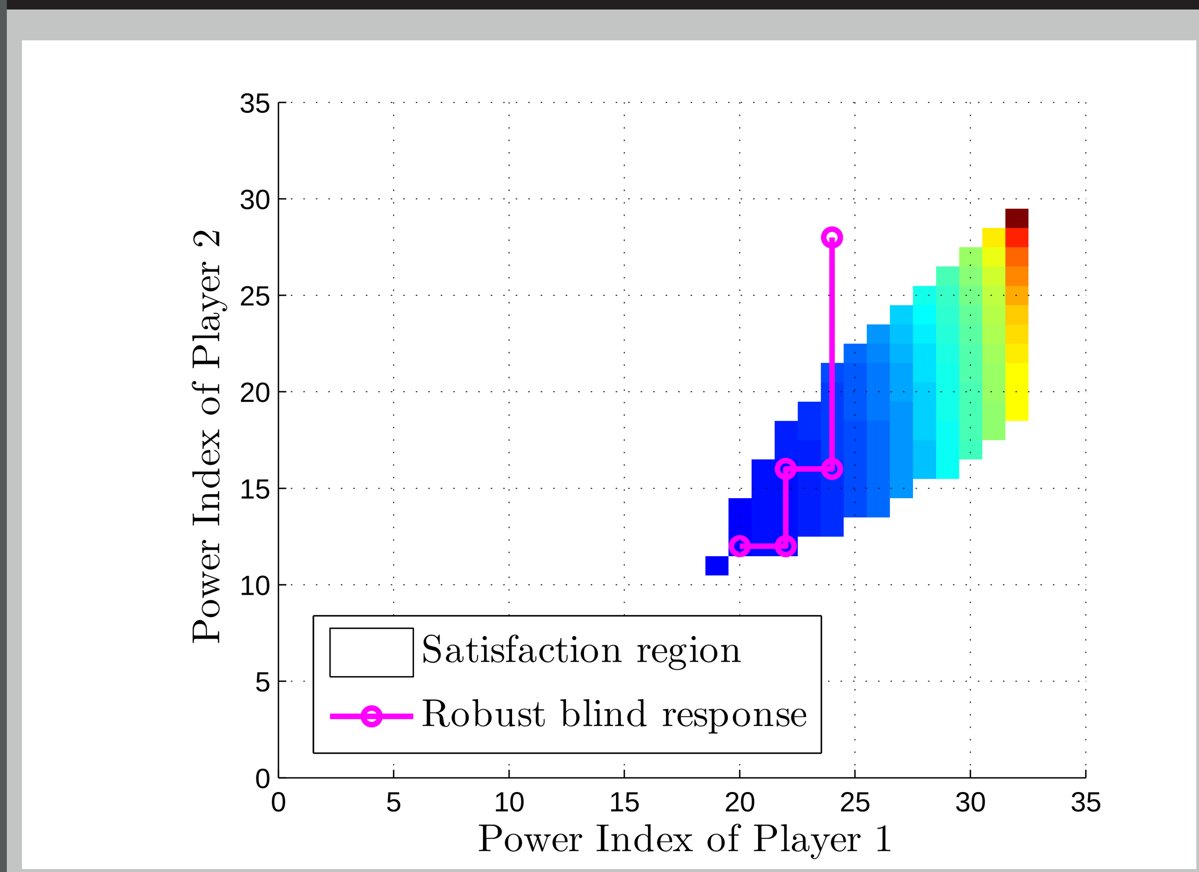
with action a'_k being randomly chosen in $\mathcal{A}_k$, such that $\forall a_k \in \mathcal{A}_k$, $\Pr(a'_k = a_k) > 0$.

Theorem (Convergence of the RBRD)

Assume that for all $k \in \mathcal{K}$, $f_k(\cdot)$ is nonempty and compact for all the values of their arguments, $f_k(\cdot)$ has the ascending property and it is continuous, and $c_k(\cdot)$ is strictly increasing. Then, the following holds:

- *(i) If the dynamics start from an SE, the sequence of RBRs converges to an ESE. It monotonically decreases in all components.*
- *(ii) If the dynamics start with the actions associated with the highest effort in $c_k(\cdot)$, $\forall k \in \mathcal{K}$, the sequence of RBRs converges monotonically to an ESE.*
- *(iii) In a two-player game, the sequence of RBRs converges to an ESE from any starting point.*

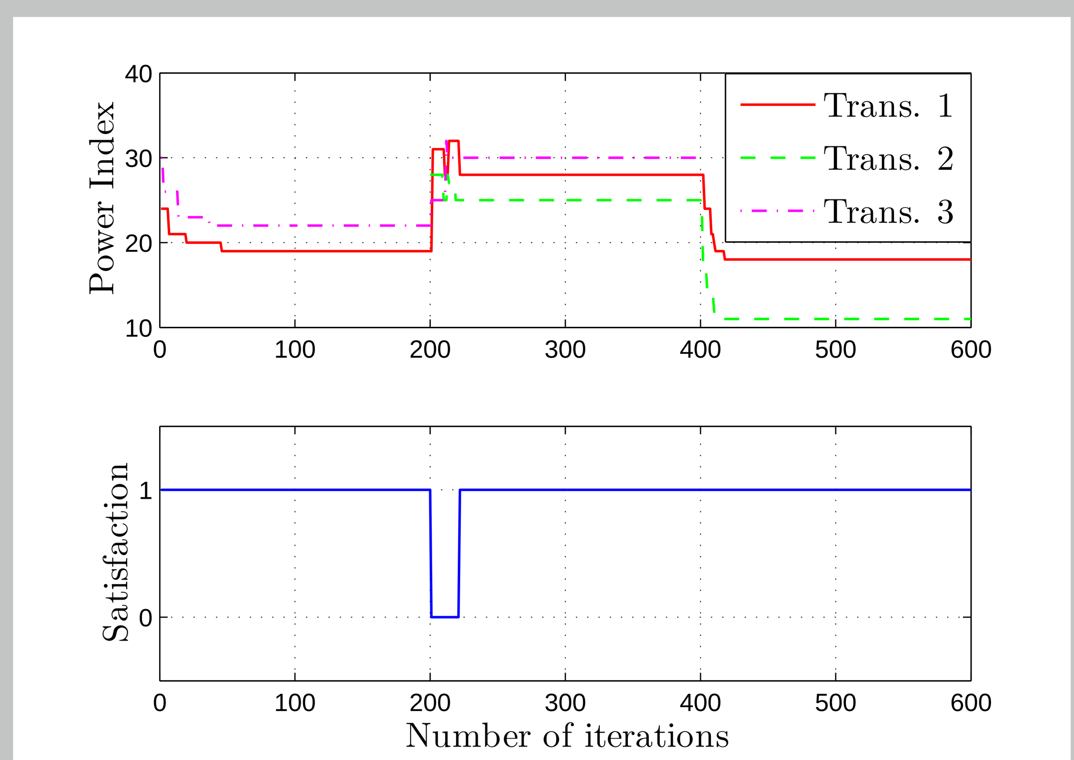
RBRD in the 2-player uplink power control game



Theorem (Discrete action sets)

In the power allocation game defined above, with discrete action sets, i.e., $\forall k \in \mathcal{K}$, $\mathcal{A}_k = \{p_k^{(1)}, \dots, p_k^{(N_k)}\}$ with N_k the number of power levels in action set $\mathcal{A}_k$, the RBRD converge to an ESE from any starting point.

RBRD in the 3-player uplink power control game



Conclusion

- RBRD is a behavior rule that converges to an ESE in the general framework of compact sublattices as actions sets.
- Compared to BRD, **less information required** but **longer convergence time**.
- **Main advantage** : In the uplink power control game with discrete actions sets, convergence from any starting action profile!

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