Effect of Saliency on a Position Observer for Permanent Magnet Synchronous Motors

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Main Contribution Determination of operating regimes for "good behavior" of a position observer for permanent magnet synchronous motors (PMSM) in spite of saliency.

Background

- A globally convergent position observer for non–salient PMSM
  - R. Ortega, L. Praly, A. Astolfi, J. Lee et K. Nam: Estimation of rotor position and speed of permanent magnet synchronous motors with guaranteed stability, *IEEE Trans. Control Syst. Technology*, Vol. 19, No. 3, pp. 601–614, May 2011.
  - J. Lee, J. Hong, K. Nam, R. Ortega, A. Astolfi et L. Praly: Sensorless control of surface-mount permanent magnet synchronous motors based on a nonlinear observer, *IEEE Transactions on Power Electronics*, Vol. 25, No. 2, pp. 290–297, 2010.
  - Korean Patent 2010: *Sensorless Control Method and System for SPMSM Using Nonlinear Observer*, (avec Kwang Hee Nam, Laurent Praly, Alessandro Astolfi, Jin Suk Hong et Jung gi Lee), Patent No. 10-1091970.
- Contract European Embedded Control Institute–Schneider Electric
  - Sabbatical year of Harish Pillai (2012)
  - Post–doc Jose Romero (2013–2014)
- Main question: Effect of saliency

Modeling of a salient PMSM

- (In $\alpha\beta$ frame) Faraday's and Ohm's law

$$\dot{\lambda} + Ri = v$$

where $\lambda, i, v \in \mathbb{R}^2$ are flux, current and voltage and $R > 0$ is stator resistance.

- Flux linkages

$$\lambda = [L_s I_2 + L_g D(\theta)]i + \lambda_m c(\theta)$$

where

$$D(\theta) := \begin{bmatrix} \cos 2n\theta & \sin 2n\theta \\ \sin 2n\theta & -\cos 2n\theta \end{bmatrix}, \; c(\theta) := \begin{bmatrix} \cos n\theta \\ \sin n\theta \end{bmatrix},$$

$\theta \in \mathbb{S}$ is rotor angle, $n > 0$ the number of pole pairs, $L_s > 0, L_g$ are the stator and "saliency" inductances and $\lambda_m > 0$ the permanent magnet flux.

cont'd

- Mechanical coordinates, Newton's law

$$J\dot{\omega} = \tau_e - \tau_L, \quad \dot{\theta} = \omega,$$

where J rotor inertia, $\tau_L \in \mathbb{R}$ is load torque and the electrical torque is

$$\tau_e = \frac{\partial}{\partial \theta}\left[\frac{1}{2} i^\top D(\theta) i + \lambda_m i^\top c(\theta)\right].$$

- $\alpha\beta$ to dq–coordinates: $\begin{pmatrix} f_d \\ f_q \end{pmatrix} = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix} \begin{pmatrix} f_\alpha \\ f_\beta \end{pmatrix}$
- dq model with $L_s = \frac{1}{2}(L_d + L_q),\ L_g = \frac{1}{2}(L_d - L_q)$

$$\begin{aligned} L_d \frac{di_d}{dt} &= -Ri_d + \omega L_q i_q + v_d \\ L_q \frac{di_q}{dt} &= -Ri_q - \omega L_d i_d - \omega \lambda_m + v_q \\ J\frac{d\omega}{dt} &= n\lambda_m i_q + n(L_d - L_q) i_d i_q - \tau_L. \end{aligned}$$

Position observer non–salient machines (Ortega, et al.,'11)

- If $L_g = 0$, $\lambda = L_s i + \lambda_m c(\theta)$. Hence, $\theta = \tan^{-1}\left(\frac{\lambda_\beta - L_s i_\beta}{\lambda_\alpha - L_s i_\alpha}\right)$.
- Now, from $\dot{\lambda} = -Ri + v$ it is clear that $\dot{\hat{\lambda}} = -Ri + v$ estimates λ up to a constant due to the unknown initial conditions.
- Idea: Add to $\dot{\hat{\lambda}}$ a correction term in the direction of the negative of the gradient of

$$J(\hat{\lambda}) := (\lambda_m^2 - |\hat{\lambda} - L_s i|^2)^2,$$

which is computable, because

$$\frac{\partial}{\partial \hat{\lambda}} J(\hat{\lambda}) = -4(\hat{\lambda} - L_s i)(\lambda_m^2 - |\hat{\lambda} - L_s i|^2).$$

- This leads to

$$\dot{\hat{\lambda}} = v - Ri + \gamma(\hat{\lambda} - L_s i)(\lambda_m^2 - |\hat{\lambda} - L_s i|^2),$$

where $\gamma > 0$ is a scaling factor.

Stability properties

P1 (Global stability) For arbitrary speeds, the disk

$$\{\tilde{\lambda} \in \mathbb{R}^2 \,|\, |\tilde{\lambda}| \leq 2\lambda_m\},$$

is globally attractive, where $\tilde{\lambda} := \hat{\lambda} - \lambda$.

P2 (Exponential stability under persistent excitation) The zero equilibrium is exponentially stable if there exists constants $T, \Delta > 0$ such that

$$\frac{1}{T}\int_t^{t+T} \omega^2(s)ds \geq \Delta,$$

for all $t \geq 0$.

P3 (Constant non–zero speed) If the speed is constant and satisfies

$$|\omega| > \frac{1}{4}\gamma\lambda_m^2,$$

then the origin is the unique equilibrium and it is globally asymptotically stable.

Ad–hoc inclusion of saliency in the observer design

- When $L_g \neq 0$

$$\lambda = [L_s I_2 + L_g D(\theta)]i + \lambda_m c(\theta)$$

- Defining the estimate of $c(\theta) = \text{col}(\cos(n\theta), \sin(n\theta))$ as

$$\hat{c} := \frac{1}{\lambda_m}(\hat{\lambda} - L_s i) \qquad (C)$$

the observer may be written as

$$\dot{\hat{\lambda}} = v - Ri + \gamma(\hat{\lambda} - L_s i)\lambda_m^2(1 - |\hat{c}|^2).$$

- Idea: Incorporate additional terms in the observer.
  - First option

  $$\dot{\hat{\lambda}} = v - Ri + \gamma(\hat{\lambda} - L_s i - L_g D(\hat{\theta}))\lambda_m^2(1 - |\hat{c}|^2).$$

  - Second option, replace (C) by

  $$\hat{c} := \frac{1}{\lambda_m}(\hat{\lambda} - L_s i - L_g D(\hat{\theta})).$$

  - Combinations of these two.

Certainty–equivalent velocity and load torque observer

- Assuming position known we can compute τ_e and design an observer for ω and τ_L for

$$
\begin{aligned}
\dot{\theta} &= \omega \\
J\dot{\omega} &= \tau_e - f\omega - \tau_L,
\end{aligned}
$$

where $f \geq 0$ is a friction coefficient, with measurable outputs $y = \text{col}(\sin\theta, \cos\theta)$.

- Alternative model of the mechanical equations. Define $\eta = \text{col}(\omega, \frac{\tau_L}{J})$. The system dynamics is

$$
\dot{\eta} = A\eta + \begin{bmatrix} \frac{1}{J}\tau \\ 0 \end{bmatrix}, \quad \dot{y} = \Phi(y)\eta,
$$

where

$$
A := \begin{bmatrix} -\frac{f}{J} & -1 \\ 0 & 0 \end{bmatrix}, \quad \Phi(y) := \begin{bmatrix} y_2 & 0 \\ -y_1 & 0 \end{bmatrix}.
$$

- Observation problem non–trivial because y is a nonlinear function of $\theta \in \mathbb{S}$.

A globally exponentially stable observer

The fifth–dimensional system

$$
\begin{aligned}
\dot{\hat{y}} &= \begin{bmatrix} y_2 \\ -y_1 \end{bmatrix} \hat{\eta}_1 - k_3 r^2 (\hat{y} - y) \\
\dot{\xi} &= \begin{bmatrix} \frac{1}{J}\tau_e - \frac{f}{J}\hat{\eta}_1 - \hat{\eta}_2 \\ 0 \end{bmatrix} + k_4[\hat{\eta}_1(1 - y^\top \hat{y}) + k_3 r^2 (y_1 \hat{y}_2 - \hat{y}_1 y_2)] \\
\dot{r} &= -\frac{k_1}{4}(r-1) + \frac{k_2}{2k_1} r (1 - y^\top \hat{y})^2, \quad r(0) \geq 1 \\
\hat{\eta} &= \xi + (y_1 \hat{y}_2 - \hat{y}_1 y_2) k_4,
\end{aligned}
$$

with $k_i > 0$ suitable tuning gains, ensures that $\hat{y}, \xi, r$ are bounded and

$$
\lim_{t \to \infty} |\hat{\eta}(t) - \begin{bmatrix} \omega(t) \\ \frac{\tau_L}{J} \end{bmatrix} | = 0 \quad (\text{exp}).
$$

Simulation: Sensitivity to saliency and parameter uncertainty

- Open loop operation, sinusoidal voltages, q weighting factor on λ_m, $\tau_L = 4.67$

q	load torque	$(\hat{\omega} - \omega)$	r value after 8 sec
1.04	4.08	- 3.5	5
1.03	4.15	- 3.3	2.3
1.02	4.20	- 2.5	1.5
1.01	4.26	- 2.0	1.37
1.00	4.32	- 1.2	1.28
0.99	4.38	- 0.5	1.25
0.98	4.44	+ 0.3	1.24
0.97	4.50	+ 1.0	1.25
0.96	4.57	+ 1.7	1.3
0.95	4.63	+ 2.5	1.37
0.94	4.69	+ 3.5	1.6
0.93	4.75	+ 4.5	2.5
0.92	4.82	+ 5.8	5.6

Variation in resistance R

- Open loop operation, sinusoidal voltages, q weighting factor on R, $\tau_L = 4.67$

q	load torque	($\hat{\omega} - \omega$)	r value after 8 sec
1.15	5.00	- 3.2	3.0
1.10	4.79	- 2.5	1.68
1.09	4.75	- 2.5	1.60
1.05	4.56	- 1.8	1.40
1.00	4.32	- 1.2	1.28
0.95	4.06	- 0.6	1.24
0.90	3.77	0.0	1.22
0.85	3.5	+ 0.6	1.22
0.80	3.15	+ 1.2	1.24
0.70	2.45	+ 2.0	1.4
0.60	1.66	+ 2.7	2.0

Simulation comparison of modified position observer

- Open loop operation, sinusoidal voltages, $\tau_L = 4.67$.
- Steady–state comparison.
- Significant improvement of model 2, marginal for others.
- Sustained (small) periodic oscillation in $\hat{\theta}$.
- Large impact on $\hat{\tau}_L$ and $\hat{\omega}$.

model	minimum error of $n\theta$ (rad)	maximum error in θ (rad)	% error in torque
1	0.015	0.08	13.63
2	0.01	0.075	13.0
3	0.01	0.09	7.34
4	0.00	0.08	5.66
5	0.01	0.09	7.34
6	0.00	0.08	5.66

Effect of saliency on position observer

Proposition Define the (scaled and rotated) error signals

$$\xi = -\frac{1}{\lambda_m} \exp(-n\theta J)\tilde{\lambda}.$$

Introduce the time–scale change $\frac{dt}{d\tau} = \frac{1}{\gamma\lambda_m^2}$. Then,

$$\begin{aligned} \frac{d\xi_1}{d\tau} &= \Omega\xi_2 - \sigma(\xi)(\xi_1 - i_d^0 - 1) \\ \frac{d\xi_2}{d\tau} &= -\Omega\xi_1 - \sigma(\xi)(\xi_2 + i_q^0) \end{aligned}$$

where

$$\sigma(\xi) := (\xi_1 - i_d^0 - 1)^2 + (\xi_2 + i_q^0)^2 - 1,$$

with the scaled currents $i_{dq}^0 := \frac{L_g}{\lambda_m} i_{dq}$ and the scaled speed $\Omega = \frac{n}{\gamma\lambda_m^2}\omega$.

Key Remark The performance degradation is determined by L_g and i_{dq}—in steady–state an equilibrium shift.

Stability properties

C1 (Global stability) For arbitrary speeds, the disk

$$\{\tilde{\lambda} \in \mathbb{R}^2 | \, |\tilde{\lambda}| \leq 1 + \sqrt{(i_d^0 + 1)^2 + (i_q^0)^2}\},$$

is globally attractive.

C2 (Equilibrium at zero) $(0, 0)$ is an equilibrium if and only if

$$(i_d^0 + 1)^2 + (i_q^0)^2 = 1.$$

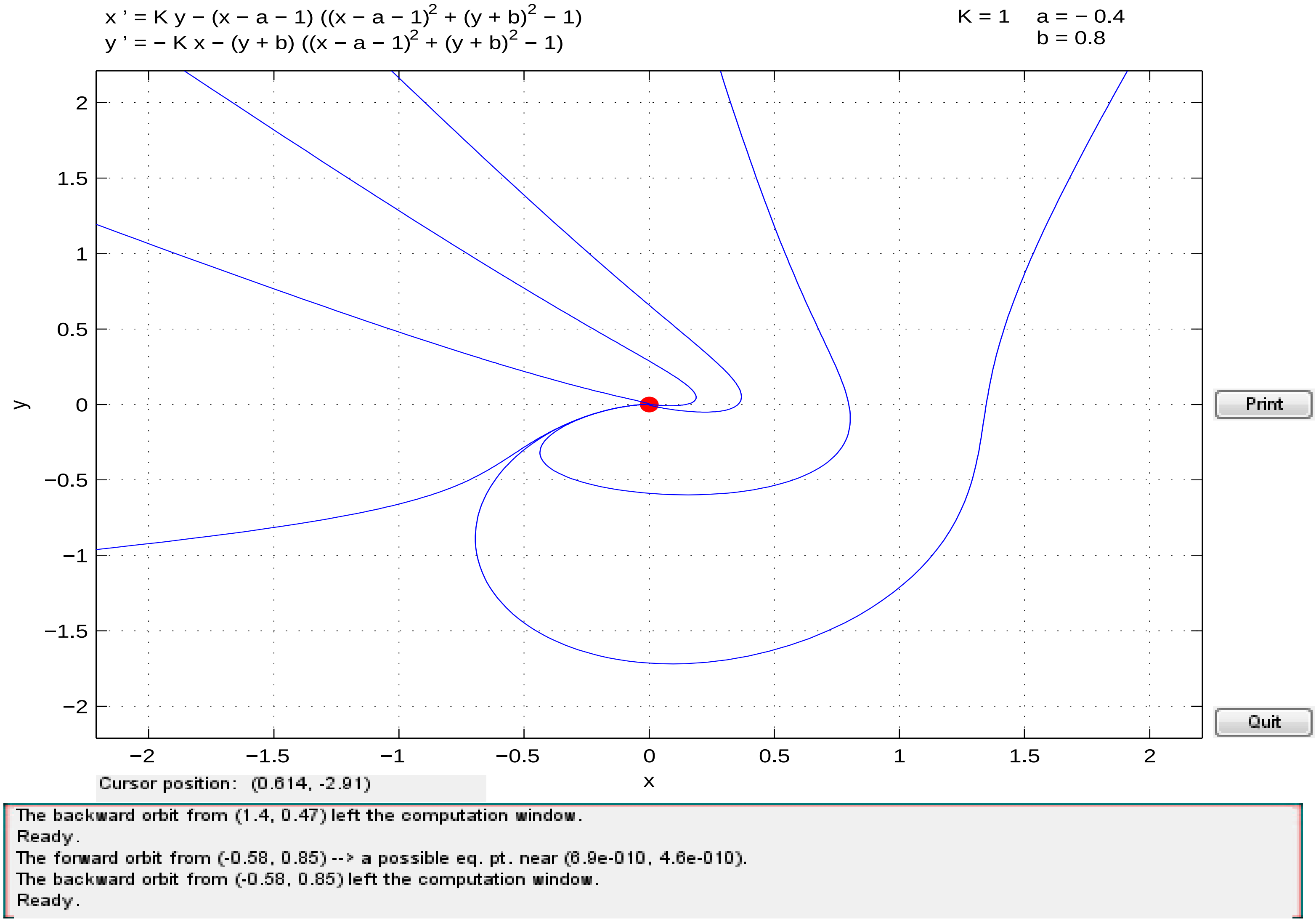
Moreover,

- If $0 < \Omega \leq \frac{1}{2}$, then there are three equilibria and the origin is a stable node.
- If $\Omega > \frac{1}{2}$, then the origin is the only equilibrium. It is a stable node for $\Omega \leq 1$ and a stable focus for $\Omega > 1$.

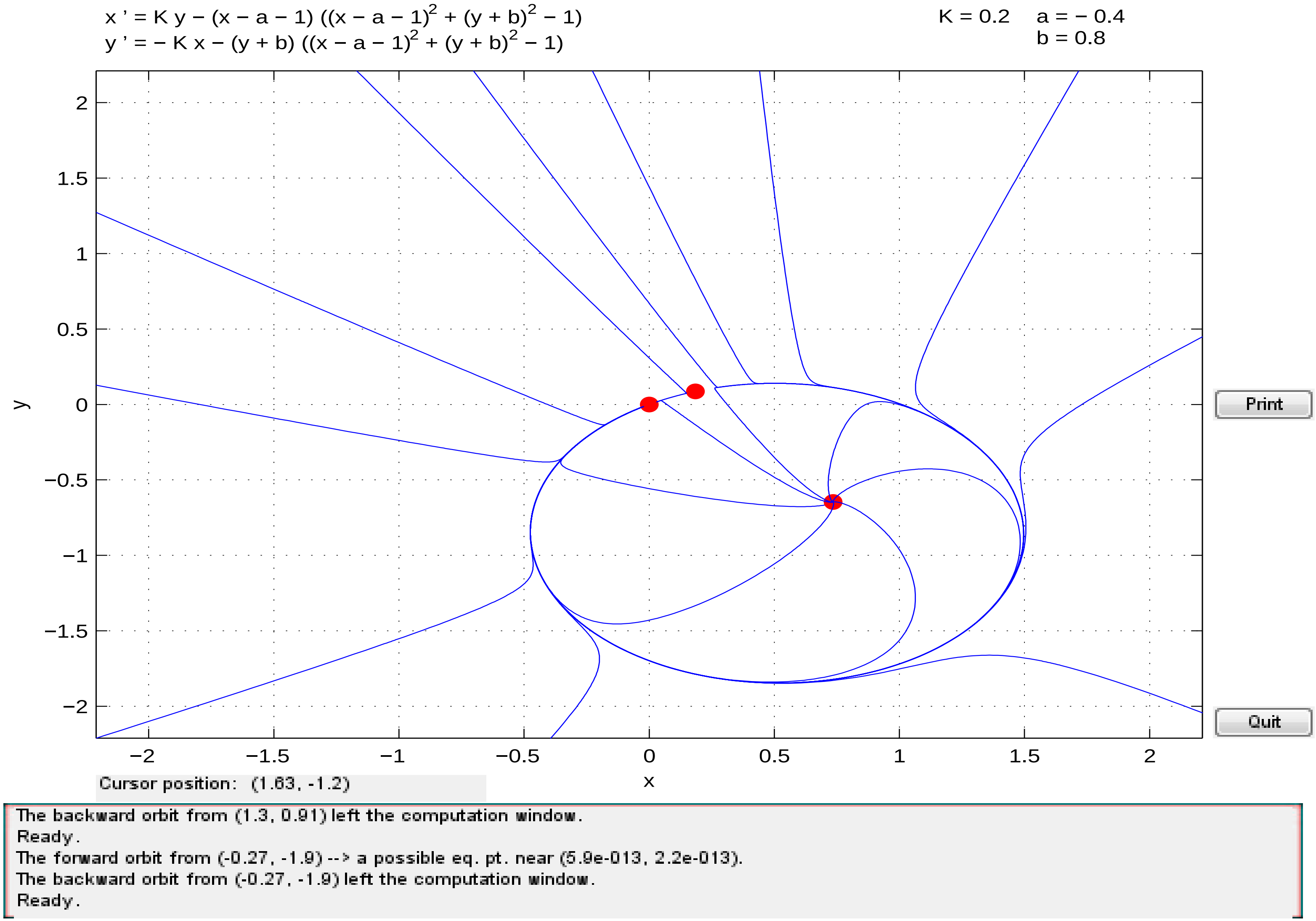
C3 (Limit cycles) If $(i_d^0 + 1)^2 + (i_q^0)^2 \leq 1/2$, there is an (almost globally) stable limit cycle.

C4 (Equilibria away from zero) If $(i_d^0 + 1)^2 + (i_q^0)^2 > 1$, there might be one or three equilibrium points.

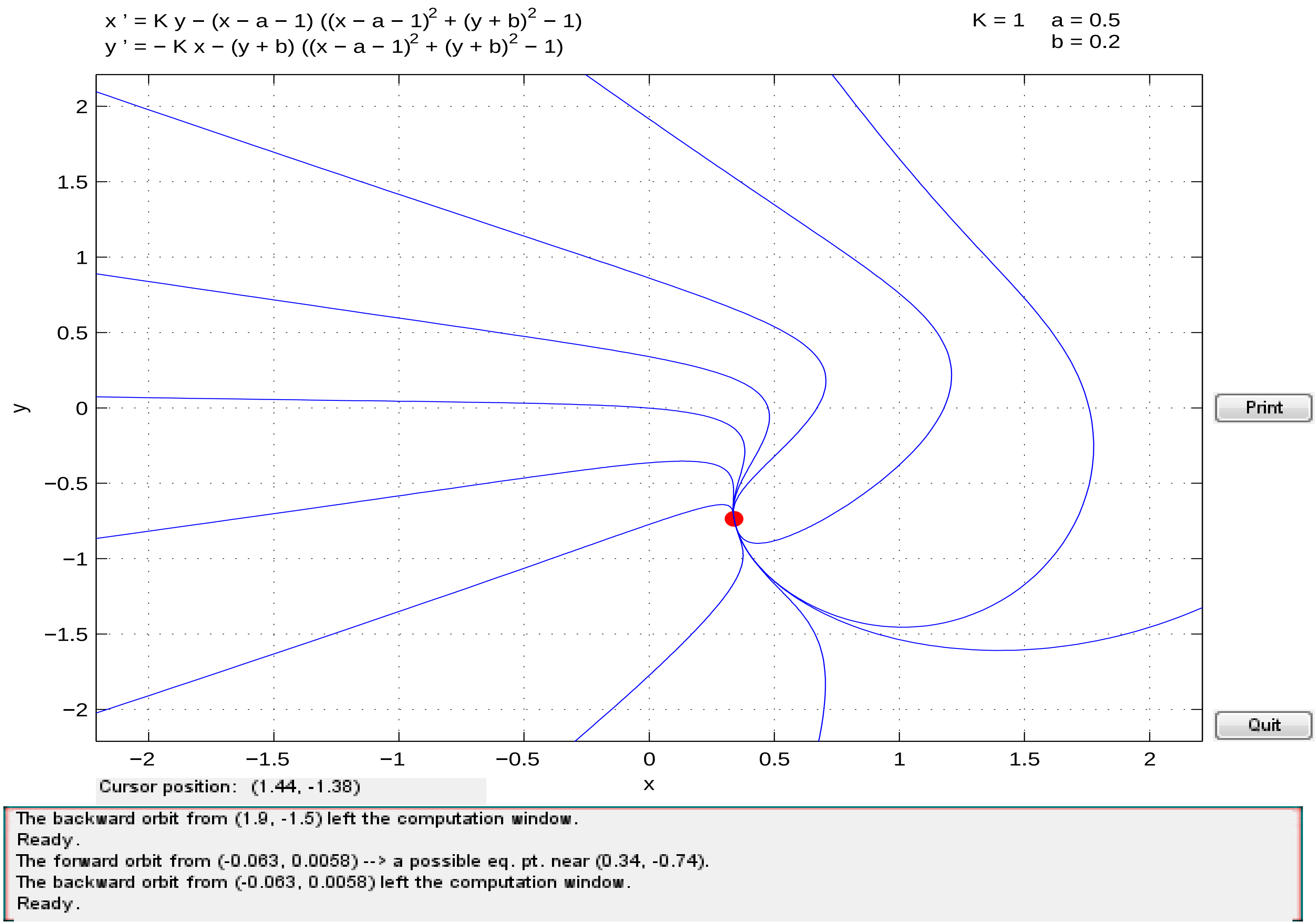
Case $(i_d^0 + 1)^2 + (i_q^0)^2 = 1$ high speed



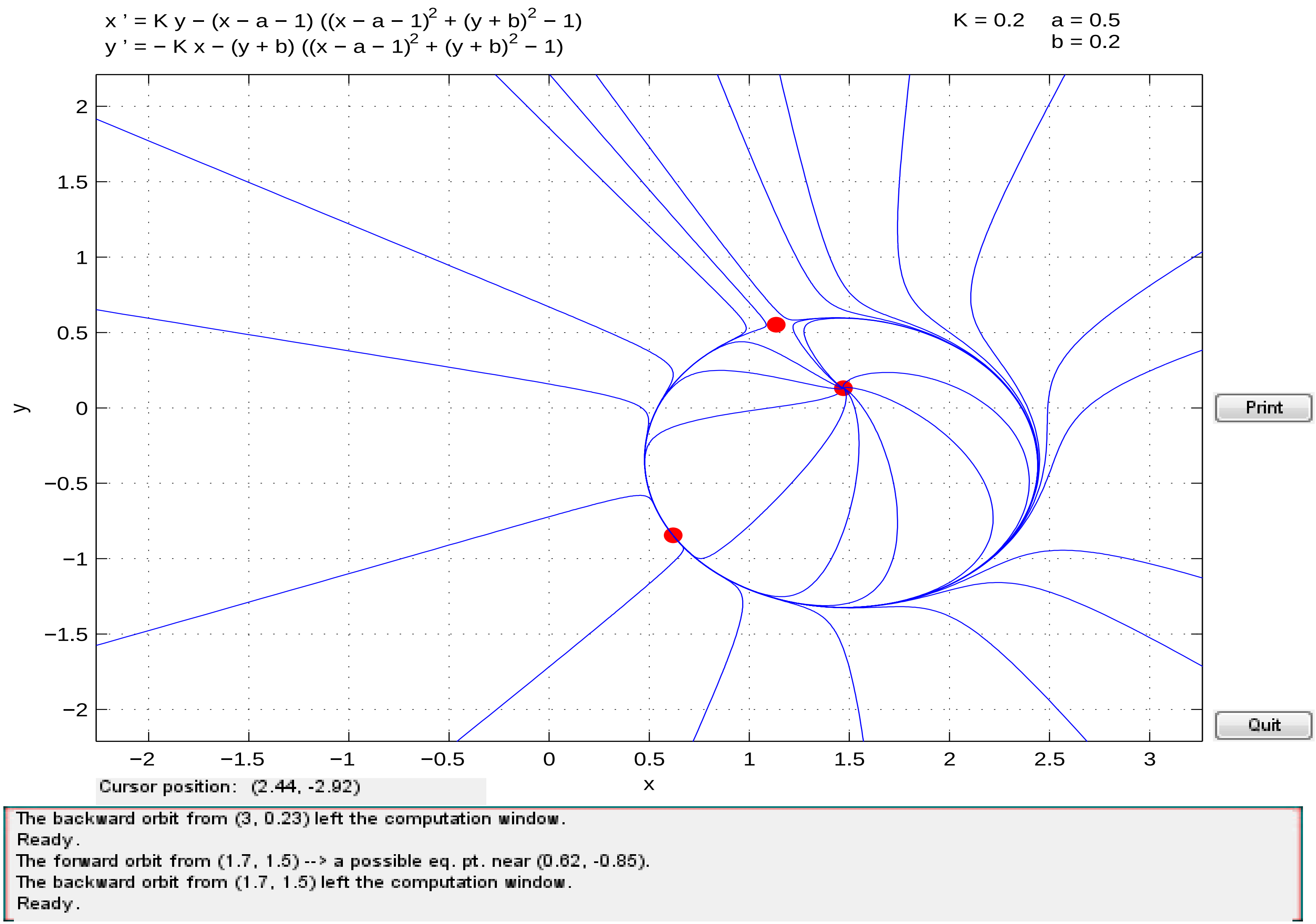
Case $(i_d^0 + 1)^2 + (i_q^0)^2 = 1$ low speed



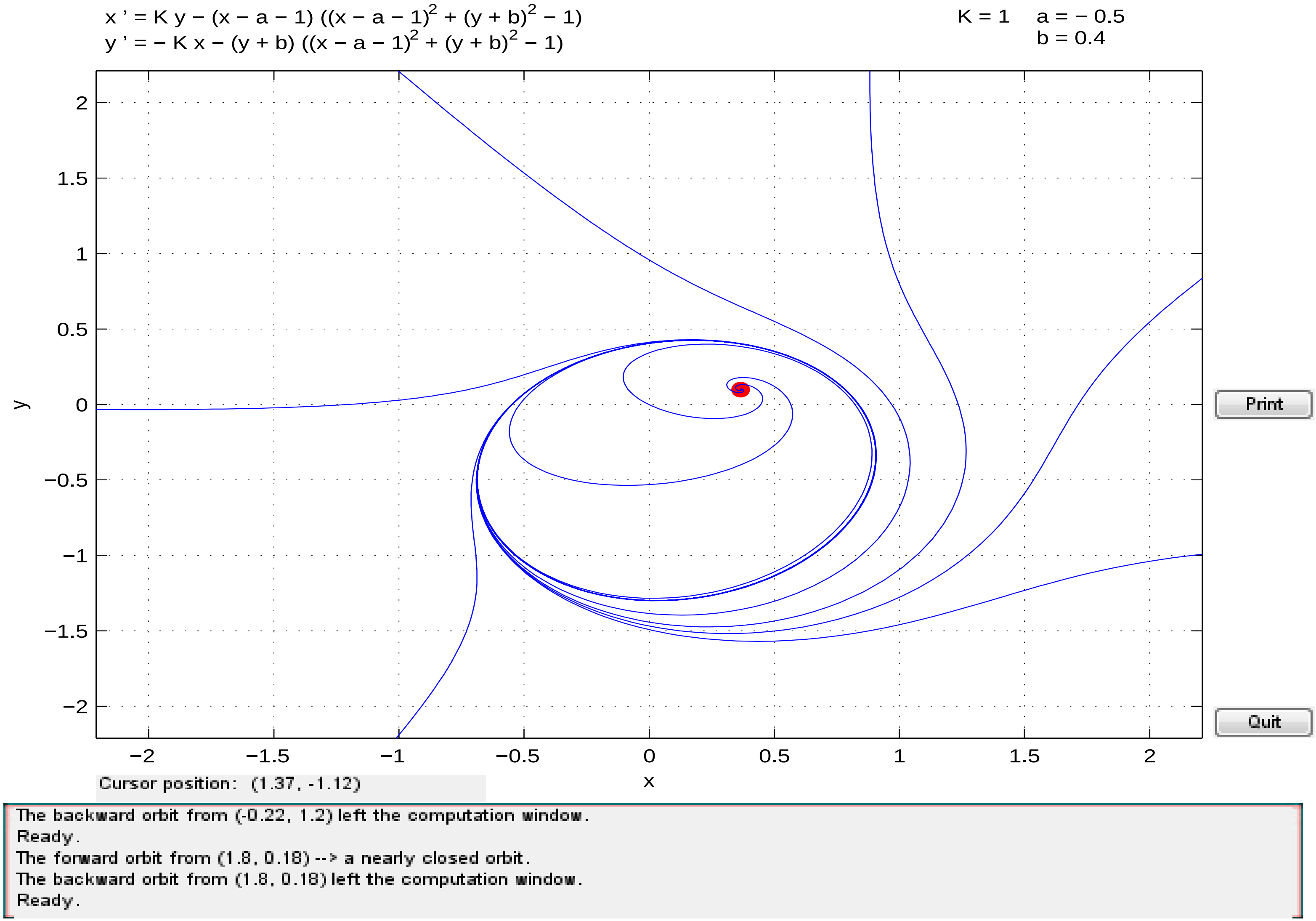
Case $(i_d^0 + 1)^2 + (i_q^0)^2 > 1$ high speed



Case $(i_d^0 + 1)^2 + (i_q^0)^2 > 1$ low speed



Case $(i_d^0+1)^2+(i_q^0)^2<1/2$: Sustained oscillations



Future research

- Final objective to propose a sensorless controller for salient PMSM:
  - robust to saliency and parameter uncertainty,
  - with flux saturation effects,
  - comparison of the proposed scheme (or combination) with high–frequency signal injection at low speeds.
- The clear understanding of the role of saliency on the position observer can guide us to the redesign of both, the velocity and load torque observers and the energy–shaping controller.
- Recent results of robustifying energy–shaping controllers developed with the phD student Jose Romero: achieved with simple (nonlinear) PI controllers.
- Same techniques applicable to observers?
- Saturation effects opens a completely new research avenue: use of the new Euler–Lagrange models.
- The final test of the performance should be carried out in a specialized experimental rig.